Tutorial Letter 101/3/2014

Models for Strategic Decision Making

DSC3704

Semesters 1 and 2

Department of Decision Sciences

Important Information:

This tutorial letter contains important information about your module.

Bar code

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1 INTRODUCTION AND WELCOME

Dear student

It is a pleasure to welcome you to the Department of Decision Sciences and to the module *Models for Strategic Decision Making*. We hope that you will find this module interesting and that you will complete it successfully.

1.1 Tutorial matter

The Despatch Department should supply you with the following tutorial matter for this module:

- one study guide
- Tutorial Letter 101 (this tutorial letter)
- Tutorial Letter 301
- Tutorial Letter 201 containing the solutions to the first compulsory assignment (will be sent after the due date)
- Tutorial Letter 202 containing the solutions to the second compulsory assignment (will only be made available on *my*Unisa)
- any other tutorial letters that may be necessary.

Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible. The study guide and tutorial letters are also available electronically in pdf format on ***my*Unisa** at `https://my.unisa.ac.za`.

It is essential that you read this Tutorial Letter 101, as well as Tutorial Letter 301, very carefully.

- Tutorial Letter 101 contains important information on this module.
- Tutorial Letter 301 contains general information relevant to all undergraduate students in the Department of Decision Sciences.

1.2 Computer requirements

Access to a computer is a prerequisite for this module.

You can use the software you prefer for the occasional instances where the volume of the computations makes it feasible to use a computer. A spreadsheet package (e.g. Excel, Open Office) is adequate but a mathematical package (e.g. Octave, MatLab, Maxima) will be better.

2 PURPOSE AND OUTCOMES OF THIS MODULE

This section contains the purpose of as well as the learning outcomes and the assessment criteria for the module. The assessment criteria specify how you will be assessed to determine whether you have attained the learning outcomes.

Purpose

The purpose of this module is to introduce you to the theory and practice of multi-criteria decision-making as applied to the problem of selecting one from a number of contenders where the most important information

is human judgment.

Learning Outcome 1: Learners can use the applicable techniques to calculate weights for a ratio scale including the case where all possible observations are not available. They know the underlying error models and can derive or justify the methods.

Assessment Criteria

- Recognise an instance where the calculation of weights based on human judgment is relevant and useful.
- Calculate the implied weights of a number of items from a complete as well as an uncompleted matrix of pairwise ratio comparisons using the relevant methods.
- Discuss the error models underlying the methods.
- Derive the LLS method for all cases and derive or justify the other methods.
- Discuss the elicitation of pairwise comparisons.
- Reproduce and manipulate the relevant theory (definitions; theorems and results; proofs, derivations and justifications).

Learning Outcome 2: Learners can use the applicable techniques to calculate a single figure (scalar) value to express the value of an item with many properties evaluated under a number of criteria for the purpose of ranking the items. They know the underlying assumptions of the methods and can compare and evaluate them. They understand the problems associated with scales and the calculation of weights for a large number of items and know how to handle such problems numerically.

Assessment Criteria

- Recognise an instance where the calculation of a scalar value (for items under a number of criteria) based on human judgment is relevant and useful.
- Use the different methods to calculate such a scalar value.
- Give the arguments for and against the different methods.
- Discuss the problems and limitations associated with the use of trees in calculating weights, and do the relevant calculations.
- Discuss the problems and limitations associated with scales and do the calculations and transformations associated with scales.
- Reproduce and manipulate the appropriate theory (definitions; theorems and results; proofs, derivations and justifications).

Learning Outcome 3: Learners can scan research papers to locate required information, and grasp the arguments, proposals and mathematics of the authors. They can partake in the practice of scientific argument and debate.

Assessment Criteria

- Reproduce, compare and evaluate modelling concepts and proposals in this field of study.
- Apply the methods and techniques proposed by the authors.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

You will find the name of the lecturer responsible for this module in Tutorial Letter 301. We suggest that you write the name and contact details of the lecturer in the space below.

Lecturer:

The lecturer will assist you with any problems you may experience with the study material. Feel free to contact the lecturer, but please phone beforehand to make an appointment if you wish to see the lecturer in person.

3.2 Department of Decision Sciences

Tel: 012 433 4684
Fax: 012 429 4898
E-mail: qm@unisa.ac.za

3.3 College of Economic and Management Sciences

Ms Portia Ngcobo, College Information Coordinator

Tel: 012 429 3925
E-mail: CEMSenquiries@unisa.ac.za

3.4 University

You may send an e-mail to info@unisa.ac.za or consult the brochure, *my Studies @ Unisa* for the contact details. Always quote your student number whenever you communicate with Unisa.

4 MODULE RELATED RESOURCES

4.1 Prescribed book

The following book has been prescribed for this module and you must buy it:

WINSTON, Wayne L
OPERATIONS RESEARCH: Applications and Algorithms
Thomson, Fourth edition, 2004.

We will refer to this book as *Winston* in the remainder of this tutorial letter.

Prescribed books can be obtained from the University's official booksellers. If you have difficulty in locating your book(s) at these booksellers, please contact the Prescribed Book Section at 012 429 4152 or e-mail `vospresc@unisa.ac.za`.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

5 STUDENT SUPPORT SERVICES FOR THE MODULE

You can consult the brochure, *my Studies @ Unisa* for detail on student support services rendered.

The lecturer can assist you with any enquiries related to the study material, the assignment questions and the solutions to them.

If you wish to find out whether your assignment has been received, registered, processed or returned, please check on *my*Unisa at `https://my.unisa.ac.za`.

For detailed information on and requirements for assignments, see the brochure *my Studies @ Unisa*.

6 MODULE SPECIFIC STUDY PLAN

Use your *my Studies @ Unisa* brochure for general time management and planning skills.

This module is presented in both semesters, which means that you write the exam in May/June if you are registered for the first semester and October/November if you are registered for the second semester. Your study time ends a month earlier if you want to have some time left to prepare for the examination. As study time is limited, you should plan your study program very carefully. Working consistently throughout the semester is the clever thing to do – as you no doubt know by now. This helps in avoiding unproductive panic and stress immediately before the examination.

Schedule your own due dates for the completion of a section. Take the due dates of the compulsory assignments into consideration, and leave about a month for revision before the start of the examination.

Work through the study guide from beginning to end and do the first two chapters as soon as possible.
Then do Assignment 01.
Thereafter you should study Chapters 3, 4, and 5.
Then do Assignment 02.

You must also read through the articles in the reader at the back of the study guide as they are referred to.

Winston is prescribed for most of the department's undergraduate modules. You can use it to revise matrix operations in Chapter 2 and for a second look at the AHP in Section 13.7. Study this once you have mastered Chapter 3 of the study guide. In this section you will also find more information on the computation of the preference vector and the consistency of the matrix of pairwise comparisons.

Each chapter in the study guide contains **activities**. Work through them all and compare your answers to the model solutions. The activities play an important role in mastering the tutorial matter.

All this work must be done in one semester, the duration of which is about four months. This is a very limited period of time and you will have to plan your studies carefully. Set aside sufficient time for studying. Do not delay, **start working immediately**!

7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Compulsory assignments

This tutorial letter contains the **two** compulsory assignments that students must do in the semester for which they are registered.

If you are registered for the **first** semester, do

- Assignment 01 provided in Section 12.1.
- Assignment 02 provided in Section 12.2.

If you are registered for the **second** semester, do

- Assignment 01 provided in Section 13.1.
- Assignment 02 provided in Section 13.2.

8.2 Semester mark

Both assignments in a semester **count** towards your semester mark.
Assignment 01 will contribute 50% and Assignment 02 will contribute 50% towards your semester mark.

The semester mark will contribute 20% and your examination mark 80% to your final mark. A sub-minimum of 40% applies to your examination. It means that your semester mark will be ignored if your examination mark is less than 40%, and your examination mark will then be your final mark. You need a final mark of at least 50% to pass.

8.3 Format and solutions

8.3.1 Assignments 01

Assignment 01 consists of multiple choice questions that must be completed on a mark reading sheet.

All registered students will receive solutions to this assignment after its due date.

8.3.2 Assignment 02

This is a written assignment. Answers to the questions of Assignment 02 must be written or typed.

If you submit this assignment before its due date, it will be marked by the lecturer and cryptic solutions to the assignment will be sent to you when your marked assignment is returned to you. Students who do not submit this assignment, or submit it late, will not receive the solutions in the mail. Late assignments will **not** be marked. Cryptic solutions will however be made available on ***my*Unisa**.

8.4 Due dates for assignments

The due date of an assignment is the last date on which the assignment must reach Unisa.

All assignments must be submitted on time.
Late assignments will **not** be marked.

The unique number that you have to fill in on the mark reading sheet and the due dates for the assignments are:

Semester	Assignment	Unique number	Due date
First semester 2014	Assignment 01	774274	4 March 2014
	Assignment 02	835432	1 April 2014
Second semester 2014	Assignment 01	880183	26 August 2014
	Assignment 02	750506	23 September 2014

All the assignments must be submitted to Unisa early in the semester to facilitate their marking. Together they cover all the study material. So we encourage you to start working immediately and on a regular basis.

8.5 Submission of assignments

Students may submit assignments either by post or mobile MCQ submission or electronically via ***myUnisa***.

To submit an assignment via ***myUnisa***:

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on Assignments in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

8.6 Assignment enquiries

The lecturer can assist you with any enquiries related to the contents of the study material.

If you wish to find out whether your assignment has been received, registered, processed or returned, please check on ***my*Unisa** at `https://my.unisa.ac.za/`.

For detailed information on and requirements for assignments, see the brochure *my Studies @ Unisa*.

9 EXAMINATION

For general requirements and guidelines on preparation for the examination, see the brochure *my Studies @ Unisa*.

9.1 Examination admission

Examination admission is obtained by submitting the **first** compulsory assignment *on time* in the semester for which you are registered, and not by the marks you obtain for it.

If the first compulsory assignment reaches the University after its due date, you will not be admitted to the examination.

9.2 Format of examination

The examination will be written during May/June (first semester) and October/November (second semester).

- The duration of the examination is *two hours* and the total of the marks is 80.
- You are allowed to use a programmable pocket calculator in the examination.
- You may take only your writing materials and a programmable pocket calculator into the examination hall.
- Your examination paper will be collected after the session and you will not be allowed to take it with you.
- Examination questions will comply with the assessment criteria set out in this tutorial letter.

To pass this module, you must obtain a final mark of at least 50%.

10 ALTERNATIVE ASSESSMENT METHODS

No other alternative assessment methods exist for this module.

11 FREQUENTLY ASKED QUESTIONS

The *my Studies @ Unisa* brochure contains an A-Z guide of the most relevant study information.

1. MATTERS PERTAINING TO REGISTRATION
   - What should I do if any of my personal details (name, address, etc) have been incorrectly captured during the registration process?

     Please use ***my*Unisa** (`https://my.unisa.ac.za`) to rectify any mistakes. Alternatively send an e-mail to `econ@unisa.ac.za`. Alternatively mail us a letter addressed to The Registrar, PO Box 392, Unisa, 0003. Please remember to quote your student number in all correspondence.
   - What do I need to do to obtain items that were out of stock when I registered?

     Please use the internet facility at `https://my.unisa.ac.za` in order to download copies of any of the study material. Alternatively send us an e-mail to `despatch@unisa.ac.za`. Please remember

to quote your student number in all correspondence. Please allow about four weeks for delivery of any outstanding items before contacting Unisa about out-of-stock items.

2. ASSIGNMENTS

- My assignment is late because ... . Can I submit it at a later date?

  It is your responsibility to ensure that your assignment is submitted at *my*Unisa or reaches the Unisa Main Campus on or before the closing date. Please do not call us to request an extension for the submission of an assignment. Once you have received the tutorial letter containing the solutions you will know that you definitely can no longer submit the assignment.

- Are the assignments of this module compulsory?

  The assignments are compulsory. The intention is that you should use the assignments to prepare yourself for the examination.

- Do the marks obtained for assignments count towards the final mark?

  The assignment marks do count towards the final marks, as explained in this tutorial letter.

- Who do I contact to find out if Unisa received my assignment(s) that I sent by mail?

  Preferably submit your assignments via ***my*Unisa**. If you do not have access to the Internet, mail your assignments. Visit `https://my.unisa.ac.za` in order to determine if Unisa received your assignment(s).

- What do I do if my assignment goes missing in the mail?

  Preferably submit your assignments via ***my*Unisa**. Always make a copy of your assignment and keep it in a safe place in case you need to submit a duplicate copy.

- Can I submit my assignments by e-mail?

  Assignments should be submitted via the ***my*Unisa** facility. Please do not e-mail assignments.

- How long will I have to wait before I can expect my assignment(s) results?

  We aim to have assignments marked within three (3) weeks after each due date. Visit `https://my.unisa.ac.za` to determine what mark you achieved. Allow an additional week or two for the post office to deliver your marked assignment to your postal address.

- Can I expect my assignment to be returned sooner if I submit it well before the due date?

  The marking of assignments normally starts after their respective due dates.

3. EXAMINATION

- What happens if I am unable to write the examination?

  Any requests for aegrotat examinations need to be done in writing and directed to the Head: Examinations, P O Box 392, Unisa, 0003 or `exams@unisa.ac.za`. A fee is payable for such requests. The amount payable is determined by the Examination Section.

- What happens if I fail any papers?

  Students who fail any of the papers may be granted admission to write the supplementary examination scheduled for the next examination, provided they achieved a final mark of at least 40%. Any enquiries in this regard need to be directed to The Head: Examinations, P O Box 392, Unisa, 0003 or `exams@unisa.ac.za`.

- What will happen if I do not write any of the papers during the supplementary examination?

In such a case you will have to obtain a registration form, complete it, pay the required fees and register again. Registration forms are available from `www.unisa.ac.za`.

- What do I need to bring to the examination venue?

  Bring your identity document (ID), student card, black pen, ruler, eraser, pencil, sharpener and calculator.

- Will it be an open-book examination?

  No.

- Could you give me an idea of the scope of the examination paper?

  The examination paper is based on the learning outcomes provided in this tutorial letter. Each learning outcome statement and assessment criterion may be used to formulate an examination question.

- Are any old examination papers available?

  Previous examination papers are published at ***my*Unisa**, but no memoranda are made available to students. Old questions will not necessarily be repeated in subsequent examination papers and students are warned not to rely on old examination papers in order to pass the module.

4. CALCULATORS

   - Am I allowed to use a calculator?

     Yes. For this module, a programmable pocket calculator is allowed.

5. RESULTS

   - How will the results be announced?

     The results are normally mailed to students. They are also published on ***my*Unisa** at `https://my.unisa.ac.za` and the MTN results facility, cellular number 083 1234.

6. STUDY GUIDE AND TUTORIAL LETTERS

   - I have lost my study guide and tutorial letters. Could you please mail or fax me a copy?

     Please download a copy from the ***my*Unisa**. Register as a user at `https://my.unisa.ac.za`. You may also order replacement copies of study guides from Unisa's Despatch Department. Please send an e-mail to `despatch@unisa.ac.za` or a letter to The Head, Despatch, PO Box 392, Unisa, 0003.

7. CONTACT WITH FELLOW STUDENTS

   - I wish to contact other students enrolled for the module in my area. Please provide me with their contact details.

     Contact may be established with fellow students at the discussion forum at ***my*Unisa**. Due to the constitutional right to privacy we may not disclose the details of students to their fellow students.

12 FIRST SEMESTER COMPULSORY ASSIGNMENTS

12.1 Assignment 01: Semester 1

Unique number: 774274 **Due date: 4 March 2014**

This assignment covers concepts explained in Chapters 1 and 2 of the study guide.

Question 1

The name of this module is

[1] Financial mathematics

[2] Nonlinear mathematical programming

[3] Simulation

[4] Models for strategic decision making

[5] Rational decision making

Question 2

Another meaningful name for this module could be the following:

[1] Financial mathematics

[2] Nonlinear mathematical programming

[3] Production modelling

[4] Simulation

[5] Mathematical modelling of human judgement

Question 3

The purpose of this module is to introduce you to the theory and practice of multi-criteria decision-making as applied to the problem of

[1] generating random numbers.

[2] allocating scarce resources.

[3] forecasting trends.

[4] selecting one from a number of contenders.

[5] minimising costs.

Question 4

A learning outcome of this module is that learners can use the applicable techniques to calculate

[1] roots of equations.

[2] weights for a ratio scale.

[3] regression coefficients.

[4] economic order quantities.

[5] minimum costs.

Question 5

Normalising the vector $\left(\frac{3}{4}; \frac{1}{2}; \frac{2}{3}; \frac{1}{6}\right)^t$ gives

[1] 0,48

[2] $(0{,}36; 0{,}24; 0{,}32; 0{,}08)^t$

[3] $\begin{bmatrix} 0{,}75 \\ 0{,}50 \\ 0{,}67 \\ 0{,}17 \end{bmatrix}$

[4] $\left(\frac{25}{16}; \frac{25}{24}; \frac{25}{18}; \frac{25}{72}\right)^t$

[5] none of the above.

In **Questions 6 and 7** consider the set of weights $\{0{,}4;\ 0{,}3;\ 0{,}2;\ 0{,}1\}$, and the set of values $\{11;\ 25;\ 38;\ 62\}$.

Question 6

The weighted arithmetical mean of the values is

[1] 8,82.

[2] 21,44.

[3] 25,70.

[4] 34,00.

[5] none of the above.

Question 7

The weighted geometrical mean of the values is

[1] 8,82.

[2] 21,44.

[3] 25,70.

[4] 34,00.

[5] none of the above.

In **Questions 8 to 13** suppose the matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & * & 0{,}50 & * \\ * & 1{,}00 & 0{,}75 & * \\ * & * & * & * \\ * & * & 0{,}75 & 1{,}00 \end{bmatrix}$$

is perfectly consistent (errorless).

Question 8

The element a_{33} is

[1] 0,25.

[2] 0,50.

[3] 0,75.

[4] 1,00.

[5] 1,50.

Question 9

The element a_{31} is

[1] 0,25.

[2] 0,50.

[3] 0,75.

[4] 2,00.

[5] none of the above.

Question 10

The element a_{34} is

[1] 0,75.

[2] 1,00.

[3] 1,25.

[4] $1{,}3\dot{3}$.

[5] none of the above.

Question 11

The normalised preference vector is

[1] $(\frac{1}{3};\ \frac{1}{2};\ \frac{2}{3};\ \frac{1}{2})^t$

[2] $(\frac{1}{6};\ \frac{1}{4};\ \frac{1}{3};\ \frac{1}{4})^t$

[3] $(2;\ 3;\ 4;\ 3)^t$

[4] $(0{,}1;\ 0{,}2;\ 0{,}3;\ 0{,}2)^t$

[5] none of the above.

Question 12

The element a_{41} is

[1] 0,75.

[2] $\frac{1}{6}$.

[3] $\frac{2}{3}$.

[4] 1,50.

[5] none of the above.

Question 13

The element a_{14} is

[1] 0,75.

[2] $\frac{1}{6}$.

[3] $\frac{2}{3}$.

[4] 1,50.

[5] none of the above.

In **Questions 14 to 16** apply the required method to find the normalised preference vector implied by the matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & 1{,}75 & 3{,}00 & 0{,}91 \\ 0{,}57 & 1{,}00 & 2{,}00 & 0{,}80 \\ 0{,}33 & 0{,}50 & 1{,}00 & 0{,}25 \\ 1{,}10 & 1{,}25 & 4{,}00 & 1{,}00 \end{bmatrix}.$$

Round off final answers to two decimals and ignore small errors that result from rounding off.

Question 14

After performing one iteration of the iterative method and using $\mathbf{v_0} = (0{,}3;\ 0{,}2;\ 0{,}1;\ 0{,}4)^t$ as the initial solution, the column vector $\mathbf{v_1}$ is

[1] $(0{,}30;\ 0{,}20;\ 0{,}10;\ 0{,}40)^t$

[2] $(0{,}31;\ 0{,}21;\ 0{,}10;\ 0{,}38)^t$

[3] $(0{,}33;\ 0{,}22;\ 0{,}10;\ 0{,}35)^t$

[4] $(0{,}33;\ 0{,}19;\ 0{,}11;\ 0{,}37)^t$

[5] $(1{,}31;\ 0{,}89;\ 0{,}40;\ 1{,}38)^t$

Question 15

The preference vector found by calculating the average of the normalised columns is

[1] $(0{,}30;\ 0{,}20;\ 0{,}10;\ 0{,}40)^t$

[2] $(0{,}31;\ 0{,}21;\ 0{,}10;\ 0{,}38)^t$

[3] $(0{,}33;\ 0{,}22;\ 0{,}10;\ 0{,}35)^t$

[4] $(0{,}33;\ 0{,}19;\ 0{,}11;\ 0{,}37)^t$

[5] $(1{,}31;\ 0{,}89;\ 0{,}40;\ 1{,}38)^t$

Question 16

The preference vector found by using the geometric means method is

[1] $(0{,}30;\ 0{,}20;\ 0{,}10;\ 0{,}40)^t$

[2] $(0{,}31;\ 0{,}21;\ 0{,}10;\ 0{,}38)^t$

[3] $(0{,}33;\ 0{,}22;\ 0{,}10;\ 0{,}35)^t$

[4] $(0{,}33;\ 0{,}19;\ 0{,}11;\ 0{,}37)^t$

[5] $(1{,}31;\ 0{,}89;\ 0{,}40;\ 1{,}38)^t$

In **Questions 17 to 19** the log least squares method is applied on the incomplete matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & * & 3{,}00 & 0{,}80 \\ * & 1{,}00 & 2{,}00 & * \\ * & * & 1{,}00 & * \\ * & * & * & 1{,}00 \end{bmatrix}$$

to determine the implied preference and weights vectors.

Question 17

The left hand side of the first equation in the system of equations $D\mathbf{b} = \mathbf{d}$ is

[1] $2b_1 - b_3 - b_4$

[2] $b_2 - b_3$

[3] $-b_1 - b_2 + 2b_3$

[4] $-b_1 + b_4$

[5] none of the above.

Question 18

The right hand side of the second equation in the system of equations $D\mathbf{b} = \mathbf{d}$ is

[1] $\ln 3 + \ln 4 - \ln 5$

[2] $\ln 2$

[3] $-\ln 2 - \ln 3$

[4] $-\ln 4 + \ln 5$

[5] none of the above.

Question 19

The weights vector is

[1] $(0{,}30;\ 0{,}20;\ 0{,}10;\ 0{,}40)^t$

[2] $(\ln 1;\ \ln \frac{2}{3};\ \ln \frac{1}{3};\ \ln \frac{5}{4})^t$

[3] $(1;\ \frac{2}{3};\ \frac{1}{3};\ \frac{5}{4})^t$

[4] $(0{,}31;\ 0{,}21;\ 0{,}10;\ 0{,}38)^t$

[5] none of the above.

Question 20

A matrix A of pairwise comparisons will be considered too inconsistent to use if

[1] the inconsistency index $OI > 0{,}1$.

[2] the random index $EI < 0{,}1$.

[3] $\frac{OI}{EI} > 0{,}1$.

[4] $\lambda_{\max} = 0$.

[5] none of the above.

12.2 Assignment 02: Semester 1

Unique number: 835432 **Due date: 1 April 2014**

This is a written assignment.

It covers all the methods explained in Chapters 2 and 3 of the study guide and Chapter 13.7 of the prescribed book by Winston, as well as concepts touched on in the rest of the study guide.

Students must answer all the questions and submit a single document in written or typed format.

The lecturer will not mark all the questions, but will select a question or sub questions to mark. The mark for this selection will be the mark for the assignment.

Feedback will be provided on *my*Unisa.

Question 1

Consider an $n \times n$ uncompleted errorless matrix of pairwise comparisons $C = [c_{ij}]$ and the weights vector $\mathbf{w}' = (w_1; \ldots; w_n)$.

1.1 Write down the definition of the element c_{ij} in terms of the weights w_i and w_j.
Use this definition to show that

(a) $c_{ii} = 1$.

(b) $c_{ij} = c_{ji}^{-1}$.

(c) any element c_{ij} can be expressed as the product of two elements c_{ik} and c_{kj}.

(d) $C_{\cdot j}$, the j-th column of an errorless C matrix, is directly proportional to the preference vector.

(e) $C\mathbf{w} = n\mathbf{w}$.

1.2 Use the expressions from Question 1.1 to complete the following errorless matrix

$$C = [c_{ij}] = \begin{bmatrix} 1 & 0{,}8 & * & * \\ * & * & * & * \\ * & * & * & * \\ * & 0{,}6 & 0{,}75 & 1 \end{bmatrix}$$

and to calculate the (normalised) preference vector.

1.3 Construct the log least squares equations for the observations in the uncompleted matrix C given in Question 1.2 and determine the implied preference vector.

1.4 Confirm that the geometric means method yields the same preference vector for the completed matrix C obtained in Question 1.2.

Question 2

Suppose you are considering buying a new cell phone.

Make a list of at least twelve characteristics that you consider important.

Order these characteristics in a tree structure.

Calculate a weight for each leaf in the tree structure.

Now compare two cell phones according to these criteria and decide which one you want to buy. Define and describe any value functions that you need in your calculations.

Eliminate or add or regroup some characteristics. Will your choice change if your tree structure changed?

Question 3

Winston Chapter 13.7 Problem 2 p.793.

Question 4

Joan needs to buy a new car as her old car was written off by the insurance company after an accident. She decides to compare four vehicle models and designs a SMART scoring function for each of four criteria (vehicle specifications).

The specifications of four vehicle models are as follows:

Specification	**Car1**	**Car2**	**Car3**	**Car4**
Price (R1 000)	130	165	187	220
Power (kW)	65	73	73	88
Capacity (cm^3)	1 198	1 339	1 339	1 496
Airbag quantity	2	4	4	6

The SMART scoring function for the retail price of the vehicle model is as follows:

$$f_{\text{Price}}(x) = \begin{cases} 100 & \text{if} \quad 0 \leq x \leq 100 \\ 100 - 0{,}1x & \text{if} \quad 100 < x \leq 150 \\ 75 & \text{if} \quad 150 < x \leq 200 \\ 110 - 0{,}3x & \text{if} \quad 200 < x \leq 300 \\ 0 & \text{if} \quad x > 300 \end{cases}$$

where x is the retail price of the vehicle model in thousands of rand.

The SMART scoring function for the engine power of the vehicle model is as follows:

$$f_{\text{Power}}(x) = \begin{cases} x & \text{if} \quad 0 \leq x \leq 100 \\ 100 & \text{if} \quad x > 100 \end{cases}$$

where x is the engine power of the vehicle model in kW.

The SMART scores for the engine capacity of the vehicle model are as follows:

Engine CC in cm^3	**Score**
0^+ to 1 200	20
$1\,200^+$ to 1 400	40
$1\,400^+$ to 1 600	60
$1\,600^+$ to 1 800	80
$1\,800^+$	100

Safety and security are important features. Her SMART scores for the number of airbags are as follows:

Airbag quantity	**Score**
2	20
4	80
6	100

4.1 Sketch the SMART scoring function for price and briefly describe it.
How would you improve this function?

4.2 Determine the SMART score for each of the four cars for each of the four specifications. Which car is her best choice if

(a) price is her only consideration?

(b) safety is her only consideration?

(c) all four specifications are equally important?

(d) price is twice as important as each of the other three specifications?

Justify your answers by deriving the relevant weights and calculating the weighted SMART score for each of the four cars in each case.

4.3 The SMART scores of Car2 and Car3 are exactly the same. What would critics of the Analytic Hierarchy Process (AHP) warn against?

4.4 The difference in price between Car2 and Car3 can be explained by the inclusion or not of visually pleasing features such as alloy wheels and the existence or not of a maintenance plan. How would you incorporate these qualitative criteria in the SMART model?

4.5 Set up the matrix of pairwise comparisons of the four cars according to the number of airbags in their specifications. Complete the matrix. Calculate the corresponding weights vector.

4.6 Suppose Joan wants to determine weights for three of the criteria, namely safety, price and power. She feels that safety is twice as important as price, and that price is three times as important as power. Capacity does not matter to her at all.
Assume that she is perfectly consistent (errorless) in her judgements of her pairwise comparisons of the criteria. Complete her pairwise comparison judgement matrix of these three criteria and use it to determine her preference and weights vector.

13 SECOND SEMESTER COMPULSORY ASSIGNMENTS

13.1 Assignment 01: Semester 2

Unique number: 880183 **Due date: 26 August 2014**

This assignment covers concepts explained in Chapters 1 and 2 of the study guide.

Question 1

The name of this module is

[1] Financial mathematics

[2] Nonlinear mathematical programming

[3] Simulation

[4] Models for strategic decision making

[5] Rational decision making

Question 2

Another meaningful name for this module could be the following:

[1] Rational decision making

[2] Nonlinear mathematical programming

[3] Production modelling

[4] Multi-criteria decision analysis

[5] Mathematical optimisation

Question 3

The purpose of this module is to introduce you to the theory and practice of multi-criteria decision-making as applied to the problem of selecting one from a number of contenders where the most important information is

[1] summary statistics.

[2] human judgment.

[3] random numbers.

[4] share prices.

[5] interest rates.

Question 4

A learning outcome of this module is that learners can use the applicable techniques to calculate a single figure (scalar) value to express the value of an item with many properties evaluated under a number of criteria for the purpose of

[1] ranking the items.

[2] optimising the scale.

[3] setting up error models.

[4] simulating human behaviour.

[5] comparing investments.

Question 5

Normalising the vector $\left(\frac{1}{4}; \frac{5}{6}; \frac{2}{3}; \frac{3}{8}\right)^t$ gives

[1] $\frac{8}{17}$

[2] 2,125

[3] $\begin{bmatrix} 0{,}25 \\ 0{,}83 \\ 0{,}67 \\ 0{,}38 \end{bmatrix}$

[4] $\left(\frac{2}{17}; \frac{20}{51}; \frac{16}{51}; \frac{3}{17}\right)^t$

[5] none of the above.

In **Questions 6 and 7** consider the set of weights $\{0{,}2;\ 0{,}3;\ 0{,}4;\ 0{,}1\}$, and the set of values $\{22;\ 55;\ 41;\ 32\}$.

Question 6

The weighted arithmetical mean of the values is

[1] 27,0.

[2] 35,5.

[3] 40,5.

[4] 47,5.

[5] none of the above.

Question 7

The weighted geometrical mean of the values is

[1] 36,6.

[2] 38,6.

[3] 40,5.

[4] 58,5.

[5] none of the above.

In **Questions 8 to 13** suppose the matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & * & 0{,}25 & * \\ * & 1{,}00 & 0{,}75 & * \\ * & * & * & * \\ * & * & 0{,}50 & 1{,}00 \end{bmatrix}$$

is perfectly consistent (errorless).

Question 8

The element a_{33} is

[1] 0,25.

[2] 0,50.

[3] 0,75.

[4] 1,00.

[5] 1,50.

Question 9

The element a_{34} is

[1] 0,25.

[2] 0,50.

[3] 0,75.

[4] 2,00.

[5] none of the above.

Question 10

The element a_{32} is

[1] 0,25.

[2] 0,30.

[3] 0,75.

[4] 2,00.

[5] none of the above.

Question 11

The normalised preference vector is

[1] $(\frac{1}{4};\ \frac{3}{4};\ 1;\ \frac{1}{2})^t$

[2] $(1;\ 3;\ 4;\ 2)^t$

[3] $(0{,}2;\ 0{,}6;\ 0{,}8;\ 0{,}4)^t$

[4] $(0{,}1;\ 0{,}3;\ 0{,}4;\ 0{,}2)^t$

[5] none of the above.

Question 12

The element a_{21} is

[1] $0{,}3\dot{3}$.

[2] 1,50.

[3] 2,00.

[4] 3,00.

[5] none of the above.

Question 13

The element a_{41} is

[1] $0{,}3\dot{3}$.

[2] 1,50.

[3] 2,00.

[4] 3,00.

[5] none of the above.

In **Questions 14 to 16** apply the required method to find the normalised preference vector implied by the matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & 2{,}00 & 6{,}00 & 12{,}00 \\ 0{,}50 & 1{,}00 & 3{,}60 & 5{,}00 \\ 0{,}17 & 0{,}28 & 1{,}00 & 2{,}00 \\ 0{,}08 & 0{,}20 & 0{,}50 & 1{,}00 \end{bmatrix}$$

Round off final answers to two decimals and ignore small errors that result from rounding off.

Question 14

After performing one iteration of the iterative method, and using $\mathbf{v_0} = (0{,}5;\ 0{,}35;\ 0{,}1;\ 0{,}05)^t$ as the initial solution, the column vector $\mathbf{v_1}$ is

[1] $(0{,}50;\ 0{,}35;\ 0{,}10;\ 0{,}05)^t$

[2] $(0{,}55;\ 0{,}25;\ 0{,}10;\ 0{,}10)^t$

[3] $(0{,}57;\ 0{,}28;\ 0{,}09;\ 0{,}06)^t$

[4] $(0{,}57;\ 0{,}29;\ 0{,}09;\ 0{,}05)^t$

[5] $(0{,}60;\ 0{,}25;\ 0{,}10;\ 0{,}05)^t$

Question 15

The implied preference vector using the average of the normalised columns is

[1] $(0{,}50;\ 0{,}35;\ 0{,}10;\ 0{,}05)^t$

[2] $(0{,}55;\ 0{,}25;\ 0{,}10;\ 0{,}10)^t$

[3] $(0{,}57;\ 0{,}28;\ 0{,}09;\ 0{,}06)^t$

[4] $(0{,}57;\ 0{,}29;\ 0{,}09;\ 0{,}05)^t$

[5] $(0{,}60;\ 0{,}25;\ 0{,}10;\ 0{,}05)^t$

Question 16

The implied preference vector using the geometric mean method is

[1] $(0{,}50;\ 0{,}35;\ 0{,}10;\ 0{,}05)^t$

[2] $(0{,}55;\ 0{,}25;\ 0{,}10;\ 0{,}10)^t$

[3] $(0{,}57;\ 0{,}28;\ 0{,}09;\ 0{,}06)^t$

[4] $(0{,}57;\ 0{,}29;\ 0{,}09;\ 0{,}05)^t$

[5] $(0{,}60;\ 0{,}25;\ 0{,}10;\ 0{,}05)^t$

In **Questions 17 to 19** the log least squares method is applied on the incomplete matrix of pairwise comparisons

$$A = [a_{ij}] = \begin{bmatrix} 1{,}00 & * & 6{,}00 & 9{,}00 \\ * & 1{,}00 & 3{,}00 & * \\ * & * & 1{,}00 & * \\ * & * & * & 1{,}00 \end{bmatrix}$$

to determine the implied preference and weights vectors.

Question 17

The left hand side of the first equation in the system of equations $D\mathbf{b} = \mathbf{d}$ is

[1] $2b_1 - b_3 - b_4$

[2] $b_2 - b_3$

[3] $-b_1 - b_2 + 2b_3$

[4] $-b_1 + b_4$

[5] none of the above.

Question 18

The right hand side of the second equation in the system of equations $D\mathbf{b} = \mathbf{d}$ is

[1] $\ln 6 + \ln 9$

[2] $\ln 3$

[3] $-\ln 6 - \ln 3$

[4] $-\ln 9$

[5] none of the above.

Question 19

The weights vector is

[1] $(0{,}40;\ 0{,}30;\ 0{,}20;\ 0{,}10)^t$

[2] $(\ln 1;\ \ln \frac{1}{2};\ \ln \frac{1}{6};\ \ln \frac{1}{9})^t$

[3] $(1;\ \frac{1}{2};\ \frac{1}{6};\ \frac{1}{9})^t$

[4] $(\frac{9}{16};\ \frac{9}{32};\ \frac{3}{32};\ \frac{1}{16})^t$

[5] none of the above.

Question 20

A matrix A of pairwise comparisons will be considered consistent enough to use if

[1] the inconsistency index $OI < 0{,}1$.

[2] the random index $EI > 0{,}1$.

[3] $\frac{OI}{EI} < 0{,}1$.

[4] $\lambda_{\max} < 0$.

[5] none of the above.

13.2 Assignment 02: Semester 2

Unique number: 750506 **Due date: 23 September 2014**

This is a written assignment.

It covers all the methods explained in Chapters 2 and 3 of the study guide and Chapter 13.7 of the prescribed book by Winston, as well as concepts touched on in the rest of the study guide.

Students must answer all the questions and submit a single document in written or typed format.

The lecturer will not mark all the questions, but will select a question or sub questions to mark. The mark for this selection will be the mark for the assignment.

Feedback will be provided on *my*Unisa.

Question 1

Consider an $n \times n$ uncompleted errorless matrix of pairwise comparisons $C = [c_{ij}]$ and the weights vector $\mathbf{w}' = (w_1; \ldots; w_n)$.

1.1 Write down the definition of the element c_{ij} in terms of the weights w_i and w_j.
Use this definition to show that

(a) $c_{ii} = 1$.

(b) $c_{ij} = c_{ji}^{-1}$.

(c) any element c_{ij} can be expressed as the product of two elements c_{ik} and c_{kj}.

(d) $C_{\cdot j}$, the j-th column of an errorless C matrix, is directly proportional to the preference vector.

(e) $C\mathbf{w} = n\mathbf{w}$.

1.2 Use the expressions from Question 1.1 to complete the following errorless matrix

$$C = [c_{ij}] = \begin{bmatrix} 1{,}00 & 0{,}80 & * & * \\ * & * & * & * \\ * & 0{,}80 & * & * \\ 0{,}75 & * & * & 1{,}00 \end{bmatrix}$$

and to calculate the (normalised) preference vector.

1.3 Construct the log least squares equations for the observations in the uncompleted matrix C given in Question 1.2 and determine the implied preference vector.

1.4 Confirm that the geometric means method yields the same preference vector for the completed matrix C obtained in Question 1.2.

Question 2

Suppose you are considering buying a new cell phone.

Make a list of at least twelve characteristics that you consider important.

Order these characteristics in a tree structure.

Calculate a weight for each leaf in the tree structure.

Now compare two cell phones according to these criteria and decide which one you want to buy. Define and describe any value functions that you need in your calculations.

Eliminate or add or regroup some characteristics. Will your choice change if your tree structure changed?

Question 3

Winston Chapter 13.7 Problem 3 p.793.

Question 4

Joan needs to buy a new car as her old car was written off by the insurance company after an accident. She decides to compare four vehicle models and designs a SMART scoring function for each of four criteria (vehicle specifications).

The specifications of four vehicle models are as follows:

Specification	**Car1**	**Car2**	**Car3**	**Car4**
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where x is the retail price of the vehicle model in thousands of rand.

The SMART scoring function for the engine power of the vehicle model is as follows:

$$f_{\text{Power}}(x) = \begin{cases} x & \text{if} \quad 0 \leq x \leq 100 \\ 100 & \text{if} \quad x > 100 \end{cases}$$

where x is the engine power of the vehicle model in kW.

The SMART scores for the engine capacity of the vehicle model are as follows:

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4.1 Sketch the SMART scoring function for price and briefly describe it.
How would you improve this function?

4.2 Determine the SMART score for each of the four cars for each of the four specifications. Which car is her best choice if

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