ASSIGNMENT 1 detailed solution

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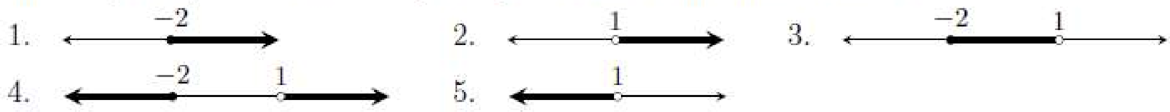
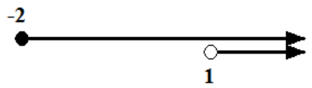
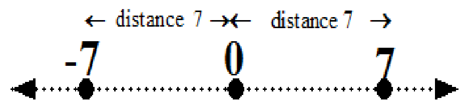
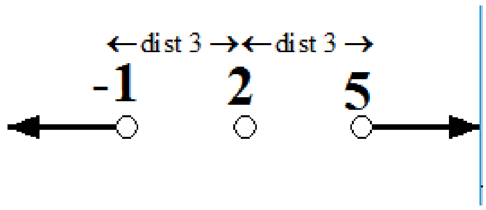
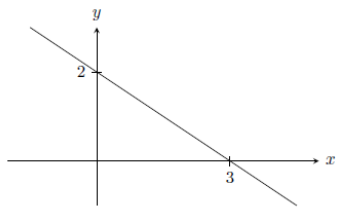
MAT 1510 - 2017

1	Suppose $P = \{-3, 1, 3\}$, $Q = \{-3, -2, -1, 1, 2, 3\}$ and $R = \{-2, 2, 3\}$. Which of the following statements is/are true? (a) $P \cap R = \{3\}$ (b) $P \cup R = Q$ (c) $P = \{-3, 1, 3, 0\}$
Solution	$P \cap R = \{-3,1,3\} \cap \{-2,2,3\} = \{3\}$ CORRECT $P \cap R = \{-3,1,3\} \cup \{-2,2,3\} = \{-3,-2,1,2,3\} \neq Q$ $P = \{-3,1,3\} \neq \{-3,1,3,0\}$
2	The set $\{x \in \mathbb{R} \| x \geq -2 \text{ or } 1 < x\}$ is represented on the number line as 
Solution	 union (OR) of the two sets gives 
3	The area of ΔABC is x cm^2 and the area of ΔXYZ is y cm^2. Which one of the following mathematical inequalities represents the statement: "The area of ΔABC is at most the same as the area of ΔXYZ"? 1. $x \geq y$ 2. $x > y$ 3. $x \leq y$ 4. $x < y$ 5. None of the preceding
Solution	" at most the same" interpreted as "$\leq$" $\therefore \quad x \leq y$
4	The sum of the largest and smallest of the numbers $-\frac{1}{4}, -\frac{1}{3}, 0, \frac{3}{2}, \frac{4}{3}$ is 1. 1 2. $\frac{4}{3}$ 3. $\frac{5}{4}$ 4. $\frac{7}{6}$ 5. $\frac{13}{12}$
Solution	largest number $= \frac{3}{2}$ smallest number $= -\frac{1}{3}$ sum of the two numbers $= \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6}$

5	If the ratio of x to y is at least $\frac{5}{6}$, and $y > 0$, then 1. $5x < 6y$ 2. $6x \leq 5y$ 3. $6x > 5y$ 4. $6x \geq 5y$ 5. $5x \geq 6y$
Solution	$x : y \geq \frac{5}{6}$ $\frac{x}{y} \geq \frac{5}{6}$ since $y > 0$ we find $6x \geq 5y$
6	Which of the following statements is/are FALSE? (a) $\lvert x \rvert = 7$ can be represented geometrically as all numbers x which are a distance of 7 units from 0. (b) $\lvert x+2 \rvert > 3$ can be represented geometrically as all numbers x whose distance from 2 is greater than 3 units. (c) The inequality $\lvert x-a \rvert \leq \delta$, where $\delta > 0$, is true if and only if $-\delta < x - a < \delta$.
Solution	$\lvert x \rvert = 7 \Rightarrow x = 7$ or $x = -7$ graphically  TRUE $\lvert x+2 \rvert > 3$ $\Rightarrow x > 1$ or $x < -5$ graphically  FALSE
7	Which of the following statements is/are true? (a) $\sqrt{4} = \pm 2$ (b) $\sqrt{6} + \sqrt{3} = \sqrt{9}$ (c) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
Solution	$\sqrt{4} = +2 \neq \pm 2$ so FALSE $\sqrt{6} + \sqrt{3} \neq \sqrt{6+3} = \sqrt{9}$ so FALSE $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ so TRUE

8	If we consider only those values of x for which the expression is defined, then $\frac{x^2-x-2}{x^2+x-6}$ is equal to 1. $\frac{1}{3}$ 2. $\frac{-x-2}{x-6}$ 3. $\frac{x+1}{x-2}$ 4. $\frac{x+1}{x+3}$ 5. None of the preceding
Solution	$\frac{x^2-x-2}{x^2+x-6}=\frac{(x-2)(x+1)}{(x-2)(x+3)}=\frac{x+1}{x+3}$
9	Which one of the following statements is/are FALSE? (a) $\frac{a+1}{b}=\frac{a-1}{b}+\frac{b+1}{a}\Longrightarrow a=\frac{b(b+1)}{2}$ (b) $\frac{l}{k}=2-\frac{k}{l}\Longrightarrow k=l$ (c) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}-1\Longrightarrow c=\frac{ab}{a(1+b)+b}$
Solution	$(a)\quad \frac{a+1}{b}=\frac{a-1}{b}+\frac{b+1}{a}$ $\Rightarrow a(a+1)=a(a-1)+b(b+1)$ $\Rightarrow a^2+a=a^2-a+b^2+b$ $\Rightarrow 2a=b(b+1)$ $\Rightarrow a=\frac{b(b+1)}{2}$ TRUE $(b)\quad \frac{\ell}{k}=2-\frac{k}{\ell}$ $\Rightarrow \ell^2=2k\ell-k^2$ $\Rightarrow \ell^2-2k\ell+k^2=0$ $\Rightarrow (\ell-k)(\ell-k)=0$ $\therefore k=\ell$ TRUE $(c)\quad \frac{1}{a}+\frac{1}{b}=\frac{1}{c}-1$ $\Rightarrow bc+ac=ab-abc$ $\Rightarrow bc+ac+abc)=ab$ $\Rightarrow c(b+a+ab)=ab$ $\Rightarrow c=\frac{ab}{ab+a+b}=\frac{ab}{a(1+b)+b}$ TRUE

10	The set of solutions of $(2x+3)(3x-4)=3$ is 1. $\{0\}$ 2. $\{-\frac{3}{2},\frac{4}{3}\}$ 3. $\{-\frac{5}{3},\frac{3}{2}\}$ 4. $\{\frac{7}{3}\}$ 5. None of the preceding
Solution	$(2x+3)(3x-4)=3$ $\Rightarrow 6x^2+x-12-3=0$ $\Rightarrow 6x^2+x-15=0$ $\Rightarrow (2x-3)(3x+5)=0$ $\therefore x=\frac{3}{2}$ or $x=-\frac{5}{3}$ $\therefore x\in\{-\frac{5}{3},\frac{3}{2}\}$
11	Which of the following statements is/are correct? (a) The solution set of $x^2-a^2=0$ is $\{-\|a\|,\|a\|\}$. (b) The solution set of $9y^3-25y=0$ is $\{-\frac{5}{3},\frac{5}{3}\}$. (c) The solution set of $0\cdot x=0$ is $\{0\}$.
Solution	(a) $x^2-a^2=0\Rightarrow(x-a)(x+a)=0\Rightarrow x=a$ or $x=-a\Rightarrow x=\|a\|$ or $x=-\|a\|$ CORRECT (b) $9y^3-25y=0\Rightarrow y(9y^2-25)=0\Rightarrow y=0$ or $y=\pm\frac{5}{3}\Rightarrow x\in\{-\frac{5}{3},0.\frac{5}{3}\}\neq\{-\frac{5}{3}.\frac{5}{3}\}$ FALSE (c) $0\cdot x=0\Rightarrow x\in\mathbb{R},x\neq 0$ FALSE
12	Which one of the following is the solution set of the equation $\frac{1}{x-4}-\frac{1}{x}=\frac{4}{5}$? 1. $\{-5,1\}$ 2. $\{-5,5\}$ 3. $\{-1,1\}$ 4. $\{-1,5\}$ 5. None of the preceding
Solution	$\frac{1}{x-4}-\frac{1}{x}=\frac{4}{5}$ $\Rightarrow 5x-5(x-4)=4x(x-4)$ $\Rightarrow 5x-5x+20=4x^2-16x$ $\Rightarrow 4(x^2-4x-5)=0$ $\Rightarrow 4(x-5)(x+1)=0$ $\therefore x\in\{-1,5\}$
13	The solution set of $\sqrt{4x+3}=2x$ is 1. $\{-\frac{1}{2}\}$ 2. $\{\frac{3}{2}\}$ 3. $\{-\frac{3}{2},\frac{1}{2}\}$ 4. $\{-\frac{1}{2},\frac{3}{2}\}$ 5. $\{-\frac{3}{2},-\frac{1}{2}\}$
Solution	$\sqrt{4x+3}=2x$ $\Rightarrow 4x+3=4x^2$ $\Rightarrow 4x^2-4x-3=0$ $\Rightarrow (2x+1)(2x-3)=0$ $\Rightarrow x\neq-\frac{1}{2}$ or $x=\frac{3}{2}$ $\therefore x\in\{\frac{3}{2}\}$

14	 The equation of the line sketched above is: 1. $2y+3x-6=0$ 2. $2y+3x+6=0$ 3. $-3y+2x-6=0$ 4. $3y-2x-6=0$ 5. $3y+2x-6=0$
Solution	$y=mx+c$ $gradient = m = -\frac{2}{3}$ $y-\text{int ercept}=2$ $y=-\frac{2}{3}x+2$ $\therefore 3y=-2x+6$ $\therefore 3y+2x-6=0$
15	Which one of the following equations represents a line that is perpendicular to the line with equation $3x-7y+2=0$? 1. $3x+7y+2=0$ 2. $-3x+7y+2=0$ 3. $7x+3y+2=0$ 4. $7x-3y+2=0$ 5. $-7x+3y+2=0$
Solution	$3x-7y+2=0 \Rightarrow y=\frac{3}{7}x+\frac{2}{7}$ $\therefore y=-\frac{7}{3}x+c$ $\therefore 3y=-7x+3c$ $\therefore 3y+7x+2=0$ possible equation
16	The solution of the inequality $2x^2+5x+3\leq 0$ is 1. $x\leq -\frac{3}{2}$ or $x\geq -1$ 2. $-\frac{3}{2}\leq x\leq -1$ 3. $x\geq -\frac{3}{2}$ 4. $x\leq -1$ 5. None of the preceding
Solution	$2x^2+5x+3\leq 0$ $\Rightarrow (2x+3)(x+1)\leq 0$ using a number line we obtain: $\therefore -\frac{3}{2}\leq x\leq -1$
17	The solution of the inequality $\frac{2-x}{\lvert 3x+1\rvert}\leq 0$ is 1. $x\geq 2$ 2. $-\frac{1}{3}\leq x\leq 2$ 3. $-\frac{1}{3}<x\leq 2$ 4. $x\leq -\frac{1}{3}$ or $x\geq 2$ 5. $x<-\frac{1}{3}$ or $x\geq 2$
Solution	$\frac{2-x}{\lvert 3x+1\rvert}\leq 0 \quad \Rightarrow 2-x\leq 0 \quad \Rightarrow x\geq 2$

18	The circle with equation $2(x+4)^2+2(y-1)^2-18=0$ has centre C and radius r units, where 1. $C=(4;-1);\ r=\sqrt{18}$ 2. $C=(-4;1);\ r=\sqrt{18}$ 3. $C=(-4;-1);\ r=3$ 4. $C=(4;-1);\ r=3$ 5. $C=(-4;1);\ r=3$
Solution	$2(x+4)^2+2(y-1)^2=18$ $\Rightarrow (x+4)^2+(y-1)^2=9$ $\therefore$ centre C=$(-4,1)$ and radius =3
19	Towns A and B are 300 km apart. Susan travels from A to B at a speed of 80 km/h. Tebogo leaves 30 minutes after Susan and also travels from A to B, at a speed of 100 km/h. Tebogo overtakes Susan t hours after Susan has left town A. Which one of the following equations can be used to solve for t? Remember that distance = speed × time.
Solution	distance after t hours (Susan) = $speed \times time = 80 \times t$ distance after t hours (Tebogo) = $speed \times time = 100 \times \left(t-\frac{1}{2}\right)$ $\therefore 80t = 100\left(t-\frac{1}{2}\right)$
20	Dhaya takes 5 hours to paint a room by himself. Working together, it takes Dhaya and Fawzia two-thirds of the time that it takes Fawzia to paint the room by herself. Suppose it takes Fawzia x hours to paint the room by herself. Which one of the following equations can be used to solve for x? 1. $5+x=\frac{2}{3}x$ 2. $\frac{2}{3}(5+x)=x$ 3. $5+x=\frac{2}{3}\times 5$ 4. $\dfrac{x+5}{5x}=\frac{2}{3}x$ 5. $\dfrac{5x}{x+5}=\frac{2}{3}x$
Solution	in 1 hour Dhaya paints $\frac{1}{5}$ of the room in 1 hour Fawzia paints $\frac{1}{x}$ of the room in 1 hour (Dhaya and Fawzia) paints $\left(\frac{1}{5}+\frac{1}{x}\right)=\frac{5+x}{5x}$ of the room but ALSO in 1 hour (Dhaya and Fawzia) paints $=\frac{1}{\frac{2}{3}x}=\frac{3}{2x}$ of the room $\therefore \frac{5+x}{5x}=\frac{3}{2x} \Rightarrow \frac{5x}{5+x}=\frac{2}{3}x$