UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

DSC3704

May/June 2010

DEPARTMENT OF DECISION SCIENCES
MODELS FOR STRATEGIC DECISION MAKING

Duration 2 Hours — 80 Marks

EXAMINERS :
FIRST : PROF MD JANKOWITZ
SECOND : MISS J LE ROUX
EXTERNAL : DR GN ENGELBRECHT

Programmable pocket calculator is permissible.

This paper consists of three pages

Answer ALL the questions fully but concisely in your answer book. Make sure that you are writing the correct paper by checking the seven character code of the module now

[redacted]

Show all workings with the necessary annotations or explanations. Do all calculations to an accuracy of four decimal places.

Question 1

1.1 The most important methods of decision making use a value function to obtain a single figure that reflects the value of a particular competitor. Give and explain the form of this value function (3)

1.2 Give the missing words
When α_i represents weights in a set, then $\sum_{i=1}^{n} \alpha_i x_i$ is a weighted .... mean and $\prod_{i=1}^{n} x_i^{\alpha_i}$ a weighted .... mean of the values x_i, $i = 1, \ldots, n$ (2)

1.3 How are errors modelled and handled in the case of the eigenvector method and in the case of the log least squares method? (4)

1.4 A least squares fit for a straight line is based on the additive error model $y_i = a + bx_i + \varepsilon_i$. Derive the aggregate error $S(a,b)$ (3)

[TURN OVER]

1.5 The error model $a_{ij} = \frac{u_i}{u_j} f_{ij}$ leads to the aggregate error $S(\mathbf{u}) = \sum(\ln a_{ij} - \ln u_i + \ln u_j)^2$. Show that $S(c\mathbf{u}) = S(\mathbf{u})$.
How is this result used in the calculation of preference vectors? (5)
[17]

Question 2

Consider the matrix of pairwise comparisons below.

$$A = (a_{ij}) = \begin{pmatrix} 1{,}00 & 0{,}83 & 0{,}64 & 0{,}44 \\ 1{,}20 & 1{,}00 & 0{,}74 & 0{,}55 \\ 1{,}56 & 1{,}35 & 1{,}00 & 0{,}77 \\ 2{,}27 & 1{,}82 & 1{,}30 & 1{,}00 \end{pmatrix}$$

Calculate the implied preference vector using the methods given below. Give the answers to four decimal places.

2.1 The average of the normalised columns (5)

2.2 The geometric mean (5)

2.3 The iterative method with initial solution $\mathbf{v_0} = (0{,}2;\ 0{,}2;\ 0{,}2;\ 0{,}2)^t$. Do one iteration only. The initial solution is not normalised. Does this matter from a computational perspective? Justify your answer. (6)
[16]

Question 3

Assume that C is an errorless $n \times n$ and A an ordinary matrix of pairwise comparisons.

3.1 Show that $C\mathbf{w} = n\mathbf{w}$ (4)

3.2 The iterative procedure for finding $\mathbf{w}$ from A is based on $C\mathbf{w} = n\mathbf{w}$. Derive the procedure. (4)
[8]

Question 4

Construct the (log least squares) equations for the observations in the matrix below and then determine the implied preference vector. (10)

$$A = (a_{ij}) = \begin{pmatrix} 1{,}0 & * & * & * \\ 1{,}5 & 1{,}0 & * & * \\ * & 1{,}0 & 1{,}0 & 1{,}5 \\ * & * & * & 1{,}0 \end{pmatrix}$$

[10]

Question 5

5 1 Five options A, .., E are compared under the four criteria C_1, C_2, C_3, C_4 The preference vectors are

$$\mathbf{w}_1 = \begin{bmatrix} 0{,}17 \\ 0{,}28 \\ 0{,}14 \\ 0{,}25 \\ 0{,}16 \end{bmatrix} \qquad \mathbf{w}_2 = \begin{bmatrix} 0{,}13 \\ 0{,}29 \\ 0{,}19 \\ 0{,}30 \\ 0{,}09 \end{bmatrix} \qquad \mathbf{w}_3 = \begin{bmatrix} 0{,}21 \\ 0{,}13 \\ 0{,}41 \\ 0{,}12 \\ 0{,}13 \end{bmatrix} \qquad \mathbf{w}_4 = \begin{bmatrix} 0{,}22 \\ 0{,}14 \\ 0{,}31 \\ 0{,}12 \\ 0{,}21 \end{bmatrix}$$

and the first three criteria are of the same importance while the weight of C_4 is 0,10 Use the AHP to arrange A, . , E from the best to the worst. (8)

5 2 Define rank reversal (2)

[10]

Question 6

A young professional is moving to the Western Cape and decides to evaluate his work opportunities formally by means of SMART His criteria C_1, C_2, C_3, C_4, and C_5 are remuneration, opportunities for promotion, opportunities to come into contact with new technologies, the safety of the office environment, and the distance to the office.

6.1 Give the value function if all criteria are of the same importance except for the first which is twice as important as any other single criterion (4)

6 2 SMART needs a scoring function for each criterion Construct scoring functions $f_1(x_1)$ and $f_5(x_5)$ for C_1 and C_5 by sketching them Mark the two axes clearly Give a short justification for each. (10)

[14]

Question 7

Consider the preference vector

$$\mathbf{w} = \begin{bmatrix} 0{,}38 \\ 0{,}40 \\ 0{,}22 \end{bmatrix}$$

Stretch the scale so that the lowest value (before normalisation) is 0,10 (5)

[5]

TOTAL: 80