edge Business School

LECTURE NOTES

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MODULE:
MAC3701

LINEAR PROGRAMMING

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OPTIMISATION

INTRODUCTION

Optimisation is a technique which provides relevant information to management in order to make strategic decisions with regards to the optimum utilisation of scarce resources.

The primary objective, obviously, is to maximise profits. Profits are maximised by maximising contribution or marginal income. The question is, how one maximises contribution when faced with limited resources.

FACTORS OF PRODUCTION

Factors of production are the essential ingredients required to produce our commodities. These factors include capital from owners, raw materials, labour, water, land, etc.

(we will not deal with capital as a limiting factor)

UNERLYING ASSUMPTIONS

The assumptions are similar to those of Cost Volume Profit analysis, the most important of which are:

- Cost relationships are **linear**.
- Units produced and resources allocated are infinitely **divisible**.
- Within the **output range**, the contribution per unit for each product and the utilisation of resources per unit are the same irrespective of the quantity produced or sold

In order to put things into perspective we will begin with a simple illustration and thereafter build on

ILLUSTRATION 1
Assume a company manufactures 2 products, viz Elsi and Mate. Both products require the same raw material which is called Sharp.

The following information is relevant to the products:

	ELSI	MATE
Demand (units)	300	200
Contribution per unit (R)	3	6
Kg of raw material required per unit (kg)	1,5	1

Assume that raw material sharp is available in unlimited quantities and so is labour.

Which product(s) will you produce in order to maximise profits?

The question that needs to be asked is whether you have any limiting factors. The possible limiting factors are raw material and labour. We have been told that there are unlimited quantities available which means no limiting factors.

So I can produce all my required units.

Let's calculate our total contribution should we produce all of Elsi and all of Mate.

		R
Elsi	300 x 3	900
Mate	200 x 6	1 200
Total contribution		2 100

Now, what if we faced with shortage of raw material? Let's say we only had 600 kg of sharp available.

Do we have a constraint? Let's check.

Raw material required to produce 300 Elsi and 200 Mate:

		kg
Elsi	1,5 kg x 300	450
Mate	1kg x 200	200
Required		650
Available		600
Shortage		50

We are 50 kg short which means we have a limiting factor. We cannot produce everything.
With our objective in mind we will produce the one which renders the highest contribution per unit, which in this case is Mate.

So if we produce all of Mate how much of Elsi can we make? Let's check.

		Kg
To produce 200 Mate	200 x 1kg	200
Available		600
Available to produce Elsi		400

Once Mate has been produced only 400 kg are available to produce Elsi, which means we can produce (400 ÷ 1,5) = 266 units of Elsi

Let's see what happens to our contribution:

		R
Elsi	266 x 3	793
Mate	200 x 6	1 200
Total contribution		1 998

Contribution
Elsie R3 1.5kg R2/kg — Mate R6 1kg R6/kg
Mate 200 x 1.5kg Available
200
600
400/1.5kg
266 units

THE ABOVE TECHNIQUE IS SIMPLY CALLED LOGIC

Let's move on:

We now introduce another resource in the form of labour.

Assume now that Elsi needs 1,2 hours to manufacture and Mate 1,5 hours. Also assume for now that we only have 610 hours available.

Is labour a limiting factor? Let's check.

Labour hours required to produce 300 Elsi and 200 Mate:

		hours
Elsi	1,2 hrs x 300	360
Mate	1,5 hrs x 200	300
Required		660
Available		610
Shortage		50

Labour is a limiting factor. We are now faced with two limiting factors. What do we produce?
With one limiting factor all we needed to look at was the total contribution. With 2 limiting factors we need to look further. We will now compare the contribution per limiting factor (also referred to as marginal income per limiting factor).

Contribution per limiting factor

Material

		ELSI	MATE
Contribution per unit	A	R3,00	R6,00
Raw material required per unit (limiting factor)	B	1,5kg	1kg
Contribution per unit of raw material required	A÷B	R2/kg	R6/kg
Ranking		2	1

What this means is that for every kg of raw material used for Elsi I can earn R2 and for every kg of raw material used for Mate I can earn R6. Hence, this limiting factor favours the production of Mate and is therefore ranked 1.

Let's check labour.

		ELSI	MATE
Contribution per unit	A	R3,00	R6,00
Labour hours required per unit (limiting factor)	B	1,2hrs	1,5hrs
Contribution per labour hour required	A÷B	R2,5/hr	R4/hr
Ranking		2	1

Again I can earn more by employing more hours on mate. Labour also favours the production of Mate.

Material
Mate 200kg
Available 600
400/1,5kg
Available Elsie 266

Labour
300 hrs
610
310/1.2
258,33

So let's produce all of Mate. Let's see what happens.

Earlier we determined our position with regards to raw materials. We determined that we could produce 200 units of Mate and 266 units of Elsi.

Let's see what labour tells us.

		Hours
To produce 200 Mate	200 × 1,5 hrs	300
Available		610
Available to produce Elsi		310

Once Mate has been produced only 310 hours are available to produce Elsi, which means we can produce (310 ÷ 1.2) = 258 units of Elsi.

Labour constraints dictate that we produce 258 units of Elsi while raw material tells us 266. What do we make? What this means is that we have sufficient raw material to make 266 units but we don't have sufficient time to make 266 units. This means, we will have to settle for 200 of Mate and 258 of Elsi.

The above technique is called marginal costing and is used when the ranking of limiting factors favours the same product.

Let us take the above scenario and change the given information so that the ranking favours both products for the different limiting factors. Assume that the amount of labour hours required to manufacture one unit of Mate is 2.5 hours. Let's see what happens to the ranking.

Labour

		ELSI	MATE
Contribution per unit	A	R3,00	R6,00
Labour hours required per unit (limiting factor)	B	1,2hrs	2,5hrs
Contribution per labour hour required	A÷B	R2,5/hr	R2,4/hr
Ranking		1	2

And remember raw materials

		ELSI	MATE
+Contribution per unit	A	R3,00	R6,00
Raw material required per unit (limiting factor)	B	1,5kg	1kg
Contribution per unit of raw material required	A÷B	R2/kg	R6/kg
Ranking		**2**	**1**

Labour favours the manufacture of Elsi while Raw materials favour the manufacture of Mate. We call this **conflicting ranking**. In such a case we have to resort to **linear programming**.
The above problem can be solved algebraically by solving simultaneous equations.

The limiting factors form the basis of our equations.

Objective function: Maximise 3e + 6m

where : **Elsi = e** **Mate = m**

Parameters: $0 \le e \le 300$
$0 \le m \le 200$

Raw material:	1,5e	+ 1m	≤	600	1
Labour	1,2e	+ 2,5m	≤	610	2

Take equation 1 and solve for m:

1,5e	+ 1m	≤	600
1m		≤	600 – 1,5e

Substitute in equation 2:

1,2e	+ 2,5m	≤	610
1,2e	+ 2,5(600 – 1,5e)	≤	610
1,2e	+ 1 500 – 3,75e	≤	610
890		≤	2,55e
E		≤	349

Alternative calc:

1 x 2,5 : 3,75e + 2,5m ≤ 1 500
(1,2e + 2,5m ≤ 610)
2,55e ≤ 890
e ≤ 349
Limited to 300

Note your parameters. The demand for Elsi is limited to 300 units which means that we will produce 300 units of Elsi.

When one of the products is limited in production by its demand, then substitute back into BOTH equations to determine the production of the other product to ensure that both e and m fall within the limits specified in the objective function. The production of the other product will be the lower of the two possible solutions

= 1m = 600 – 1.5e —— (3)
(1) in (2): = 1.2e + 2.5(600 – 1.5e) = 610
1.2e + 1500 – 3.75e = 610
2.55e = 890
e = 349
< 300 as per parameter
Rm m = 600 – 1.5(300) = 150
Labour 1.2e + 2.5m = 610
1.2(300) + 2.5m = 610
360 + 2.5m = 610
m = 100

Substitute e = 300 into equation 1:

1,5e	+ 1m	≤	600
M		≤	600 – 1,5(300)
M		=	150

Substitute e = 300 into equation 2:

1,2e	+ 2,5m	≤	610
2,5m		≤	610 – 1,2(300)
M		=	100

Our optimum solution is 300 Elsi and 100 Mate

GENERAL STEPS IN SOLVING AN OPTIMISATION PROBLEM

Remember - NB!!!

- Limiting factor is just another word for a constraint! (Denoted below as "LF" in order to save space)
- Marginal income and contribution mean the same thing! (Denoted below as "MI" in order to save space)
- Please write the terms out in full in the exam – do not use "LF" or "MI".

AT THE BEGINNING OF THE QUESTION YOU WILL ALWAYS:

- **State the objective function** (which is really to maximise total marginal income).
- **State the parameters**

(we can usually only do the above after we have worked out marginal income per unit in step 2 so leave a space for this at the beginning of your question – a few lines needed)

EG:

The objective function is: **maximise 5p + 30q,**
Where p is the optimal production amount of product P.
Where q is the optimal production amount of product Q.

The limiting values of p and q have to be specified:
p ≥ 0 and p ≤ demand in units
q ≥ 0 and q ≤ demand in units

BASIC STEPS TO FOLLOW:

1. Identify potential constraints (these are factors of production that are available in limited quantities).

2. Calculate marginal income (contribution) per unit for each product (MI p.u. = SP p.u. – VC p.u)
This may be provided in the question but usually you will have to calculate it. Don't forget to include ALL variable costs when calculating your marginal income p.u. This includes variable selling and admin expenses and variable portion of semi-variable costs.

NB: Split semi variable costs

3. Identify which of those potential constraints in 1 are actual constraints. In order to do this you need to determine the following for each potential constraint:
    - the quantity **required** to meet demand
    - the total quantity of the constraint that is **available**
    - the shortage/surplus between **required** and **available**

4. If there is only one limiting factor use MI/ contribution per unit and produce all of the product with the highest MI/contribution per unit. (There will be no step 5 and 6)

If there is more than one limiting factor calculate the marginal income (contribution) per limiting factor relating to each actual limiting factor for each product.

= Marginal income (MI) per unit of product ÷ limiting factor (LF) per unit of product
= MI per LF

This will give you some amount of Rands per LF.
EG: R/kg (if LF is amount of material in kg);
R/Machine Hour (if LF is no. of machine hours);
R/R (if LF is an amount that can be spent)

5. Rank the products according to their marginal income per limiting factor (LF), for each actual LF. This is an indication of the product's profitability per LF; therefore the highest is ranked first.

NB! IT IS AT THIS POINT THAT YOU NEED TO DECIDE *WHICH TECHNIQUE* YOU ARE USING (based primarily on your rankings): you can either use ***marginal costing*** (if rankings favour the same product) or ***linear programming*** (if rankings favour different products). Note that steps 1-5 are identical for both techniques.

USING MARGINAL COSTING TECHNIQUES TO SOLVE OPTIMISATION QUESTIONS

When do we use marginal costing techniques?
We are able to use marginal costing techniques when there is only ONE actual constraint, or, if there is more than one actual constraint, we use marginal costing when the ranking of the different constraints favours the **SAME** product.

6. Work out the optimal product mix (i.e. how much of each product you will produce) using the ranking as determined above.

Keep in mind that you cannot produce more of a product than the amount that is demanded of that product. Also remember that production will stop as soon as one of the actual constraints has been exhausted despite a surplus of the other constraints/LFs.

If you were required to calculate net profit from making the optimal product mix, then you would simply subtract fixed costs from the total marginal income (total contribution) earned. In this case the total marginal income would be the marginal income per unit multiplied by the number of units in the optimal product mix for each product.

OR

USING LINEAR PROGRAMMING TECHNIQUES TO SOLVE OPTIMISATION QUESTIONS

When do we use linear programming techniques?

We use linear programming techniques if there is more than one actual constraint AND the ranking of the different constraints favours **DIFFERENT** products.

6. Express **each** actual constraint in terms of a linear equation.

   [LF per unit of P x p] + [LF per unit of Q x q] ≤ total ***available*** LF

7. Solve the equations for the different constraints simultaneously in order to establish the optimal product mix (i.e. we are solving for p and q). When one of the products is limited in production by its demand, then substitute back into BOTH equations to determine the production of the other product to ensure that both p and q fall within the limits specified in the objective function. The production of the other product will be the lower of the two possible solutions.

Keep in mind that you cannot produce more of a product than the amount that is demanded of that product. Also remember that production will stop as soon as one of the actual constraints has been exhausted despite a surplus of the other constraints.

If you were required to calculate net profit from making the optimal product mix, then you would simply subtract fixed costs from the total marginal income (total contribution) earned. In this case the total marginal income would be the marginal income per unit multiplied by the number of units in the optimal product mix for each product.