Tutorial Letter 202/1/2017

Elementary Quantitative Methods
QMI1500

Semester 1

Department of Decision Sciences

IMPORTANT INFORMATION:
Solutions to Assignment 02

Bar code

Solutions to compulsory Assignment 02

Question 1

We see that price p is a function of the number of sets N (in thousands), so the slope is given by

$$m = \frac{\text{change in } p}{\text{change in } N} = \frac{-10.40}{1\,000} = -0.0104$$

A point on the line is $(N_1, p_1) = (6485, 504.39)$. We use the point-slope form adapted to the variables N and p.

$$\begin{aligned} p - p_1 &= m(N - N_1) \\ p - 504.39 &= -0.0104(N - 6485) \\ p - 504.39 &= -0.0104N + 67.444 \\ p &= -0.0104N + 571.834 \end{aligned}$$

Answer: Option 1

Question 2

Using the formula $y = mx + c$; for monthly charge the base charge per month is R5.19.

$$\therefore \; c = 5.19$$

Now monthly charge per hundred m^3 is 51.91 cents, which is R0.5191.

$$\begin{aligned} \therefore \; m &= 0.5191 \\ \therefore \; y &= 0.5191x + 5.19 \end{aligned}$$

Answer: Option 1

Question 3

The standard form is $y = -x^2 + 4x + 0$, so $a = -1$. Thus the parabola opens downward, and the vertex is the highest (maximum) point.

The vertex occurs at $x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$.

The y-coordinate of the vertex is $f(2) = -(2)^2 + 4(2) = 4$.

Thus the vertex is (2; 4).

Answer: Option 3

Question 4

The zeros of the function are solution to

$$\begin{aligned} -x^2 + 4x &= 0 \\ x(-x + 4) &= 0 \\ x = 0 \text{ or } x &= 4 \end{aligned}$$

Thus the x-intercepts are $(0; 0)$ and $(4; 0)$. **Answer: Option 2**

Question 5

The revenue function is given by:

$$R(x) = (50 + x)(30 - 0.50x)$$

(see Question 6)

Expanding this function gives a form from which we can find the vertex of its graph:

$$R(x) = 1\,500 + 5x - 0.5x^2$$

The vertex of the graph of this function is $x = \frac{-b}{2a} = \frac{-5}{2(-0.50)} = 5, R(5) = 1512.50$.

This means that the revenue will be maximized at R1 512.50 when $50 + 5 = 55$ people are in the group, with each paying $30 - 0.50(5) = 27.50$ rands.

Answer: Option 3

Question 6

The revenue function is described by:

$$\text{Revenue function}(R) = \text{Price per quantity}(P) \times \text{Total quantity}(Q)$$

In the case of Ace Cruise, the price is R30 per person for a group of 50 people. However, for a group greater than 50 people, the price is reduced by R0.50c for each additional person above 50. The price function can by represented by

$$\text{Price} P(x) = 30 - 0.5x \text{ [where } x \text{ represents the number of additional people above 50}$$

The minimum number of people for the cruise is 50. But the cruise allows additional people. Therefore, the quantity function can be represented by this function.

$$\text{Quantity}(Q(x)) = 50 + x$$

To complete the revenue function $(R(x))$

$$\begin{aligned} R(x) &= P(x) \times Q(x) \\ &= (30 - 0.5x)(50 + x) \\ &= 1\,500 + 30x - 25x - 0.5x^2 \\ &= -0.5x^2 + 5x + 1\,500 \end{aligned}$$

Answer: Option 2

Question 7

$$\begin{aligned} \text{Since } x &= 5, \\ S &= 10\,000(1.005)^{12\times 5} \\ &= 10\,000(1.005)^{60} \\ &= \text{R}13\,488.50 \text{ (nearest cent)} \end{aligned}$$

Answer: Option 3

Question 8

$$\begin{aligned} \text{Since } x &= 30 \\ S &= 10\,000(1.005)^{12\times 30} \\ &= 10\,000(1.005)^{360} \\ &= \text{R}60\,225.75 \text{ (nearest cent)} \end{aligned}$$

Answer: Option 4

Question 9

The boy's height from the formula is $H = 2.31(9) + 31.26 = 52.05$ inches. For a 9-year-old boy to be considered of normal height, H would have to be within $\pm 5\%$ of 52.05 inches. That is, the boy's height H is considered normal if $H \geq 52.5 - (0.05)(52.05)$ and $H \leq 52.05 + (0.05)(52.05)$. We can write this range of normal height by the compound inequality

$$52.05 - (0.05)(52.05) \leq H \leq 52.05 + (0.05)(52.05)$$

or

$$49.45 \leq H \leq 54.65$$

Answer: Option 3

Question 10

If x represents the amount invested at 9% and y represents the amount invested at 8%, then $x + y$ is the total investment,

$$x + y = 200\,000 \quad (1)$$

and $0.09x + 0.08y$ is the total income earned, i.e.

$$0.09x + 0.08y = 17\,200 \quad (2)$$

We solve these equation as follows:

$$\begin{array}{rrll} -8x - 8y = & -1\,600\,000 & (3) & \text{Multiply equation (1) by} -8. \\ \underline{9x + 8y =} & \underline{1\,720\,000} & (4) & \text{Multiply equation (2) by 100.} \\ x = & 120\,000 & & \text{Add (3) and (4).} \end{array}$$

We find y by substituting $x = 120\,000$ in equation (1).

$120\,000 + y = 200\,000$ gives $y = 80\,000$.

Thus R120 000 is invested at 9% and R80 000 is invested at 8%.

As a check, we note that equation (1) is satisfied and

$$0.09(120\,000) + 0.08(80\,000) = 10\,800 + 6\,400 = 17\,200$$

Answer: Option 4

Question 11

The total revenue for x MP3 players is R$50x$, so the total revenue function is $R(x) = 50x$. The fixed costs are R200 000, so the total cost for x players is R$(10x + 200\,000)$. Hence, $C(x) = 10x + 200\,000$. The profit function is given by $P(x) = R(x) - C(x)$. Thus

$$\begin{aligned} P(x) &= 50x - (10x + 200\,000) \\ P(x) &= 40x - 200\,000 \end{aligned}$$

Answer: Option 2

Question 12

$$\begin{aligned} x + y = -1 \text{ means } x = -y - 1, \text{ so } 3x - 4y &= -24 \text{ becomes} \\ 3(-y - 1) - 4y &= -24 \\ -3y - 3 - 4y &= -24 \\ -7y &= -21 \\ y &= 3 \end{aligned}$$

Hence $x = -3 - 1 = -4$, and the solution is $x = -4$, $y = 3$.

Answer: Option 2

Question 13

$$\begin{aligned} -5x+3 &\geq x-15 \\ -6x &\geq -18 \\ x &\leq 3 \end{aligned}$$

Answer: Option 3

Question 14

If x represents the number of acres of corn and y represents the number of acres of soybeans, then the total number of acres planted is limited by $x + y \leq 6\,000$. The limitation on fertilizer/herbicide is given by $9x + 3y \leq 40\,500$, and the labor constraint is given by $\frac{3}{4}x + y \leq 5\,250$. Because the number of acres planted in each crop must be nonnegative, the system of inequalities that describes the constraints is as follows.

$$\begin{array}{rcrcrl} x & + & y & \leq & 6\,000 & (1) \\ 9x & + & 3y & \leq & 40\,500 & (2) \\ \frac{3}{4}x & + & y & \leq & 5\,250 & (3) \\ & & x \geq 0,\ y \geq 0 & & & \end{array}$$

Answer: Option 3

Question 15

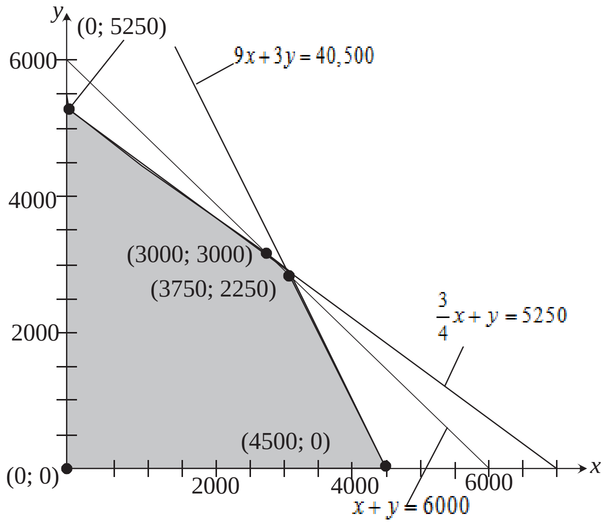
The solution region is shaded in figure below, with three of the corners at $(0;0)$, $(4\,500;0)$, and $(0;5250)$. The corner $(3\,750;2\,250)$ is found by solving the following equations simultaneously, as follows.

$$\begin{cases} 9x + 3y = 40\,500 & (4) \\ x + y = 6\,000 & (5) \end{cases}$$

$$\begin{array}{rcrcrl} 9x & + & 3y & = & 40\,500 & (4) \\ -(3x & + & 3y & = & 18\,000) & [3 \times \text{Eq.}(5)] \\ \hline 6x & & & = & 22\,500 & \end{array}$$

$$x = \frac{22\,500}{6} = 3\,750$$

$$y = 6\,000 - x = 2\,250$$



The corner $(3\,000, 3\,000)$ is found similarly by solving the following equations simultaneously:

$$\begin{cases} x + y = 6\,000 \\ \frac{3}{4}x + y = 5\,250. \end{cases}$$

Answer: Option 4

Question 16

In question 15 you were asked to find the most number of acres, for corn and soybeans, that the farm could plant treat with fertilizer and harvest. It is at these maximum points that the farm will realize maximum profits.

In question 15 it was found that the maximum acres were 3 750 acres for corn and 2 250 soybeans.

The profit function can be written as:

$$P = 60x + 40y$$

but

$$x = 3\,750 \text{ (number of acres of corn)}$$

and

$$y = 2\,250 \text{ (number of acres of soybeans).}$$

Therefore

$$\begin{aligned} P &= 60 \times 3\,750 + 40 \times 2\,250 \\ &= 315\,000 \end{aligned}$$

Maximum profit is R315 000.

Answer: Option 3

Question 17

The marginal profit function is

$$\bar{M'}P = P'(x) = 20 \cdot \frac{1}{2}(x+1)^{-1/2} - 2 = \frac{10}{\sqrt{x+1}} - 2$$

Answer: Option 4

Question 18

If 15 units are produced, the marginal profit is

$$P'(15) = \frac{10}{\sqrt{15+1}} - 2 = \frac{10}{4} - 2 = \frac{1}{2}$$

This means that the profit from the sale of the 16th unit is approximately $\frac{1}{2}$ (thousand dollars), or R500.

Answer: Option 1

Question 19

$$\bar{M'}C \quad = \quad C'(x) = 0.03x^2 - 1.8x + 33$$

Answer: Option 3

Question 20

$$C'(50) = 18$$

Hence the marginal cost is R18 is per unit.

Answer: Option 2