ASSIGNMENT 2 TIPS

Students often experience challenges with **Functions & representation of functions, Linear systems and Application of differentiation** topics of this module. We will explain the principles required to solve questions in these study units using the examples below. You will also realize that most of the questions will require you to formulate a linear equation, a system of linear equations or a system of linear inequalities and then solve. I hope the examples will enable you answer assignment 2 questions with deep understanding of the underlying principles.

Question 1

A children's play ground has just been opened in "Leisure City" and access cards cost R60 per day. In addition, each activity in the play ground costs 75 cents. Express the amount spent in rand (y) by a child in a day in terms of the number of activities (x) he/she has taken part in.

Solution

Note that every child must first pay a fixed cost of R60 for the access card.

Secondly if a child takes part in x activities, he/she will have to pay a total of R 0.75x for the activities (we have divided 75 cents by 100 to get R 0.75 before multiplying by x)

Hence the total amount spent by a child in a day is **y = 0.75x + 60**

Question 2

Electrocars is a company which manufactures electric cars. When the price of petrol increases by 50 cents, the number of electric cars sold in that month increases by 10. Last month, the price of petrol was R13.20 and the number of electric cars sold was 300. Let x denotes the price of petrol and y denotes the number of electric cars sold in a month. Express the number of electric cars (y) sold in a month as a linear function of the price of petrol (x).

Solution

The linear function is of the form y = ax + b, where "a" is the gradient and "b" the y-intercept.

The gradient, $a = \frac{Increase\ in\ the\ number\ of\ elecrtic\ cars\ sold}{Increase\ in\ the\ price\ of\ petrol} = \frac{10}{0.50} = 20$

Hence the equation now becomes y = 20x + b.

Note that last month, x = 13.2 and y = 300.

Substituting these values into the equation above will enable us to find "b" as follows

$$300 = 20(13.2) + b$$

$$300 = 264 + b$$

$$300 – 246 = b$$

$$b = 36$$

Hence, the linear function is y = 20x + 36

Questions 3 and 4 are based on the following information:

Consider the quadratic function $y = 2x - \frac{1}{3}x^2$ *and its associated graph*

Question 3

Determine the vertex of the graph

Solution

If we compare the above equation with the general quadratic function $y = ax^2 + bx + c$, we can see that

$$a = -\frac{1}{3}, \ b = 2 \text{ and } c = 0$$

The x value at the vertex is obtained from $x = -\frac{b}{2a}$

Hence $$x = -\frac{2}{\frac{-2}{3}} = -2\left(-\frac{3}{2}\right) = 3$$

In order to find the y value at the vertex, we will substitute the x value of the vertex into the quadratic function above as follows

$$y = 2(3) - \frac{1}{3}(3^2) = 3$$

Hence the coordinates of the vertex are (3, 3)

Question 4

Find the coordinates of the x-intercepts of the graph.

Solution

The x-intercepts are determined by equation the quadratic function to zero and solving for the values of x. Hence

$$2x - \frac{1}{3}x^2 = 0 \qquad \text{where } a = -\frac{1}{3}, \ b = 2 \text{ and } c = 0$$

(We can factorize out x in order to solve the equation easily but I will use the quadratic formula to show the general principle). We can determine the x-intercepts using the quadratic formula as follows

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4\left(-\frac{1}{3}\right)(0)}}{2(-\frac{1}{3})}$$

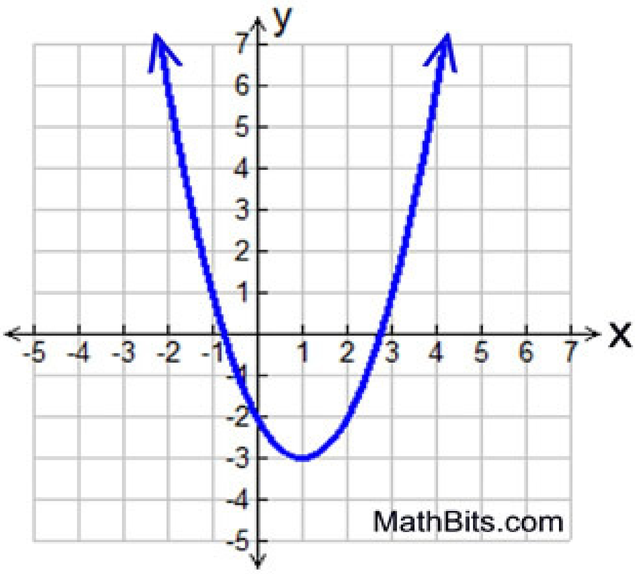
$$x = \frac{-2 \pm \sqrt{4}}{-\frac{2}{3}}$$

$$x = 0 \text{ or } x = 6$$

Note that the curve graph cuts the x-axis when y = 0. Hence the coordinates of the x-intercepts are

(0, 0) and (6, 0)

Questions 5 and 6 are based on the following information:



Question 5

Determine the vertex of the graph.

Solution

The vertex here is the coordinates of the minimum turning point of the graph which is (1, -3)

Question 6

Determine the y-intercept of the graph.

Solution

The coordinates of the y-intercept are read from the point where the parabola cuts through the y-axis. In this case, the y-intercept has coordinates (0, -2).

Questions 7 and 8 are based on the following information:

Pedro invested an amount of money p, in dollars at a continuously compounded interest rate r for time t years.

Question 7

Find the time taken for the amount invested to treble in logarithmic form.

Solution

Present value = P

Future value S = 3P (since the amount trebled)

Rate = r

Using formula for continuous compounding, we have

$$S = P\,e^{rt}$$

$$3P = P\,e^{rt}$$

Cancelling out P gives

$$3 = e^{rt}$$

Taking logs of both sides give

$\ln 3 = \ln e^{rt}$ (Ln brings down the exponent "rt" and removes the exponential function e)

$$\ln 3 = rt$$

$$t = \frac{\ln 3}{r}$$

Question 8

In how many years will the investment trebled if the investment earns 5.5% compounded continuously.

Solution

Substituting the interest rate 5.5% = 0.055 into $t = \frac{\ln 3}{r}$ gives the time taken for the investment to trebled.

$$t = \frac{\ln 3}{0.055}$$

$$t = 19.97 \sim 20 \text{ years}$$

Question 9

Thato has R 200 000 to invest but in order to diversify his risk exposure, he has chosen two investment options. The first investment option pays 5% per annum and the second investment option pays 8% per annum. He intends to make an annual profit of R 12 400. How much will he invest in each investment option?

Solution

The future value of the payments at the end of 1 year must be equal to the value of R 212,400 (200 000 + 12400 interest) at the end of 1 year.

Let's assume that an amount 'P' is invested at 5% per annum while (200 000 - P) is invested at 8% per annum.

Hence, using the formula $S = P(1 + R)^T$, we will have:

$$212\,400 = P(1 + 0.05)^1 + (200\,000 - P)(1 + 0.08)^1$$

$$212\,400 = P(1.05) + (200\,000 - P)(1.08)$$

Expanding the brackets give

$$212\,400 = 1.05P + (200\,000 \times 1.08) - 1.08P$$

$$212\,400 = 1.05P + 216\,000 - 1.08P$$

$$212\,400 = 216\,000 - 0.03P$$

$$0.03P = 216\,000 - 212\,400$$

$0.03P = 3\ 600$

$P = 3\ 600/0.03$

$P = 120\ 000$

Hence, R120 000 is invested at 5% per annum while R80 000 is invested at 8% per annum.

Question 10

A chemist has two chemical solutions that contain different concentrations. He wants to use these two solutions to prepare a cleaning chemical that will be effective in removing grease from plastics. One solution has a 16% concentration and the other has a 7% concentration. How many litres (l) of each solution should he mix to obtain 30 litres (l) of a 10% concentration.

Solution

Let x = number of litres of 16% concentration

Let y = number of litres of 7% concentration

We also know that he needs to prepare 30 litres of 10% concentration. Hence

$16x + 7y = 30 \times 10$

$16x + 7y = 300$ (equation 1)

Also the total volume of solution to be prepared is 30 litres. Hence

$x + y = 30$ (equation 2)

We will then solve the two equations simultaneously as follows:

From equation 2, $y = 30 - x$

Substituting into equation 1 gives

$16x + 7(30 - x) = 300$

$16x + 210 - 7x = 300$

$16x - 7x = 300 - 210$

$9x = 90$

$x = \frac{90}{9} = 10$ litres.

But $y = 30 - x$

$y = 30 - 10$

$y = 20$ litres.

Hence we need 10 litres of 16% concentration and 20 litres of 10% concentration in order to prepare the cleaning chemical.

Questions 11 and 12 are based on the following information:

The University senate has decided to be supplying Apple laptops to staff. A decision has been taken to buy 200 laptops per year if the price is R4 000 and 150 laptops per year if the price is R5 000. Apple Company is only willing to supply 150 laptops per year if the price is R4 400 and 200 laptops per year if the price is R4 900.

Question 11

Determine the market equilibrium point.

Solution

Market equilibrium is the price at which the quantity demanded is equal to the quantity supplied. We will need to find the demand equation and supply equation. These equations will be solved simultaneously in order to obtain the price and quantity at equilibrium point. The demand and supply equation will be in the form p = aq + b where p is the equilibrium price and q is the quantity at equilibrium, "a" is the gradient and b is the y-intercept.

For the demand function, a = $\frac{5000-4000}{150-200} = \frac{1000}{-50}$ = -20

The demand equation now becomes p = -20q + b. Substituting p = 4000 and q = 200 into the equation, we can find b as follows

$$4000 = -20(200) + b$$

$$4000 = -4000 + b$$

$$8000 = b$$

Hence the demand equation is p = -20q + 8000.

For the supply function, a = $\frac{4900-4400}{200-150} = \frac{500}{50}$ = 10

The supply equation now becomes p = 10q + b. Substituting p = 4400 and q = 150 into the equation, we can find b as follows

$$4400 = 10(150) + b$$

$$4400 = 1500 + b$$

$$2900 = b$$

Hence the supply equation is p = 10q + 2900.

We can solve the two equations simultaneously as follows

$$-20q + 8000 = 10q + 2900$$

$$-20q - 10q = 2900 - 8000 \qquad \text{(collecting like terms)}$$

$$-30q = -5100$$

$$q = \frac{-5100}{-30} = 170$$

But p = -20q + 8000, substituting q = 170 into the equation gives

$$p = -20(170) + 8000$$

$$p = 4600.$$

Question 12

If Apple is taxed 15% per unit sold, find the new equilibrium point.

Solution

The tax affects the supply function but not the demand function. Hence, Apple will be willing to supply 150 laptops per year if the price is R5 060 (4400 + (15% of 4400)) and 200 laptops per year if the price is R5 635 (4900 + (15% of 4900)).

For the supply function, $a = \frac{5635-5060}{200-150} = \frac{575}{50} = 11.5$

The supply equation now becomes p = 11.5q + b. Substituting p = 5060 and q = 150 into the equation, we can find b as follows

$$5060 = 11.5(150) + b$$

$$5060 = 1725 + b$$

$$3335 = b$$

Hence the supply equation is p = 11.5q + 3335.

We can solve the two equations simultaneously as follows

$$-20q + 8000 = 11.5q + 3335$$

$$-20q - 11.5q = 3335 - 8000 \qquad \text{(collecting like terms)}$$

$$-31.5q = -4665$$

$$q = \frac{-4665}{-31.5} = 148$$

But p = -20q + 8000, substituting q = 148 into the equation gives

$$p = -20(148) + 8000$$

$$p = 5040.$$

Question 13

Solve the inequality -27 + 5x > 3(8 +7x)

Solution

$-27 + 5x > 24 + 21x$ (expanding the brackets on the right)

$5x - 21x > 24 + 27$ (collecting like terms together give)

Hence

$-17x > 51$

Dividing all through by the -17 will reverse the sign of the inequality and give

$x < -3$

Questions 14, 15 and 16 are based on the following information:

A furniture manufacturer makes two types of furniture – chairs and sofas. The production of the sofas and chairs requires three operations – carpentry, finishing, and upholstery. Manufacturing a chair requires 3 hours of carpentry, 9 hours of finishing, and 2 hours of upholstery. Manufacturing a sofa requires 2 hours of carpentry, 4 hours of finishing, and 10 hours of upholstery. The factory has allocated at most 66 labour hours for carpentry, 180 labour hours for finishing, and 200 labour hours for upholstery. The profit per chair is R90 and the profit per sofa is R75. How many chairs and how many sofas should be produced each day to maximize the profit? Let *x* be the number of chairs produced and *y* be the number of sofas produced.

Question 14

Write the system of inequalities that describe the furniture manufacturer's constraints.

Solution

$3x + 2y \leq 66$ (carpentry hours constraint)

$9x + 4y \leq 180$ (finishing hours constraint)

$2x + 10y \leq 200$ (upholstery hours constraint)

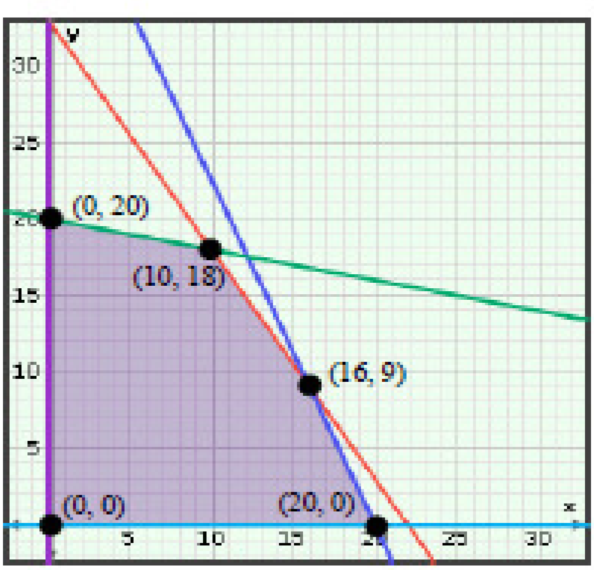
$X \geq 0, y \geq 0$ (number of chairs and sofas produced must be greater than zero)

Question 15

How many chairs and how many sofas should be produced each day to maximize the profit?

Solution

This can be obtained by solving the system of equations. The easiest way is to plot these inequalities and determine the region that satisfies the inequalities.



We will use the points of intersection to determine the number of chairs and sofas that will produce the maximum profit

p = 90x + 75y as follows;

When x = 0 and y = 0, p = 90(0) + 75(0) = 0

When x = 0 and y = 20, p = 90(0) + 75(20) = 1500

When x = 10 and y =18, p = 90(10) + 75(18) = 2250

When x = 16 and y = 9, p = 90(16) + 75(9) = 2115

When x = 20 and y = 0, p = 90(20) + 75(0) = 1800

The manufacturer must produce 10 chairs and 18 sofas in order to maximize the profit.

Question 16

What is the maximum profit realized in a day?

Solution

The maximum profit is R2250 as per the calculation above.

Questions 17 and 18 are based on the following information:

A Company specializes in the production of washing machines. The cost in rand to produce a washing machine is

$$C(x) = 300$$

and the revenue in rand obtained from selling a washing machine is

$$R(x) = 800 - x + 0.05x^2$$

Where x is the number of washing machines produced.

Question 17

Determine the marginal profit function

Solution

The cost in rand to produce x washing machines is 300x.

The revenue from selling x washing machines is $800x - x^2 + 0.05x^3$

First, find the profit function by subtracting the cost function from the revenue function.

$$p(x) = (800x - x^2 + 0.05x^3) - 300x$$

$$p(x) = 0.05x^3 - x^2 + 500x$$

Then, determine the marginal profit function by differentiating the profit function as follows

$$p^1(x) = 0.15x^2 - 2x + 500$$

Question 18

What is the marginal profit obtained by selling 50 washing machines?

Solution

The marginal profit is obtained by substituting 50 into the marginal profit function as follows

$$p^1(x) = 0.15x^2 - 2x + 500$$

$$p^1(50) = 0.15(50)^2 - 2(50) + 500$$

$$p^1(50) = 775$$

Questions 19 and 20 are based on the following information:

The cost in rand for the daily production of x kilograms of gold by a mining company is given by the function

$$C(x) = -\frac{1}{50}x^2 + 200x + 3000.$$

Question 19

Determine the marginal cost to produce these kilograms of gold.

Solution

The marginal cost function is obtained by differentiating the cost function as follows:

$$C^1(x) = -(2)\frac{1}{50}x + 200$$

$$C^1(x) = -\frac{1}{25}x + 200$$

Question 20

Calculate the marginal cost to produce 150 kilograms of gold.

Solution

The marginal cost (MC) to produce 150 kilograms of gold is obtained by substituting 150 into the marginal cost function as follows

$$C^1(150) = -\frac{1}{25}(150) + 200$$

$$C^1(150) = 194$$