Tutorial letter 101/3/2018

Linear Algebra

MAT1503

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:
This tutorial letter contains important information about your module.
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1 INTRODUCTION AND WELCOME

Welcome to this module. We trust that you will find it both interesting and rewarding. This tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as exam admission. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignment(s), preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed study material and other resources and how to obtain it. Please study this information carefully and make sure that you obtain the prescribed material as soon as possible.

You will receive a number of tutorial letters during the year. A tutorial letter is our way of communicating with you about teaching, learning and assessment.

We have also included certain general and administrative information about this module. Please study this section of the tutorial letter carefully. Right from the start we would like to point out that you must read all the tutorial letters you receive during the semester immediately and carefully, as they always contain important and, sometimes urgent information.

We hope that you will enjoy this module and wish you all the best!

Tutorial matter
The Department of Despatch should supply you with the Study Guide and Tutorial Letter 101 at registration and others later.

You will receive an inventory letter that will tell you what you have received in your study package and also show items that are still outstanding. Also see the booklet entitled My Studies @ Unisa. Check the study material that you have received against the inventory letter.

PLEASE NOTE: Your lecturers cannot help you with missing study material.

Apart from Tutorial Letter 101, you will also receive other tutorial letters during the semester. These tutorial letters will not necessarily be available at the time of registration. Tutorial letters will be despatched to you as soon as they are available or needed.

If you have access to the Internet, you can view the study guide and tutorial letters for the modules for which you are registered on the University's online campus, myUnisa, at http://my.unisa.ac.za

Tutorial Letter 101 contains important information about the scheme of work, resources and assignments for this module. I urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In this tutorial letter you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides information with regard to other resources and where to obtain them. Please study this information carefully.
Certain general and administrative information about this module has also been included. Please study this section of the tutorial letter carefully.

You must read all the tutorial letters you receive during the semester immediately and carefully, as they always contain important and, sometimes, urgent information.

2 PURPOSE AND OUTCOMES OF THE MODULE

2.1 Purpose

This module will be useful to students interested in developing the basic skills in linear algebra which can be applied in the natural sciences and social sciences. Students credited with this module will have an understanding of the basic ideas of linear algebra and be able to apply the basic techniques for handling systems of linear equations, matrices, determinants and vectors.

2.2 Outcomes

The broad outcomes for this module are

2.2.1 To solve systems of linear equations.
2.2.2 To perform basic matrix operations.
2.2.3 To evaluate determinants and use them to solve certain systems of linear equations and to find inverses of invertible matrices.
2.2.4 To perform various operations in 2–space, 3–space and \( n \)–space and to find equations for lines and planes in 3–space.
2.2.5 To express complex numbers in Polar form, solve polynomial equations of a complex variable.
2.2.6 To extract \( nth \) roots of any complex number where \( n \in \mathbb{N} \).
2.2.7 To express relationships between trigonometric functions using complex numbers.
3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

The lecturers responsible for this module are as follows:

Dr. L. Godloza
Tel: (011) 670 9096
Room no: 642
GJ Gerwel Building
Florida Campus
e-mail: godlол@unisa.ac.za

Dr. ZE Mpono
Tel: (011) 670 9161
Room no: 643
GJ Gerwel Building
Florida Campus
e-mail: mponoze@unisa.ac.za

All queries that are not of a purely administrative nature but are about the content of this module should be directed to us. Please have your study material with you when you contact us.

You are always welcome to come and discuss your work – please make sure that we are free to help you by making an appointment well before the time. Please come to these appointments well prepared with specific questions that indicate your own efforts to have understood the basic concepts involved. You are also free to write to us about any of the difficulties you encounter with your work. If these difficulties concern exercises which you are unable to solve, you must send us your attempts so that we can see where you are going wrong.

Address:

The MAT1503 Lecturers
Department of Mathematical Sciences
P O Box 392
UNISA
0003

PLEASE NOTE: Letters to lecturers may not be enclosed with or inserted into assignments.
3.2 Department

Fax number: 011 670 9171 (RSA) +27 11 670 9171 (International)
Departmental Secretary: 011 670 9147 (RSA) +27 11 670 9147 (International)

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication Study @ Unisa that you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

4 RESOURCES

4.1 Prescribed book

The prescribed textbook is


Please refer to the list of official booksellers and their addresses in the Study @ Unisa brochure. Prescribed books can be obtained from the University’s official booksellers. If you have difficulty in locating your book(s) at these booksellers, please contact the Prescribed Book Section at Tel: 012 429-4152 or e-mail vospresc@unisa.ac.za.

4.2 Recommended books

The following is a publication that you may consult in order to broaden your knowledge of MAT1503. A limited number of copies is available in the Library.


Recommended books may be requested telephonically from the Main Library in Pretoria at telephone (012) 429-3133 (08:00 – 18:00 weekdays; 08:00 – 13:00 Saturdays) by supplying the request numbers and your student number.
The following books are also available at the Unisa Library. However, there is a limited number of copies of these books.

- Johnson, Lee W.: *Introduction to Linear Algebra* (2\textsuperscript{nd} or earlier editions), Addison-Wesley, Reading, MASS., 1989.

**NOTE:** Do not feel that you should study from these books, simply because we have provided you with this list. Sometimes, however, if one really gets bogged down on a particular section or part of the work, a different presentation might just be what is needed to get going again.

4.3 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information, go to www.unisa.ac.za/brochures/studies

For detailed information, go to http://www.unisa.ac.za/library. For research support and services of personal librarians, click on "Research support".

The library has compiled a number of library guides:
• finding recommended reading in the print collection and e-reserves
  – http://libguides.unisa.ac.za/request/undergrad
• requesting material
  – http://libguides.unisa.ac.za/request/request
• postgraduate information services
  – http://libguides.unisa.ac.za/request/postgrad
• finding, obtaining and using library resources and tools to assist in doing research
  – http://libguides.unisa.ac.za/Research_Skills
• how to contact the library/finding us on social media/frequently asked questions
  – http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

For information on the various student support systems and services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication Study @ Unisa that you received with your study material.

Study groups
It is advisable to have contact with fellow students. One way to do this is to form study groups. The addresses of students in your area may be obtained from the following department:

    Directorate: Student Administration and Registration
    PO Box 392
    UNISA
    0003

myUnisa
If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The myUnisa learning management system is Unisa’s online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa all through the computer and the internet.

To go to the myUnisa website, start at the main Unisa website, www.unisa.ac.za, and then click on the “myUnisa” link below the orange tab labelled “Current students”. This should take you to the myUnisa website. You can also go there directly by typing my.unisa.ac.za in the address bar of your browser.

Please consult the publication Study @ Unisa which you received with your study material for more information on myUnisa.

Group Discussions
You will receive a tutorial letter 301 with dates, venues and times for group discussions in this module.

Tutorial Classes
Tutorial classes will be given at some learning centres. This will depend on how many students are registered at that learning centre. You have to pay for these classes. You may contact the learning centre in connection with tutorial classes. The information and telephone numbers are in the Study @ Unisa brochure.
6 STUDY PLAN

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<th>Semester 2</th>
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<td>19 February</td>
<td>12 August</td>
</tr>
<tr>
<td>Outcomes 2.2.1 to 2.2.4 to be achieved by</td>
<td>4 March</td>
<td>26 August</td>
</tr>
<tr>
<td>Outcomes 2.2.4 - 2.2.7</td>
<td>18 March</td>
<td>9 September</td>
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7 PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment plan

In each semester there are three assignments for MAT1503, two of which are multiple-choice assignments (Assignment 01 and 02) and the other one (Assignment 03) is a written assignment. The questions for the assignments are given at the end of this tutorial letter. For each assignment there is a FIXED CLOSING DATE; the date by which the assignment must reach the university. Solutions for each assignment as Tutorial Letter 201, 202, etc. will be posted a few days after the closing date. They will also be made available on myUnisa.

Late assignments will be marked, but will be awarded 0%.

Written assignment

Not all the questions in the written assignment will be marked and you will also not be informed beforehand which questions will be marked. The reason for this is that Mathematics is learnt by “doing Mathematics”, and it is therefore extremely important to do as many problems as possible. You can self assess the questions that are not marked by comparing your solutions with the printed solutions that will be sent to you.

8.2 Assignment numbers

8.2.1 General assignment numbers

The assignments are numbered as 01, 02 and 03 for each semester.
8.2.2 Unique assignment numbers

Please note that each assignment also has its own unique assignment number which has to be written on the cover of your assignment upon submission.

8.3 Assignment due dates

The due dates for the submission of the assignments in 2018 are:

<table>
<thead>
<tr>
<th>Assignment no.</th>
<th>Type</th>
<th>Fixed closing date</th>
<th>Unique Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Multiple choice</td>
<td>20 February 2018</td>
<td>805792</td>
</tr>
<tr>
<td>02</td>
<td>Written</td>
<td>13 March 2018</td>
<td>733231</td>
</tr>
<tr>
<td>03</td>
<td>Written</td>
<td>10 April 2018</td>
<td>860091</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assignment no.</th>
<th>Type</th>
<th>Fixed closing date</th>
<th>Unique Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>Multiple choice</td>
<td>15 August 2018</td>
<td>772051</td>
</tr>
<tr>
<td>02</td>
<td>Written</td>
<td>14 September 2018</td>
<td>793816</td>
</tr>
<tr>
<td>03</td>
<td>Written</td>
<td>21 September 2018</td>
<td>667714</td>
</tr>
</tbody>
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8.4 Submission of assignments

You may submit written assignments and assignments completed on mark-reading sheets either by post or electronically via myUnisa. Assignments may not be submitted by fax or e-mail. For detailed information on assignments, please refer to the Study @ Unisa brochure, which you received with your study package. Assignments should be sent to

The Registrar  
P.O. Box 392  
UNISA  
0003

To submit an assignment via myUnisa:

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the menu on the left-hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

PLEASE NOTE: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. In other words, you must submit your own calculations in your own words. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of
these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

8.5 The assignments

Assignment for Semester 1 and 2 are in the Addendum section of this tutorial letter.

8.6 Other assessment methods

There are no other assessment methods for this module.

8.7 The examination

8.7.1 Examination admission

To be admitted to the examination you must submit the compulsory assignment, i.e. Assignment 01, by the due date (20 February 2018 for Semester 1, and 15 August 2018 for Semester 2).

8.7.2 Examination period

This module is offered in a semester period of fifteen weeks. This means that if you are registered for the first semester, you will write the examination in May/June 2018 and the supplementary examination will be written in October/November 2018. If you are registered for the second semester you will write the examination in October/November 2018 and the supplementary examination will be written in May/June 2019. During the semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.

8.7.3 Examination paper

The textbook forms the basis of this course. The study outcomes are listed under 2.2 of this tutorial letter. The examination will be a single written paper of two hours duration. Refer to the Study @ Unisa brochure for general examination guidelines and examination preparation guidelines.

You are not allowed to use a calculator in the exam. Previous examination paper(s) will be available to students. Students will not be supplied with solutions to these examination papers in the form of a tutorial letter. They may, discuss some problems from these with their lecturers and tutors.

9 FREQUENTLY ASKED QUESTIONS

For any other study information see the brochure Study @ Unisa.
10 SOURCES CONSULTED
No other sources were consulted in preparing this tutorial letter.

11 IN CLOSING
We hope that you will enjoy this module and we wish you success with your studies.

Your MAT1503 lecturers
Choose the correct option in each of the following questions, i.e. choose 1 or 2.

(a) A homogeneous linear system, can have no solution.

1. True 2. False

(b) The following linear system has no solutions

\[
\begin{align*}
4x + 3y &= 4 \\
3x + 3y &= 6.
\end{align*}
\]

1. True 2. False

(c) If

\[
A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}
\]

then \( AB = BA \) or \( AB = \begin{bmatrix} -1 & 2 \\ 7 & 4 \end{bmatrix} \).

1. True 2. False

(d) If

\[
A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ -2/3 & 1/3 \end{bmatrix}
\]

then \( AB \neq BA \).

1. True 2. False

(e) Let \( A \) and \( B \) be any matrices such that both \( AB \) and \( BA \) exist. If \( AB = BA \), then \( ABA^{-1} = B \).

1. True 2. False
(f) The following matrix is singular
\[
\begin{bmatrix}
4 & 0 & 0 \\
0 & \sin \theta & -\cos \theta \\
0 & \cos \theta & +\sin \theta \\
\end{bmatrix}.
\]

1. True 2. False

(g) If \( A \) is a \( 4 \times 4 \) matrix, then \( \det (kA) = k^4 \det (A) \).

1. True 2. False

(h) If \( A \) is a square matrices that is invertible, then \( \det (A^{-1}A) = 1 \).

1. True 2. False

(i) If \( C \) is a \( 4 \times 4 \) matrix and \( \det (C) = -3 \), then \( \det (3C) = -243 \).

1. True 2. False

(j) Cramer’s Rule can be applied to the following system
\[
\begin{align*}
x_1 \cos \theta - x_2 \sin \theta &= 1 \\
x_1 \sin \theta + x_2 \cos \theta &= -2.
\end{align*}
\]

1. True 2. False
QUESTION 1
If each component of a non-zero vector in $\mathbb{R}^3$ is tripped then the length of that vector is tripped. Prove this statement. (5)

QUESTION 2
Suppose that $u$ and $v$ are two vectors such that $||u|| = 2$, $||v|| = 1$ and $u \cdot v = 1$. Find the angle between $u$ and $v$ in radians. (5)

QUESTION 3
(a) Suppose the relationship $\text{Proj}_a u = \text{Proj}_a v$ is true for some vectors $a$, $u$ and $w$.
   (i) Verify that $u \cdot a = v \cdot a$. (5)
   (ii) Provide a counter example to show that $u$ need not be equal to $v$ in your example. (5)

QUESTION 4
Suppose $u$, $v$ and $w$ are vectors in $\mathbb{R}^3$. Show by means of a counter example that
   (i) $(u \times v) \times w \neq u \times (v \times w)$ sometimes and that (5)
   (ii) if $u \neq 0$, $u \times v = u \times w$ then $u$ need not be equal to $w$. (5)
      [Hint: Use $i = (1,0,0)$, $j = (0,1,0)$ and $k = (0,0,1)$]

QUESTION 5
Let $u = (1,0,2)$, $v = (0,-1,2)$ and $w = (2,1,0)$. Compute
   (i) the area of the parallelogram bounded by $u$ and $v$ (5)
   (ii) the equation of the plane parallel to $v$ and $w$ passing through the tip of $u$. (5)

QUESTION 6
Let $z_1 = x + iy$ and $z_2 = a + ib$ with $z_1 = z_2$. Prove that
   (i) $3x^2 - 3a^2 = (b - y)(b + y)$ and (5)
   (ii) The arguments of $z_1$ and $z_2$ differ by a multiple of $2\pi$ (5)
QUESTION 1

(a) (i) Show that \((x, y, z) = (t + 2, -2t - 1, t)\) is not a general solution of the system

\[
\begin{align*}
2x + 4y + 6z &= 0 \\
4x + 5y + 6z &= 3 \\
7x + 8y + 9z &= 4.
\end{align*}
\]

(ii) Solve the following system by Gauss-Jordan elimination

\[
\begin{align*}
3x + 4y + z &= 1 \\
2x + 3y &= 0 \\
4x + 3y - z &= -2.
\end{align*}
\]

(b) Solve the following system by Gauss elimination:

\[
\begin{align*}
x + 2y - z &= 2 \\
2x + 5y - 2z &= -1 \\
7x + 17y + 5z &= -1.
\end{align*}
\]

(c) Solve the system in 1(b) by using Cramer’s rule. First discuss the applicability of Cramer’s rule to the system.

(d) Consider the system

\[
\begin{align*}
x - 2y &= 1 \\
ax + by &= 5
\end{align*}
\]

where \(a\) and \(b\) are arbitrary constants.

(i) For which values of \(a\) and \(b\) does the system have a unique solution?

(ii) When is there no solution?

(iii) When are there infinitely many solutions?
QUESTION 2

Given some numbers $a, b, c, d, e$ and $f$ such that

$$
\begin{vmatrix}
  a & 1 & d \\
  b & 1 & e \\
  c & 1 & f
\end{vmatrix} = 7 \quad \text{and} \quad
\begin{vmatrix}
  a & 1 & d \\
  b & 2 & e \\
  c & 3 & f
\end{vmatrix} = 11
$$

(a) find

$$
\begin{vmatrix}
  a & 3 & d \\
  b & 3 & e \\
  c & 3 & f
\end{vmatrix}
$$

(b) find

$$
\begin{vmatrix}
  a & 3 & d \\
  b & 5 & e \\
  c & 7 & f
\end{vmatrix}
$$

(c) Consider an invertible $2 \times 2$ matrix $A$ with integer entries.

(i) Show that if the entries of $A^{-1}$ are integers, then $\det A = 1$ or $\det A = -1$.

(ii) Show the converse: If $\det A = 1$ or $\det A = -1$, then the entries of $A^{-1}$ are integers.

(d) Find the determinant of the following matrix

$$
E = \begin{bmatrix}
  1 & 0 & 2 & 1 \\
  5 & 2 & 4 & 1 \\
  1 & 0 & 2 & 1 \\
  1 & 2 & -4 & 3
\end{bmatrix}
$$

(e) Find all possible values of $c$ for which the following matrix

$$
F = \begin{bmatrix}
  1 & 1 & 1 \\
  1 & 9 & c \\
  1 & c & 3
\end{bmatrix}
$$

is singular.

QUESTION 3

Let $u = (-1, 0, 2)$, $v = (0, 1, 2)$, $w = (1, -2, 0)$ be vectors in standard position. Compute

(a) $3v - 2u$

(b) $\|u + v + w\|$

[25]
(c) the distance between \(-3\mathbf{u}\) and \(\mathbf{v} + 5\mathbf{w}\) 

(d) \(\text{Proj}_w\mathbf{v}\) 

(e) the area of the parallelogram bounded by \(\mathbf{u}\) and \(\mathbf{v}\) 

(f) the equation of the plane parallel to \(\mathbf{v}\) and \(\mathbf{w}\) and passing through the tip of \(\mathbf{u}\) 

QUESTION 4 

(a) Let \(z_1 = 1 + \sqrt{2}i\) and \(1 - \sqrt{2}i\).

(i) Determine the polar form of \(z_1\). 

(ii) Determine that the polar form of \(z_2\). 

(iii) Use the polar forms of \(z_1\) and \(z_2\) to verify that \(z_1 \cdot z_2 = 3\) 

(iv) Use the polar forms of \(z_1\) and \(z_2\) to verify that \(\frac{-1}{3} + \frac{2}{3}\sqrt{2}i = \frac{z_1}{z_2}\). 

(b) Use de Moivre’s Theorem to derive a formula for the 4th roots of 8.
Choose the correct option in each of the following questions, i.e. choose 1 or 2.

(a) If $A, B, C$ are matrices of the same size such that $A + C = B + C$ then $A = B$ and $A^{-1} = B^{-1}$

1. True  
2. False

(b) If $A, B, C$ are square matrices of the same size such that $AC = BC$, then $A$ and $B$ are identical and $C$ is invertible

1. True  
2. False

(c) A square matrix containing a row or column of zeros is singular

1. True  
2. False

(d) Let $A$ and $B$ be $n \times n$ matrices. If $AB$ is invertible, then $A$ and $B$ are invertible

1. True  
2. False

(e) Given

\[
A = \begin{bmatrix} 3 & 0 \\ 2 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 \\ 4 & 1 \end{bmatrix}
\]

it is true that

\[
AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

1. True  
2. False

(f) Two square matrices $A$ and $B$ can have the same determinant if and only if they are equal.

1. True  
2. False
(g) Let $A$ be a $3 \times 3$ matrix and $B$ be obtained from $A$ by adding 5 times the first row to each of the second and third rows, then $\det (B) = 5^3 \det (A)$

1. True  2. False

(h) A square matrix $D$ is invertible if and only if $\frac{1}{\det (D)}$ exists as a real number

1. True  2. False

(i) If a square matrix $B$ is invertible, then its inverse has non-zero determinant.

1. True  2. False

(j) In order to apply Cramer’s Rule, the coefficient matrix of a system must be invertible and the determinant of the inverse of the coefficient matrix must be different from 0.

1. True  2. False
QUESTION 1

Let \( \mathbf{u} \) and \( \mathbf{v} \) be non-zero vectors in \( \mathbb{R}^3 \) in standard position. Prove that if \( \mathbf{u} \) and \( \mathbf{v} \) are of length \( r \) cm each, where \( r \in \mathbb{R} \) and \( r > 0 \), then their tips lie on the surface of a sphere of radius \( r \) cm. (5)

QUESTION 2

Determine the components of the vector \( \mathbf{v} = \overrightarrow{P_1P_2} \) with initial point \( P_1(3, 1, 4) \) and terminal point \( P_2(3, 2, 4) \). Show all your calculations. (5)

QUESTION 3

Let \( \mathbf{v} = (1, 1, 1) \) be a vector in \( \mathbb{R}^3 \). Verify that the unit vector in the direction of \( \mathbf{v} \) is \( \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \). (5)

QUESTION 4

Suppose \( \mathbf{u}, \mathbf{v} \) and \( \mathbf{w} \) are vectors in \( \mathbb{R}^3 \) such that \( \mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w} \). Prove that \( (u_2v_3 - v_2u_3) = (u_2w_3 - w_2u_3) \). (5)

QUESTION 5

Let \( \mathbf{u} = (2, 0, 1) \), \( \mathbf{v} = (1, 2, 0) \) and \( \mathbf{w} = (0, 2, 1) \). Compute

(i) \( 3\mathbf{v} - 2\mathbf{u} \)
(ii) \( ||\mathbf{u} + \mathbf{v} - \mathbf{w}|| \)
(iii) \( (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} \)
(iv) \( \text{Proj}_\mathbf{w} \mathbf{v} \)
(v) the area of the parallelogram bounded by \( \mathbf{u} \) and \( \mathbf{v} \)
(vi) the equation of the plane parallel \( \mathbf{u} \) and \( \mathbf{v} \) and passing through the tip of \( \mathbf{w} \)

QUESTION 6

Let the plane \( V \) be defined by \( ax + by + cz + d = 0 \) with \( a, b, c, d, \in \mathbb{R} \) and the vector \( (a, b, c) \) a unit vector. Verify that the distance of the plane from the origin is \( |d| \). (5)


**QUESTION 1**

(a) Find a system of linear equations with three unknowns \(x, y, z\) whose solutions are \(x = 9t + 12\), \(y = 4t - 5\) and \(z = t\) where \(t\) is an arbitrary constant. (5)

(b) Solve the following system using Gaussian elimination

\[
\begin{bmatrix}
x + 2y - z &= 2 \\
2x + 5y + 2z &= -1 \\
7x + 17y + 5z &= -1
\end{bmatrix}
\]

(c) Solve the system

\[
\begin{bmatrix}
x + 10z &= 5 \\
3x + y - 4z &= -1 \\
4x + y + 6z &= 1
\end{bmatrix}
\]

(c) Use Gauss-Jordan elimination to solve the linear system:

\[
\begin{align*}
x + y - z &= 3 \\
-x + 4y + 5z &= -2 \\
x + 6y + 3z &= 4
\end{align*}
\]

(10) [20]

**QUESTION 2**

(a) Consider the system

\[
\begin{align*}
ax + y &= 1 \\
2x + y &= 1
\end{align*}
\]

where \(a\) and \(b\) are constants. Find \(a\) and \(b\) such that the system has

(i) No solution

(ii) exactly one solution

(iii) infinitely many solutions

(8)
(b) Find the circle \( x^2 + y^2 + ax + by + c = 0 \) passing through the points \((1, 0)\), \((5, 0)\) and \((4, 1)\).

(c) We define the vectors

\[
\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

in \( \mathbb{R}^3 \).

(i) For

\[
A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}
\]

compute \( A\vec{e}_1 \), \( A\vec{e}_2 \) and \( A\vec{e}_3 \).

(ii) If \( B \) is an \( n \times 3 \) matrix with columns \( \vec{e}_1 \), \( \vec{e}_2 \) and \( \vec{e}_3 \) what is \( B\vec{e}_1 \), \( B\vec{e}_2 \), \( B\vec{e}_3 \)?

QUESTION 3

With \( u = (0, 3, 0), v = (1, 0, 4) \) and \( (2, 4, 0) \) compute

(a) \( 2v - 2u \)

(b) \( \|2u + 3v - w\| \)

(c) the distance between \( -3u \) and \( w - 4v \).

(d) \( \text{Proj}_w w \)

(e) the area of the parallelogram bounded by \( v \) and \( w \).

(f) The equation of the plane parallel to \( v \) and \( w \) and passing through the tip of \( u \).
QUESTION 4

(a) Let \( z_1 = 1 + i\sqrt{3} \) and \( z_2 = 1 - i\sqrt{3} \)

(i) Determine the polar form of \( z_1 \)  
(ii) Determine the polar form of \( z_2 \)  
(iii) Use the polar of \( z_1 \) and \( z_2 \) to find \( z_1 \cdot z_2 \)  
(iv) Use the polar forms of \( z_1 \) and \( z_2 \) to determine the modulus of \( \frac{z_1}{z_2} \).

(b) Use the Moivres theorem to derive a formula for the 4\(^{th}\) roots of \( -8 \).