Assignment 1 Tips and Hints

Question 1 example

Simplify:

$$\frac{25a^3b^3}{5a^2b + 10ab} \times \frac{a^2 + 4a + 4}{15a^5b^2}$$

Tips:

- *Common Factors:*
  Always remove common factors from an equation.
  e.g. $\frac{16}{20}$, the common factor is 4, so $\frac{16}{20} = \frac{4}{5}$
- *Product of power:*
  When multiplying two terms with the same base, add the exponents.
  e.g. $a^6 \times a^2 = a^{6+2} = a^8$
- *Quotient of power:*
  When dividing two terms with the same base, subtract the exponent in the denominator from the exponent in the numerator.
  e.g. $\frac{a^6}{a^2} = a^6 \times a^{-2} = a^{6-2} = a^4$

Solution

$$\rightarrow \frac{25a^3b^3}{5a^2b + 10ab} \times \frac{a^2 + 4a + 4}{15a^5b^2}$$

$$\rightarrow \frac{25a^3b^3}{5ab(a+2)} \times \frac{(a+2)(a+2)}{15a^5b^2}$$

$$\rightarrow \frac{5a^2b^2}{(a+2)} \times \frac{(a+2)(a+2)}{15a^5b^2}$$

- ✓ *25 cancelled out with the 5 below leaving 5.*
- ✓ *"Quotient of power" 3-1 =2 for both a and b exponents.*

$$\rightarrow \frac{5a^2b^2(a+2)(a+2)}{(a+2)15a^5b^2}$$

$$\rightarrow \frac{a+2}{3a^3}$$

- ✓ *The one (a+2) cancelled out with the one at the bottom.*
- ✓ *The "Quotient of power" cancelled the b out and left the exponent of 3 on the a.*

Question 2 example

Simplify:

$$\frac{b - \frac{1}{b}}{b - \frac{4}{b}}$$

Tips:

- *Please see question 1 tips.*
- *Adding a fraction to a whole number or variable:*
  There are many methods to go about doing this, I find this one the quickest and easiest:

Multiply the whole number/variable by the denominator of the fraction and divide it by the denominator. The rest is easy since both terms will have the same denominator.

e.g. $1 + \frac{1}{2} = (1 \times \frac{2}{2}) + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$

Solution

$$\rightarrow \frac{b - \frac{1}{b}}{b - \frac{4}{b}}$$

$$\rightarrow \frac{\frac{b^2}{b} - \frac{1}{b}}{\frac{b^2}{b} - \frac{4}{b}}$$

✓ $b \times \frac{b}{b} = \frac{b^2}{b}$

$$\rightarrow \frac{\frac{b^2 - 1}{b}}{\frac{b^2 - 4}{b}}$$

✓ *They now have the same denominator so they can be added or subtracted above one denominator.*

$$\rightarrow \frac{\frac{(b-1)(b+1)}{b}}{\frac{(b-2)(b+2)}{b}}$$

$$\rightarrow \frac{(b-1)(b+1)}{b} \times \frac{b}{(b-2)(b+2)}$$

$$\rightarrow \frac{(b-1)(b+1)}{(b-2)(b+2)}$$

Question 3 example

Simplify:

$$\frac{3^{-2}(3^{-1})^1}{(4^0 + 2)^{-1}}$$

Tips:

- *Exponent of 0 or 1:*
  Any number or variable raised to a power of 1 is the number itself.
  e.g. $2^1 = 2$
  Any non-zero number or variable raised to a power of 0 is equal to 1.
  e.g. $2^0 = 1$
- *Negative exponents:*
  Any number or variable raised to a power of a negative number changes its sign when moved from the numerator to the denominator and vice versa.
  e.g. $\frac{3^{-2}}{2^{-1}} = \frac{2}{3^2}$
- *Power of a power:*
  To raise a power to a power, multiply the exponents. E.g. $(2^1)^2 = 2^{1x2} = 2^2$

Solution

$\rightarrow \dfrac{3^{-2}(3^{-1})^{1}}{(4^{0}+2)^{-1}}$

$\rightarrow \dfrac{3^{-2}\times 3^{-1}}{(1+2)^{-1}}$

- ✓ *We used "power of a power" on the 3 and "exponent of zero" on the 4.*

$\rightarrow \dfrac{3^{-2+(-1)}}{(3)^{-1}}$

$\rightarrow \dfrac{3^{-3}}{3^{-1}}$

- ✓ *We used "Quotient of power" -2 + (-1) = -2 – 1 = -3*

$\rightarrow \dfrac{1}{3^{-1+3}}$

$\rightarrow \dfrac{1}{3^{2}}$

$\rightarrow \dfrac{1}{9}$

Question 4 example

Simplify:

$$\left(\frac{x^2yz^2}{x^4y^2}\right)^2$$

Tips:

- *Please see example 1 to 3 tips.*

Solution

$\rightarrow \left(\dfrac{x^2yz^2}{x^4y^2}\right)^2$

$\rightarrow \left(\dfrac{z^2}{x^2y}\right)^2$

$\rightarrow \dfrac{z^4}{x^4y^2}$

Question 5 example

Find c if

$$\sqrt{4c+3}-2=3$$

Tips:

- Square roots can always be written as exponents, for example:
  - $\sqrt{x}=x^{\frac{1}{2}}$

- $\sqrt[4]{x} = x^{\frac{1}{4}}$

- To remove a square root, you square both sides of an equation, e.g.

$$\sqrt{x} = 2 \rightarrow \left(\sqrt{x}\right)^2 = 2^2 \rightarrow \left(x^{\frac{1}{2}}\right)^2 = 2^2 \rightarrow x = 4$$

Solution

$\rightarrow \sqrt{4c+3} - 2 = 3$

$\rightarrow \sqrt{4c+3} - 2 + 2 = 3 + 2$

$\rightarrow \sqrt{4c+3} = 5$

$\rightarrow (4c+3)^{\frac{1}{2}} = 5$

$\rightarrow ((4c+3)^{\frac{1}{2}})^2 = 5^2$

✓ *We can now square both side to remove the ½ term*

$\rightarrow 4c + 3 = 25$

$\rightarrow 4c + 3 - 3 = 25 - 3$

$\rightarrow 4c = 22$

$\rightarrow c = \frac{22}{4} = 5.5$

Question 6 example

Solve for x:

$$2\left(\frac{x-1}{2}\right) - \frac{1}{2} = \frac{3+x}{4}$$

Tips:

- A number multiplied by a bracket, is all contents of the brackets multiplied by that number e.g.
$2\left(\frac{x+1+4}{3}\right) = \frac{2 \times x + 2 \times 1 + 2 \times 4}{2 \times 3} = \frac{2x+2+8}{6}$
- When two fractions are on different sides of the equal sign, you can always multiply the denominator of the one side with the numerator of the other side e.g.
$\frac{2}{x} = \frac{3}{2} \rightarrow 2 \times 2 = 3 \times x \rightarrow 4 = 3x$

Solution

$\rightarrow 2\left(\frac{x-1}{2}\right) - \frac{1}{2} = \frac{3+x}{4}$

$\rightarrow \frac{2x-2}{4} - \frac{1}{2} = \frac{3+x}{4}$

$\rightarrow \frac{2x-2-2}{4} = \frac{3+x}{4}$

$$\rightarrow \frac{2x-4}{4} = \frac{3+x}{4}$$

$$\rightarrow (2x-4) \times 4 = (3+x) \times 4$$

$$\rightarrow 8x - 16 = 12 + 4x$$

$$\rightarrow 8x - 4x = 12 + 16$$

$$\rightarrow 4x = 28$$

$$\rightarrow x = \frac{28}{4} = 7$$

- ✓ *Always put like terms on one side of the equation*
- ✓ *When a number moves from one side of the equal side to another, it changes its sign i.e. from positive to negative and vice versa.*

<u>Question 7 example</u>

A club has 24 members. In how many ways can 4 officers, a president, vice-president, secretary and coordinator be chosen from the members of the club?

Tips:

- First consider if the order is important, if it is then we know to use a permutation and if it is not then we know to use a combination.

<u>Solution</u>

This is a permutation because there can only be one president, one secretary, etc. In other words the order is important. Therefore,

$$\rightarrow \ {}_{24}p_4 = \frac{24!}{(24-4)!} = \frac{24!}{20!} = 255024 \ ways$$

<u>Question 8 example</u>

On each delivery, a postman delivers letters to 3 of the 12 suburbs in his territory. In how many different ways can he schedule his route?

<u>Solution</u>

Tips:

- The order in which he delivers to the suburbs is not mentioned, meaning that this is a combination.

<u>Solution</u>

The number of ways in which he can schedule his route is:

$$\rightarrow \ {}_{12}C_3 = \frac{12!}{(12-3)! \times 3!} \times \frac{12!}{9! \times 3!} = 220 \ ways$$

Question 9 & 10 example

1. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of 3 faculty members from the mathematics department and 4 from the computer science (CS) department, if there are 9 faculty members of the math department and 11 of the CS department?
2. In how many ways can they be selected if the first faculty member of the maths department is to be the senior maths faculty member, the second to be the deputy and the third to be the junior?

Tips:

- First consider if the order is important, if it is then we know to use a permutation and if it is not then we know to use a combination.
- Consider the number of ways to choose the faculty members from the mathematics department, and then consider the number of ways to choose the faculty members from the CS department

Solution

1. The order in which the faculty members are chosen is not important for both departments.

✓ The order is not important, so we can pick the 3 faculty members out of the total 9 from the maths department as follows:

$${}_9C_3 = \frac{9!}{(9-3)! \times 3!} = 84\ ways$$

✓ The order is not important so we can pick the 4 faculty members out of the total 11 from the CS department as follows:

$${}_{11}C_4 = \frac{11!}{(11-4)! \times 4!} = 330\ ways$$

Therefore, there are:

$${}_9C_3 \quad \times \quad {}_{11}C_4 = \frac{9!}{(9-3)! \times 3!} \times \frac{11!}{(11-4)! \times 4!} = 84 \times 330 = 27720\ ways$$

2. The order in which the maths faculty members are chosen is important but not important for the CS faculty members. So,

- ✓ We can pick the 3 faculty members out of the total 9 from the maths department as follows:

$$ {}_9P_3 = \frac{9!}{(9-3)!} = 504\ ways $$

- ✓ We can pick the 4 faculty members out of the total 11 from the CS department in the same way as 1:

$$ {}_{11}C_4 = \frac{11!}{(11-4)! \times 4!} = 330\ ways $$

Therefore, there are:

$$ {}_9P_3 \quad \times \quad {}_{11}C_4 = \frac{9!}{(9-3)} \times \frac{11!}{(11-4)! \times 4!} = 504 \times 330 = 166320\ ways $$

<u>Question 11 example</u>

A tree trunk is 150cm long. It is cut into three pieces P, Q and R in the ratio of 4: 6: 2. Determine the length of each piece.

Tips:

- It's easier to work with ratios when they are converted into fractions.

<u>Solution</u>

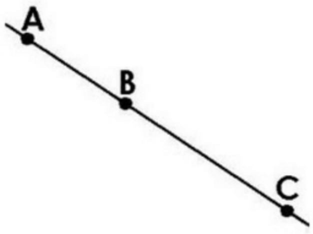
- ✓ Add all ratios to get the total

$4 + 6 + 2 = 12$

$$ \rightarrow Piece\ P\ is\ \frac{4}{12} \times 150cm = 50cm\ long $$
$$ \rightarrow Piece\ Q\ is\ \frac{6}{12} \times 150cm = 75cm\ long $$
$$ \rightarrow Piece\ R\ is\ \frac{2}{12} \times 150cm = 25cm\ long $$

$\boldsymbol{To\ check: 50 + 75 + 25 = 150.}$

<u>Question 12 example</u>



The length of AC is 30cm. The ratio of AB : AC is 1 : 4. What is the length of AB and BC?

Tips:

- It's easier to work with ratios when they are converted into fractions.

Solution

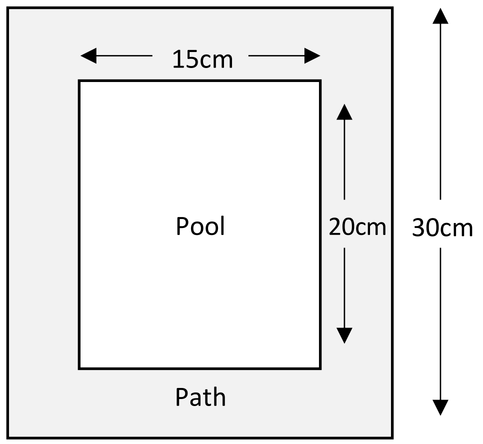
AC is the total length of the line = 4, so we do not need to add up the ratios to get the total in this instance.

$\rightarrow AB\ is\ \frac{1}{4} \times 30cm = 7.5\ cm$

$\rightarrow BC\ is\ \frac{4-1}{4} \times 30cm = \frac{3}{4} \times 30cm = 22.5cm$

$\boldsymbol{To\ check: 7.5cm + 22.5cm = 30cm}$

Question 13 example



A rectangular shaped pool has a path around it. The path has uniform width on all sides. The pool is 20cm long and 15cm wide. The total length of the path and pool is 30cm.
Calculate the area of the path.

Tips:

- Border area formula:
  $Area = Area\ of\ outer\ rectangle - Area\ of\ inner\ rectangle$
- Area of rectangle:
  $Area\ of\ rectangle = length \times width$

Solution

To find the area of the outer rectangle we must first figure out its width

$\rightarrow Uniform\ width\ of\ path = 30cm - 20cm = 10cm$

$\rightarrow Since\ there\ are\ two\ sides, it\ is\ equal\ to\ \frac{10cm}{2} = 5cm\ on\ each\ side.$

Therefore, the width of the <u>outer rectangle</u> is:
$\rightarrow 15cm + 5cm + 5cm = 25cm$

So,
$Area\ of\ outer\ rectangle = 25cm \times 30cm = 750cm^2$
$Area\ of\ inner\ rectangle = 15cm \times 20cm = 300cm^2$

$$\rightarrow Area\ of\ path = Area\ of\ outer\ rectangle - Area\ of\ inner\ rectangle$$
$$= 750cm^2 - 300cm^2$$
$$= 450cm^2$$

<u>Question 14 example</u>

A rectangular tank measures 4m long, 2m wide and 4.8m high. Find the depth of the water in the tank after 3 hours if 4000 litres of water is added to the tank every hour.

Tips:

- $Volume\ of\ a\ prism = Area\ of\ base \times height$
- $Area\ of\ rectangle = length \times width$
- $1000l = 1m^3$

<u>Solution</u>

✓ After 3 hours, the tank will have:
$\rightarrow 4000 \times 3 = 12000\ litres = Volume$

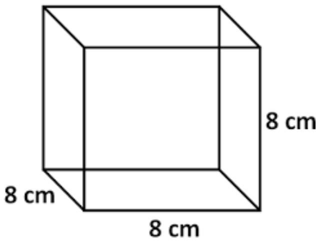
✓ Now we know the volume, length and width. Therefore, the depth after 2 hours will be:
$\rightarrow Volume\ of\ a\ prism = Area\ of\ base \times height$
$\rightarrow V = length \times width \times height/depth$
$\rightarrow 12000l = 4m \times 2m \times depth$

✓ We first need to change the 12000 litres to cubic metres for the formula to make sense, so:
$$\rightarrow \frac{12000l}{1000l} = 12m^3$$

$$\rightarrow 12m^3 = 4m \times 2m \times depth$$
$$\rightarrow depth = \frac{12m^3}{4m \times 2m} = \frac{12m^3}{8m^2} = 1.5m$$

Question 15 example

Find the volume of the following cube in mm^3:



Tips:

- $Volume\ of\ a\ cube = a^3$
- Where "a" is the length of any edge of the cube.
- $1cm^3 = 1000mm^3$

Solution

$\rightarrow V = a^3 = (8cm)^3 = 512cm^3$

$\rightarrow 512cm^3 \times 1000 = 512\ 000mm^3$

Question 16 example

A rectangular floor that has the length of 9m and width 3m must be covered with square tiles with the length of the side equal to 30cm. How many tiles are needed to cover this floor?

Tips:

- $Area\ of\ rectangle\ =\ length\ \times\ width$
- $Area\ of\ a\ square = a^2$
- Where "a" is the length of any side of the square.
- $1m = 100cm$

Solution

✓ The area of the floor has to be covered by square tiles (which have their own areas)

✓ So the area of the floor and the area of the tile must be in the same unit (in cm)

$\rightarrow 9m\ =\ 9 \times 100 = 900cm$

$\rightarrow 4m\ =\ 4 \times 100 = 400cm$

Area of the floor:

$\rightarrow A = l \times w = 900cm \times 400cm = 360\ 000cm^2$

Area of the one tile:

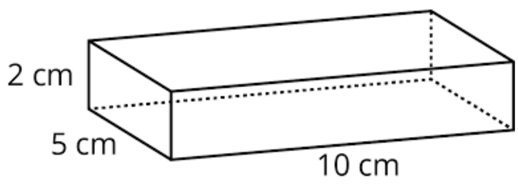
$\rightarrow A = a^2 = (30cm)^2 = 900cm^2$

Hence, the following number tiles are needed:

$$\rightarrow \frac{Area\ of\ floor}{Area\ of\ a\ single\ tile} = \frac{360\ 000cm^2}{900cm^2} = 400\ tiles$$

Question 17 example

Consider the following rectangular tank which is open at the top:



This tank is 10cm long, 5cm wide and 2cm deep. What is the cost of plastering the inside of its walls and bottom (floor) at R12 per square meter?

Tips:

- $Area\ of\ rectangle\ =\ length\ \times\ width$

Solution

✓ There are two ways we can solve this. We could calculate total surface area of the prism and then subtract the area of the missing top rectangle or we can calculate the total area of the 5 rectangles separately that make up the tank.

✓ Let us use the latter method.

Area of the rectangles that are 10cm by 2cm:

$\rightarrow A\ =\ l\ \times\ w = 10cm \times 2cm = 20cm^2$

There are **two** of these, so total area of these two:

$\rightarrow A\ =\ 2(l\ \times\ w) = 2 \times 10cm \times 2cm = 40cm^2$

Area of the rectangles that are 5cm by 2cm:

$\rightarrow A\ =\ l\ \times\ w = 5cm \times 2cm = 10cm^2$

There are **two** of these, so total area of these two:

$\rightarrow A\ =\ 2(l\ \times\ w) = 2 \times 5cm \times 2cm = 20cm^2$

Area of the rectangle at the bottom (floor):

$\rightarrow A\ =\ l\ \times\ w = 5cm \times 10cm = 50cm^2$

Total area to be plastered:

$\rightarrow 40cm^2 + 20cm^2 + 50cm^2 = 110cm^2$

Hence, cost of plastering:

$\rightarrow \mathbf{110cm^2 \times 12 = R1320}$

Question 18 example

At a certain point in time the South African rand (R) is converted to the Hong Kong dollar (HKD) at a rate of:

$\mathbf{1HKD = R1.67}$

If Luke has R50 000 when he arrives in Hong Kong, how much will he have to spend in Hong Kong?

Solution

$\rightarrow HKD : Rand$

$\rightarrow 1 : 1.67$

$\rightarrow \frac{1}{1.67} : \frac{1.67}{1.67}$

$\rightarrow \frac{1}{1.67} : 1$

Hence 1 rand is equal to $\frac{1}{1.67}$ HKD.

Therefore, Luke has:

$\rightarrow 50000 \times \frac{1}{1.67} = 29940.12HKD$ to spend in Hong Kong.

Question 19 & 20 example

Thato wants to buy a marble phone cover online. She finds a seller in Hong Kong selling the cover for 6HKD. She also finds another seller in china selling the same cover for 4CNY. When she looks up the exchange rate she finds that 1HKD = R1.54 and 1CNY = R1.88. Shipping is 4HKD and 3CNY respectively.

1. How much will she pay in rands is she buys the cover from the Hong Kong seller?
2. How much will she pay in rands if she buys the cover from the seller in China?

Solution

1. $Total\ purchase\ price = 6HKD + 4HKD = 10HKD$

   $\rightarrow 10 \times 1.54 = R15.40$

2. $Total\ purchase\ price = 4CNY + 3CNY = 7CNY$

$\rightarrow 7 \times 1.88 = R13.60$

Question 21 example

The number of lions in South Africa dropped from 43 000 in 1994 to an estimated 20 000 in 2014. What is the index for the number of lions in 2014 based on the number in 1994?

Solution

$$\rightarrow \frac{20000}{43000} \times 100 = 46.51$$

Question 22 & 23 example

The City Ice and Beverage Store sells a complete line of beer, wine and soft drink products. Listed below are the quantities sold and the prices for each beverage for 2008 and 2010.

Item	2008		2010	
	Price	Quantity	Price	Quantity
Beer	19.00	6000	21.50	5000
Wine	32.00	3000	35.00	5000
Soft drinks	6.00	9000	7.50	11000

Calculate the Paasche price index and the Laspeyres price index for 2010 with 2008 as the base year.

Solution

Item	$p_0 \times q_n$ $p_{2008} \times q_{2010}$	$p_n \times q_n$ $p_{2010} \times q_{2010}$	$p_n \times q_0$ $p_{2010} \times q_{2008}$	$p_0 \times q_0$ $p_{2008} \times q_{2008}$
Beer	19 x 5000 = 95000	21.5 x 5000 = 107500	21.50 x 6000 = 129000	19 x 6000 = 114000
Wine	32 x 5000 = 160000	35 x 5000 = 175000	35 x 3000 = 105000	32 x 3000 = 96000
Soft drinks	6 x 11000 = 66000	7.5 x 11000 = 82500	7.5 x 9000 = 67500	6 x 9000 = 54000

Total	$\sum p_0 q_n$ $= 321000$	$\sum p_n q_n$ $= 365000$	$\sum p_n q_0$ $= 301500$	$\sum p_0 q_0$ $= 264000$

The Paasche price index = $\frac{\sum p_n q_n}{\sum p_0 q_n} \times \mathbf{100} = \frac{\mathbf{365000}}{\mathbf{321000}} \times \mathbf{100} = \mathbf{113.71}$

The Laspeyres price index = $\frac{\sum p_n q_0}{\sum p_0 q_0} \times \mathbf{100} = \frac{\mathbf{301500}}{\mathbf{264000}} \times \mathbf{100} = \mathbf{114.20}$

Question 24 & 25 example

The take-home pay of Mr Gaza and the CPI for 2016 and 2017 are:

Year	Take-Home Pay	CPI
2016	R250 000	102.80
2017	R420 000	111.5

(a) What was Mr Gaza's real income in 2016?

(b) What was his real income in 2017?

Solution

Year	Real income
(a) 2016	$\frac{250\ 000}{102.80}(100) = 243\ 190.66$
(b) 2017	$\frac{420\ 000}{111.5}(100) = 376\ 681.61$