Question 1: Example

Consider the following sequence:

$$x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 + x_5y_5$$

Write it in summation notation.

Answer

Tips:

- First notice that the x and y subscripts change from 1 to 5 in the sequence.
- Also notice that the subscript is the only thing that changes in the sequence while all else remains the same
- Writing something in summation notation means using the summation symbol $\sum_{i=1}^{n}$ or $\sum_{i=0}^{n}$ or $\sum_{i=-1}^{n}$ depending on what changing number/subscript your given sequence starts from.

✓ So we already know that our "i" starts at 1 and ends at 5. Meaning our symbol will look something like this:

$\sum_{i=1}^{5}$

✓ From the summation symbol we know that the "i" represents the subscripts that are changing meaning that the general term of our sequence is x_iy_i

✓ Putting these two together, we finally get:

$$\sum_{i=1}^{5} x_iy_i$$

Question 2: Example

The number of girls studying at UNISA in second semester has increased in the ratio 3:2. If the number of girls at the start of first semester were only 20, how many of girls do we have now?

Answer

Tips:

- Another way of writing a ratio is as a fraction.

e.g. $5 : 2 \rightarrow \frac{5}{2} = 2.5$

- Sometimes they can give the ratio as words:

e.g. the ratio of 5 buttons to 2 shirts → $\mathbf{5:2} \rightarrow \frac{5}{2} = \mathbf{2.5}$

- ✓ Convert your ratio to a fraction:

  $$\mathbf{3:2} \rightarrow \frac{\mathbf{3}}{\mathbf{2}} = \mathbf{1.5}$$

- ✓ The question says the number has increased by this ratio. When something increases by something, we must multiply the two:

  So we get: $\mathbf{20 \times 1.5 = 30}$

The number of girls in second semester has now increased to 30.

<u>Question 3: Example</u>

How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of 3 faculty members from the mathematics department and 4 from the computer science(CS) department, if there are 9 faculty members of the math department and 11 of the CS department?

<u>Answer</u>

<u>Tips:</u>

- First consider if the order is important, if it is then we know to use a permutation and if it is not then we know to use a combination.
- Consider the number of ways to choose the faculty members from the mathematics department, and then consider the number of ways to choose the faculty members from the CS department

- ✓ We can pick the 3 faculty members out of the total 9 from the maths department as follows:

  $${}_{9}\boldsymbol{C}_{3} = \frac{\mathbf{9!}}{\mathbf{(9-3)! \times 3!}} = \mathbf{84}$$

- ✓ We can pick the 4 faculty members out of the total 11 from the CS department as follows:

  $${}_{11}\boldsymbol{C}_{4} = \frac{\mathbf{11!}}{\mathbf{(11-4)! \times 4!}} = \mathbf{330}$$

Therefore, there are:

$${}_{9}\boldsymbol{C}_{3} \times {}_{11}\boldsymbol{C}_{4} = \frac{\mathbf{9!}}{\mathbf{(9-3)! \times 3!}} \times \frac{\mathbf{11!}}{\mathbf{(11-4)! \times 4!}} = \mathbf{84 \times 330 = 27720}\ \boldsymbol{ways}$$

Question 4: Example

On each delivery, a postman delivers letters to 3 of the 12 suburbs in his territory. In how many different ways can he schedule his route?

Answer

Tips:

- The order in which he delivers to the suburbs is not mentioned, meaning that this is a combination

✓ The number of ways in which he can schedule his route is:

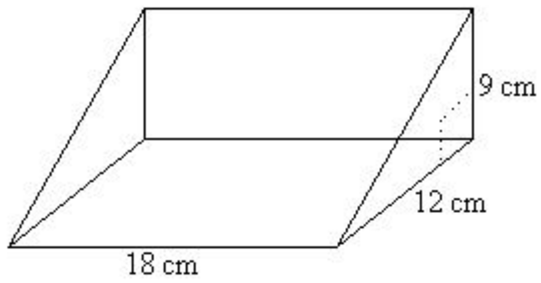
$$_{12}C_3 = \frac{12!}{(12-3)!\times 3!} \times \frac{12!}{9!\times 3!} = 220\ ways$$

Question 5 & 6: Examples

Please see the hints posted on the main QMI1500 site posted by the lecturer before proceeding to the examples below.

Example 1

Find the volume of the triangular prism shown in the diagram:



Solution:

✓ $Volume\ of\ a\ prism = Area\ of\ base \times height$

✓ The base is a triangle, so we must find the area of the triangle which is:

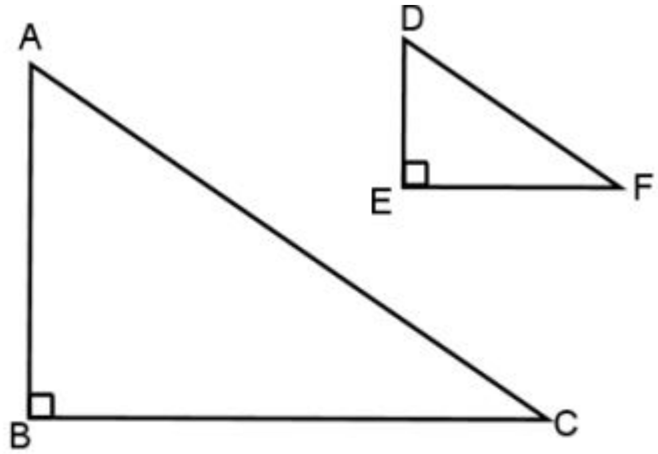
$$A = \frac{1}{2}\times base \times height = \frac{1}{2}\times 9\times 12 = 54cm^2$$

Hence,

$$V = A\times h = 54\times 18 = 972cm^3$$

Example 2

Consider the following two similar triangles:



If $AB = 6, DE = 3$ $and\ the\ area\ of\ DEF = 6cm^2$. What is the area of ABC

Solution:

- ✓ The ratio of the given sides is:

$$\frac{\boldsymbol{AB}}{\boldsymbol{DE}} = \frac{\mathbf{6}}{\mathbf{3}} = \mathbf{2}$$

- ✓ If you follow the hints posted by the lecturer you will find out that if ratio of the sides of two similar triangles is $\boldsymbol{x}$ then the ratio of the areas of the triangles is $\boldsymbol{x^2}$

Therefore,

$$\boldsymbol{Area\ of\ ABC = 2^2 \times Area\ of\ DEF}$$

$$\boldsymbol{= 4 \times 6 = 24cm^2}$$

Question 7: Example

There are 3 grade 12 classes at John's high school, class one has 50 students, class two has 60 students and class 3 has 80 students. An examiner decides to take a sample of $\frac{1}{2}$ of the entire grade 12 to test the new syllabus. All classes must be represented in the sample. Show how many students will be sampled in each class

Answer

- ✓ $\boldsymbol{Total\ grade\ 12\ students = 50 + 60 + 80 = 190}$
- ✓ $\boldsymbol{Proportional\ sample\ size = 190 \times 0.5 = 95}$
- ✓ Representation of each class will be:

$$\boldsymbol{Class\ 1 = \frac{50}{190} \times 95 = 25}$$

$$Class\ 2 = \frac{60}{190} \times 95 = 30$$

$$Class\ 3 = \frac{80}{190} \times 95 = 40$$

✓ We can check if this is correct by adding the values:

$$25 + 30 + 40 = 95$$

✓ This is 0.5 of grade 12.

Question 8-15 Example

The manager of Pies for a Palace keeps record of the number of each type of pie sold each day. The number of pepper steak pies sold for the past 7 days is:

Day	1	2	3	4	5	6	7
Number	26	35	45	28	73	52	37

Calculate the following:

1. Average
2. Median
3. Mode
4. Standard deviation
5. Quartile deviation

Answer

Tip: Always remember to arrange the data in ascending order (from small to large)

✓ 26 28 35 37 45 52 73

1. **The average is also called the mean**. It is calculated as:

$$\bar{x} = \frac{\sum x}{n}$$

$$\rightarrow where \sum x \text{ is the sum of all your observations (data)}$$

$$\rightarrow and\ n \text{ the number of your observations}$$

$$\bar{x} = \frac{\sum x}{n} = \frac{26 + 28 + 35 + 37 + 45 + 52 + 73}{7}$$

$$= \frac{296}{7} = 42.29$$

2. Determine the position of **the median** first:

- There are 7 values in the dataset, thus n =7. The position of the median is determined as:

$$\frac{n+1}{2} = \frac{8}{2} = 4$$

- The median is the value in the 4th position after arranging the data. Count up to value number 4 on the data.

$$Value\ number\ 4 = 37$$

- **The median** is therefore **37.**

3. **The mode** is the value that appears most often. This data set does not have a mode.
4. **The standard deviation is the square root of the variance,** given by**:**

$$S^2 = \frac{\sum(x-\bar{x})^2}{n-1} = \frac{(26-42.29)^2 + (28-42.29)^2 + \cdots + (73-42.29)^2}{7-1}$$

$$= \frac{1595.43}{6} = 265.90$$

Hence, standard deviation = $\sqrt{S^2} = \sqrt{265.90} = 16.30$

5. **The quartile deviation** is given by:

$$Q_D = \frac{Q_3 - Q_1}{2}$$

$$where\ Q_1 = \frac{1}{4}(n+1)\ \ and\ Q_3 = \frac{3}{4}(n+1)$$

$$\rightarrow Q_1 = \frac{1}{4}(n+1) = \frac{1}{4}(7+1) = \frac{8}{4} = 2$$

$$\rightarrow Q_3 = \frac{3}{4}(n+1) = \frac{3}{4}(7+1) = \frac{24}{4} = 6$$

- The first quartile is **the value in the 2nd position** after arrangement and the third quartile is the **value in the 6th position**. Count up to value number 2 and 4 on the data.

$$\rightarrow Value\ number\ 2 = 28$$

$$\rightarrow Value\ number\ 6 = 52$$

$$Hence, Q_D = \frac{Q_3 - Q_1}{2} = \frac{52-28}{2} = 12$$

Question 16 and 17 example

The City Ice and Beverage Store sells a complete line of beer, wine and soft drink products. Listed below are the quantities sold and the prices for each beverage for 2008 and 2010.

Item	**2008**		**2010**	
	Price	**Quantity**	**Price**	**Quantity**
Beer	19.00	6000	21.50	5000
Wine	32.00	3000	35.00	5000
Soft drinks	6.00	9000	7.50	11000

Calculate the the Paasche price index and the Laspeyres price index for 2010 with 2008 as the base year.

Solution

Item	$p_0 \times q_n$ $p_{2008} \times q_{2010}$	$p_n \times q_n$ $p_{2010} \times q_{2010}$	$p_n \times q_0$ $p_{2010} \times q_{2008}$	$p_0 \times q_0$ $p_{2008} \times q_{2008}$
Beer	19 x 5000 = 95000	21.5 x 5000 = 107500	21.50 x 6000 = 129000	19 x 6000 = 114000
Wine	32 x 5000 = 160000	35 x 5000 = 175000	35 x 3000 = 105000	32 x 3000 = 96000
Soft drinks	6 x 11000 = 66000	7.5 x 11000 = 82500	7.5 x 9000 = 67500	6 x 9000 = 54000
Total	$\sum p_0 q_n = \mathbf{321000}$	$\sum p_n q_n = \mathbf{365000}$	$\sum p_n q_0 = \mathbf{301500}$	$\sum p_0 q_0 = \mathbf{264000}$

The Paasche price index $= \frac{\sum p_n q_n}{\sum p_0 q_n} \times \mathbf{100} = \frac{\mathbf{365000}}{\mathbf{321000}} \times \mathbf{100} = \mathbf{113.71}$

The Laspeyres price index $= \frac{\sum p_n q_0}{\sum p_0 q_0} \times \mathbf{100} = \frac{\mathbf{301500}}{\mathbf{264000}} \times \mathbf{100} = \mathbf{114.20}$

Question 18 example

The number of items produced by Goodness Co. Products for 2000 and 2008 and the wholesale prices for the two periods are:

Item produced	Price (Rands)		Number Produced	
	2000	**2008**	**2000**	**2008**
Laptops	3	4	10 000	9000
Maps	1	5	600	200
Calendars	10	8	3000	5000

Find the index of the value of production for 2008 using 200 as the base period.

Answer

Item	$p_n \times q_n$ $p_{2008} \times q_{2008}$	$p_0 \times q_0$ $p_{2000} \times q_{2000}$
Notepads	$4 \times 9000 = 36000$	$3 \times 10000 = 30000$
Keyboards	$5 \times 200 = 1000$	$1 \times 600 = 600$
Printers	$8 \times 5000 = 40000$	$10 \times 3000 = 30000$
Total	$\sum p_n q_n = 77000$	$\sum p_0 q_0 = 60600$

The value index $= \frac{\sum p_n q_n}{\sum p_0 q_0} \times 100 = \frac{77000}{60600} \times 100 = 127.06$

Question 19 example

Suppose the Consumer Price Index this month is 125.0 (2002 as 100). What is the purchasing power of the dollar?

Answer

The Consumer Price Index (CPI) is used to determine the purchasing power of any currency.

$$\textit{Purchasing power of the dollar} = \frac{\$1}{125}(100) = \$0.80$$

Question 20 example

At a certain point in time the South African rand (R) is converted to the Hong Kong dollar (HKD) at a rate of

$$1.67HKD = R1.00$$

If Luke has R50 000 when he arrives in Hong Kong, how much will he have to spend in Hong Kong?

Answer

$\boldsymbol{Rand : HKD}$

$\mathbf{1 : 1.67}$

$$\frac{\mathbf{1}}{\mathbf{1.67}} : \frac{\mathbf{1.67}}{\mathbf{1.67}}$$

$$\frac{\mathbf{1}}{\mathbf{1.67}} : \mathbf{1}$$

Hence **1** rand is equal to $\frac{\mathbf{1}}{\mathbf{1.67}}$ HKD. This is our exchange rate.

Therefore, Luke has:

$$\mathbf{50000} \times \frac{\mathbf{1}}{\mathbf{1.67}} = \mathbf{29940.12}\boldsymbol{HKD}$$

Question 21 example

The take-home pay of Mr Gaza and the CPI for 2003 and 2007 are:

Year	Take-Home Pay	CPI
2016	R250 000	102.80
2017	R420 000	111.5

(a) What was Mr Gaza's real income in 2016?

(b) What was his real income in 2017?

(c) Interpret your findings

Answer

Year	Real income
(a) 2016	$\frac{\mathbf{250\,000}}{\mathbf{102.80}}(\mathbf{100}) = \mathbf{243\,190.66}$
(b) 2017	$\frac{\mathbf{420\,000}}{\mathbf{111.5}}(\mathbf{100}) = \mathbf{376\,681.61}$

(c) Mr Gaza's take-home pay increased by:

$\mathbf{376\,681.61 - 243\,190.66} = \boldsymbol{R}\,\mathbf{133\,490.95}$