CHAPTER 14 ALTERNATING VOLTAGES AND CURRENTS

Exercise 77, Page 218

1. Determine the periodic time for the following frequencies: (a) 2.5 Hz  (b) 100 Hz  (c) 40 kHz

(a) Periodic time, \( T = \frac{1}{f} = \frac{1}{2.5} = 0.4 \text{ s} \)

(b) Periodic time, \( T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s} \) or \( 10 \text{ ms} \)

(c) Periodic time, \( T = \frac{1}{f} = \frac{1}{40 \times 10^3} = 25 \mu\text{s} \)

2. Calculate the frequency for the following periodic times: (a) 5 ms  (b) 50 \( \mu\text{s} \)  (c) 0.2 s

(a) Frequency, \( f = \frac{1}{T} = \frac{1}{5 \times 10^{-3}} = 200 \text{ Hz} \) or \( 0.2 \text{ kHz} \)

(b) Frequency, \( f = \frac{1}{T} = \frac{1}{50 \times 10^{-6}} = 20 \text{ kHz} \)

(c) Frequency, \( f = \frac{1}{T} = \frac{1}{0.2} = 5 \text{ Hz} \)

3. An alternating current completes 4 cycles in 5 ms. What is its frequency?

Time for one cycle, \( T = \frac{5}{4} \text{ ms} = 1.25 \text{ ms} \)

Hence, frequency, \( f = \frac{1}{T} = \frac{1}{1.25 \times 10^{-3}} = 800 \text{ Hz} \)
Exercise 78, Page 221

1. An alternating current varies with time over half a cycle as follows:

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>0</th>
<th>0.7</th>
<th>2.0</th>
<th>4.2</th>
<th>8.4</th>
<th>8.2</th>
<th>2.5</th>
<th>1.0</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (ms)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

The negative half cycle is similar. Plot the curve and determine: (a) the frequency (b) the instantaneous values at 3.4 ms and 5.8 ms (c) its mean value, and (d) its r.m.s. value.

The graph is shown plotted below.

(a) Periodic time, \( T = 2 \times 10 \text{ ms} = 20 \text{ ms} \), hence, frequency, \( f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz} \)

(b) At 3.4 ms, current, \( i = 5.5 \text{ A} \)

and at 5.8 ms, \( i = 3.1 \text{ A} \)

(c) Mean value = \( \frac{\text{area under curve}}{\text{length of base}} \) \( \text{Using the mid-ordinate rule,} \)

\[
\text{area under curve} = (1 \times 10^{-3})(0.3 + 1.4 + 3.1 + 6.0 + 8.8 + 5.5 + 1.6 + 0.8 + 0.3 + 0.2)
\]

\[
= (1 \times 10^{-3})(28) = 28 \times 10^{-3}
\]
Hence, mean value = \( \frac{28 \times 10^{-3}}{10 \times 10^{-3}} = 2.8 \, \text{A} \)

(d) r.m.s. value = \( \sqrt{\frac{0.3^2 + 1.4^2 + 3.1^2 + 6.0^2 + 8.8^2 + 5.5^2 + 1.6^2 + 0.8^2 + 0.3^2 + 0.2^2}{10}} \)

= \( \sqrt{\frac{158.68}{10}} = 3.98 \, \text{A} \) or \( 4.0 \, \text{A} \), correct to 2 significant figures.

2. For the waveforms shown below, determine for each (i) the frequency (ii) the average value over half a cycle (iii) the r.m.s. value (iv) the form factor (v) the peak factor.

(a) (i) \( T = 10 \, \text{ms} \), hence, frequency, \( f = \frac{1}{T} = \frac{1}{10 \times 10^{-3}} = 100 \, \text{Hz} \)

(ii) Average value = \( \frac{\text{area under curve}}{\text{length of base}} = \frac{1}{2} \left( 5 \times 10^{-3} \right) \left( 5 \right) = 2.50 \, \text{A} \)
(iii) R.m.s. value = \( \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + i_4^2}{5}} \) = \( \sqrt{\frac{0.5^2 + 1.5^2 + 2.5^2 + 3.5^2 + 4.5^2}{5}} \) = 2.87 A

(iv) Form factor = \( \frac{\text{r.m.s.}}{\text{average}} \) = \( \frac{2.87}{2.50} \) = 1.15

(v) Peak factor = \( \frac{\text{maximum value}}{\text{r.m.s.}} \) = \( \frac{5}{2.87} \) = 1.74

(b) (i) \( T = 4 \) ms, hence, frequency, \( f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = 250 \) Hz

(ii) Average value = \( \frac{\text{area under curve}}{\text{length of base}} = \frac{20 \times 2}{2} = 20 \) V

(iii) R.m.s. value = \( \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}} \) = \( \sqrt{\frac{20^2 + 20^2 + 20^2 + 20^2}{4}} \) = 20 V

(iv) Form factor = \( \frac{\text{r.m.s.}}{\text{average}} \) = \( \frac{20}{20} \) = 1.0

(v) Peak factor = \( \frac{\text{maximum value}}{\text{r.m.s.}} \) = \( \frac{20}{20} \) = 1.0

(c) (i) \( T = 8 \) ms, hence, frequency, \( f = \frac{1}{T} = \frac{1}{8 \times 10^{-3}} = 125 \) Hz

(ii) Average value = \( \frac{\text{area under curve}}{\text{length of base}} = \frac{\left(\frac{1}{2} \times 1 \times 24\right) + (2 \times 24) + \left(\frac{1}{2} \times 1 \times 24\right)}{4} \) = \( \frac{72}{4} = 18 \) A

(iii) R.m.s. value = \( \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + i_4^2 + \ldots}{8}} \)

\[ = \sqrt{\frac{3^2 + 9^2 + 15^2 + 21^2 + 24^2 + 24^2 + 24^2 + 24^2}{8}} = 19.56 \] A
(iv) Form factor = \[ \frac{\text{r.m.s.}}{\text{average}} = \frac{19.56}{18} = 1.09 \]

(v) Peak factor = \[ \frac{\text{maximum value}}{\text{r.m.s.}} = \frac{24}{19.56} = 1.23 \]

(d) (i) \[ T = 4 \text{ ms}, \text{ hence, frequency, } f = \frac{1}{T} = \frac{1}{4 \times 10^{-3}} = 250 \text{ Hz} \]

(ii) Average value = \[ \frac{\text{area under curve}}{\text{length of base}} = \frac{0.5 \times 100}{2} = 25 \text{ V} \]

(iii) R.m.s. value = \[ \sqrt{\left(\frac{v_1^2 + v_2^2 + v_1^2 + v_2^2}{4}\right)} = \sqrt{\left(\frac{0^2 + 2^2 + 100^2 + 0^2}{4}\right)} = 50 \text{ V} \]

(iv) Form factor = \[ \frac{\text{r.m.s.}}{\text{average}} = \frac{50}{25} = 2.0 \]

(v) Peak factor = \[ \frac{\text{maximum value}}{\text{r.m.s.}} = \frac{100}{50} = 2.0 \]

3. An alternating voltage is triangular in shape, rising at a constant rate to a maximum of 300 V in 8 ms and then falling to zero at a constant rate in 4 ms. The negative half cycle is identical in shape to the positive half cycle. Calculate (a) the mean voltage over half a cycle, and (b) the r.m.s. voltage

The voltage waveform is shown below.
(a) Average value = \[ \frac{\text{area under curve}}{\text{length of base}} = \frac{1}{2} (8 \times 10^{-3}) (300) + \frac{1}{2} (4 \times 10^{-3}) (300) = 150 \text{ V} \]

(b) R.m.s. value = \[ \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2}{6}} = \sqrt{\frac{37.5^2 + 112.5^2 + 18.5^2 + 262.5^2 + 225^2 + 75^2}{6}} = 170 \text{ V} \]

4. An alternating e.m.f. varies with time over half a cycle as follows:

<table>
<thead>
<tr>
<th>E.m.f. (V)</th>
<th>0</th>
<th>45</th>
<th>80</th>
<th>155</th>
<th>215</th>
<th>320</th>
<th>210</th>
<th>95</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (ms)</td>
<td>0</td>
<td>1.5</td>
<td>3.0</td>
<td>4.5</td>
<td>6.0</td>
<td>7.5</td>
<td>9.0</td>
<td>10.5</td>
<td>12.0</td>
</tr>
</tbody>
</table>

The negative half cycle is identical in shape to the positive half cycle. Plot the waveform and determine

(a) the periodic time and frequency (b) the instantaneous value of voltage at 3.75 ms (c) the times when the voltage is 125 V (d) the mean value, and (e) the r.m.s. value

The waveform is shown plotted below.
(a) Half the waveform is shown, hence periodic time, \( T = 2 \times 12.0 \text{ ms} = 24 \text{ ms} \)

Frequency, \( f = \frac{1}{T} = \frac{1}{24 \times 10^{-3}} = 41.67 \text{ Hz} \)

(b) The instantaneous value of voltage at 3.75 ms = 115 V

(c) The times when the voltage is 125 V = 4 ms and 10.0 ms

(d) Mean value = \( \frac{\text{area under curve}}{\text{length of base}} \)

Using the mid-ordinate rule with 12 intervals,

\[
\text{area under curve} = \left(1 \times 10^{-3}\right) \left(15 + 45 + 68 + 100 + 145 + 190 + 250 + 320 + 260 + 160 + 95 + 25\right)
\]

\[
= \left(1 \times 10^{-3}\right) \left(1673\right) = 1.673
\]

Hence, mean value = \( \frac{1.673}{12 \times 10^{-3}} = 139 \text{ V} \)

(e) R.m.s. value = \( \sqrt{\frac{\left(15^2 + 45^2 + 68^2 + 100^2 + 145^2 + 190^2 + 250^2 + 320^2 + 260^2 + 160^2 + 95^2 + 25^2\right)}{12}} \)

\[
= \sqrt{\frac{341749}{12}} = 169 \text{ V}
\]

5. Calculate the r.m.s. value of a sinusoidal curve of maximum value 300 V.

R.m.s. value = \( 0.707 \times \text{peak value} = 0.707 \times 300 = 212.1 \text{ V} \)

6. Find the peak and mean values for a 200 V mains supply.

200 V is the r.m.s. value

r.m.s. value = \( 0.707 \times \text{peak value} \), from which, peak value = \( \frac{\text{r.m.s.}}{0.707} = \frac{200}{0.707} = 282.9 \text{ V} \)

Mean value = \( 0.637 \times \text{peak value} = 0.637 \times 282.9 = 180.2 \text{ V} \)

7. Plot a sine wave of peak value 10.0 A. Show that the average value of the waveform is 6.37 A over half a cycle, and that the r.m.s. value is 7.07 A
A sine wave of maximum value 10.0 A is shown below.

Over half a cycle, mean value = \frac{\text{area under curve}}{\text{length of base}}

Using the mid-ordinate rule with 12 intervals,

\text{area under curve} = \left( \frac{\pi}{6} \right) (1.3 + 3.8 + 6.1 + 7.9 + 9.2 + 9.9 + 9.9 + 9.2 + 7.9 + 6.1 + 3.8 + 1.3)

= \left( \frac{\pi}{12} \right) (76.4) = 20.0

Hence, mean value = \frac{20.0}{\pi} = 6.37 \text{ A}

\text{R.m.s. value} = \sqrt{\frac{1.3^2 + 3.8^2 + 6.1^2 + 7.9^2 + 9.2^2 + 9.9^2 + 9.9^2 + 9.2^2 + 7.9^2 + 6.1^2 + 3.8^2 + 1.3^2}{12}}

= \sqrt{\frac{596.8}{12}} = 7.05 \text{ A}

With a larger scale and taking values to greater than 1 decimal place, it may be shown that the r.m.s. value is 7.07 A

8. A sinusoidal voltage has a maximum value of 120 V. Calculate its r.m.s. and average values.
**R.m.s. value** = 0.707 \times \text{peak value} = 0.707 \times 120 = 84.8 \text{ V}

**Average value** = 0.637 \times \text{peak value} = 0.637 \times 120 = 76.4 \text{ V}

9. A sinusoidal current has a mean value of 15.0 A. Determine its maximum and r.m.s. values.

Mean value = 0.637 \times \text{maximum value},

from which,  \text{maximum value} = \frac{\text{mean value}}{0.637} = \frac{15.0}{0.637} = 23.55 \text{ A}

**R.m.s. value** = 0.707 \times \text{maximum value} = 0.707 \times 23.55 = 16.65 \text{ A}
Exercise 79, Page 224

1. An alternating voltage is represented by $v = 20 \sin 157.1t$ volts. Find (a) the maximum value (b) the frequency (c) the periodic time. (d) What is the angular velocity of the phasor representing this waveform?

(a) **Maximum value** = 20 V

(b) $157.1 = \omega = 2\pi f$, from which, frequency, $f = \frac{157.1}{2\pi} = 25$ Hz

(c) **Periodic time**, $T = \frac{1}{f} = \frac{1}{25} = 0.04$ s or 40 ms

(d) **Angular velocity** = 157.1 rad/s

2. Find the peak value, the r.m.s. value, the frequency, the periodic time and the phase angle (in degrees and minutes) of the following alternating quantities:

(a) $v = 90 \sin 400\pi t$ volts  (b) $i = 50 \sin(100\pi t + 0.30)$ amperes  
(c) $e = 200 \sin(628.4t - 0.41)$ volts

(a) **Peak value** = 90 V

**R.m.s. value** = $0.707 \times$ peak value = $0.707 \times 90 = 63.63$ V

$400\pi = \omega = 2\pi f$, from which, frequency, $f = \frac{400\pi}{2\pi} = 200$ Hz

**Periodic time**, $T = \frac{1}{f} = \frac{1}{200} = 5$ ms

**Phase angle** = 0°

(b) **Peak value** = 50 A

**R.m.s. value** = $0.707 \times$ peak value = $0.707 \times 50 = 35.35$ A

$100\pi = \omega = 2\pi f$, from which, frequency, $f = \frac{100\pi}{2\pi} = 50$ Hz
Periodic time, \( T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s} \) or \( 20 \text{ ms} \)

Phase angle = 0.30 radians = \( 0.3 \times \frac{180}{\pi} = 17.19^\circ \) leading

(c) Peak value = 200 V

R.m.s. value = \( 0.707 \times \) peak value = \( 0.707 \times 200 = 141.4 \text{ V} \)

\[ 628.4 = \omega = 2\pi f, \] from which, frequency, \( f = \frac{628.4}{2\pi} = 100 \text{ Hz} \)

Periodic time, \( T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s} \) or \( 10 \text{ ms} \)

Phase angle = 0.41 radians = \( 0.41 \times \frac{180}{\pi} = 23.49^\circ \) lagging

3. A sinusoidal current has a peak value of 30 A and a frequency of 60 Hz. At time \( t = 0 \), the current is zero. Express the instantaneous current \( i \) in the form \( i = I_m \sin \omega t \).

\[ i = 30 \sin \left[ 2\pi(60)t + \phi \right] \]

If \( t = 0 \) when \( i = 0 \), thus \( 0 = 30 \sin \phi \) i.e. \( 0 = \sin \phi \)

from which, \( \phi = \sin^{-1} 0 = 0 \)

Hence, \( i = 30 \sin 120\pi t \text{ A} \)

4. An alternating voltage \( v \) has a periodic time of 20 ms and a maximum value of 200 V. When time \( t = 0 \), \( v = -75 \) volts. Deduce a sinusoidal expression for \( v \) and sketch one cycle of the voltage showing important points.

Frequency, \( f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz} \)

Hence, \( v = 200 \sin \left[ 2\pi(50)t + \phi \right] = 200 \sin (100\pi t + \phi) \)

If \( t = 0 \) when \( v = -75 \), thus \( -75 = 200 \sin \phi \)

from which, \( \phi = \sin^{-1} \left( -\frac{75}{200} \right) = -0.384 \)
Hence, \[ v = 200 \sin(100\pi t - 0.384) \text{ volts} \]

5. The voltage in an alternating current circuit at any time \( t \) seconds is given by \( v = 60 \sin 40t \text{ volts} \). Find the first time when the voltage is (a) 20 V  (b) -30 V

Voltage, \( v = 60 \sin 40t \text{ volts} \)

(a) When \( v = 20 \text{ V} \), \( 20 = 60 \sin 40t \)

from which, \( \frac{20}{60} = \sin 40t \) and \( 40t = \sin^{-1}\left( \frac{20}{60} \right) = 0.3398 \)

Hence, time, \( t = \frac{0.3398}{40} = 8.496 \times 10^{-3} \text{ s} = 8.496 \text{ ms} \)

(b) When \( v = -30 \text{ V} \), \( -30 = 60 \sin 40t \)

from which, \( \frac{-30}{60} = \sin 40t \) and \( 40t = \sin^{-1}\left( \frac{-30}{60} \right) \)

Sine is negative in the 3\(^{rd}\) and 4\(^{th}\) quadrants as shown in the diagram.

\[
\sin^{-1}\left( \frac{30}{60} \right) = 0.5236 \text{ rad and the first time this occurs is in the 3\(^{rd}\) quadrant. Measuring from zero, the angle is } \pi + 0.5236 = 3.6652 \text{ rad} \\
\text{Hence, time, } t = \frac{3.6652}{40} = 0.09163 \text{ s} = 91.63 \text{ ms} \]
6. The instantaneous value of voltage in an a.c. circuit at an time t seconds is given by

\[ v = 100 \sin(50\pi t - 0.523) \text{ V}. \]

Find:

(a) the peak-to-peak voltage, the frequency, the periodic time and the phase angle
(b) the voltage when t = 0
(c) the voltage when t = 8 ms
(d) the times in the first cycle when the voltage is 60 V
(e) the times in the first cycle when the voltage is –40 V, and
(f) the first time when the voltage is a maximum.

Sketch the curve for one cycle showing relevant points

(a) **Peak to peak voltage** = 2 \times \text{maximum value} = 2 \times 100 = \textbf{200 V}

\[ 50\pi = \omega = 2\pi f, \quad \text{from which, frequency, } f = \frac{50\pi}{2\pi} = 25 \text{ Hz} \]

Periodic time, \( T = \frac{1}{f} = \frac{1}{25} = 0.04 \text{ s or } 40 \text{ ms} \)

Phase angle = 0.523 rad lagging = 0.523 \times \frac{180}{\pi} = \textbf{29.97° lagging} or \textbf{29°58'} lagging

(b) When \( t = 0, \quad v = 100 \sin[50\pi(0) - 0.523] = -49.95 \text{ V} \)

(c) When \( t = 8 \text{ ms}, \quad v = 100 \sin[50\pi(8 \times 10^{-3}) - 0.523] \]

\[ = 100 \sin 0.7336 = \textbf{66.96 V} \]

(d) When \( v = 60 \text{ V}, \quad 60 = 100 \sin[50\pi t - 0.523] \)

from which, \[ \frac{60}{100} = \sin[50\pi t - 0.523] \]

i.e. \[ 50\pi t - 0.523 = \sin^{-1} 0.60 = 0.6435 \quad \text{or} \quad \pi - 0.6435 \quad \text{(sine is positive in the 1st and 2nd quadrants, as shown)} \]
Hence, \( 50\pi t = 0.6435 + 0.523 \) and \( t = \frac{0.6435 + 0.523}{50\pi} = 7.426 \text{ ms} \)

or \( 50\pi t = \pi - 0.6435 + 0.523 \) and \( t = \frac{\pi - 0.6435 + 0.523}{50\pi} = 19.23 \text{ ms} \)

(e) When \( v = -40 \text{ V}, \quad -40 = 100 \sin[50\pi t - 0.523] \)

from which, \( \frac{-40}{100} = \sin[50\pi t - 0.523] \)

i.e. \( 50\pi t - 0.523 = \sin^{-1}(-0.40) = \pi + 0.4115 \) or \( 2\pi - 0.4115 \) (sine is negative in the 3\text{rd} and 4\text{th} quadrants, as shown)

Hence, \( 50\pi t = \pi + 0.4115 + 0.523 \) and \( t = \frac{\pi + 0.4115 + 0.523}{50\pi} = 25.95 \text{ ms} \)

or \( 50\pi t = 2\pi - 0.4115 + 0.523 \) and \( t = \frac{2\pi - 0.4115 + 0.523}{50\pi} = 40.71 \text{ ms} \)

(f) The first time when the voltage is a maximum is when \( v = 100 \text{ V} \)

i.e. \( 100 = 100 \sin[50\pi t - 0.523] \)

i.e. \( 1 = \sin[50\pi t - 0.523] \)

i.e. \( 50\pi t - 0.523 = \sin^{-1}1 = 1.5708 \)

from which, \( t = \frac{1.5708 + 0.523}{50\pi} = 13.33 \text{ ms} \)
A sketch of $v = 100 \sin(50\pi t - 0.523)$ is shown below.
Exercise 80, Page 227

1. The instantaneous values of two alternating voltages are given by \( v_1 = 5 \sin \omega t \) and

\[ v_2 = 8 \sin \left( \omega t - \frac{\pi}{6} \right) \]

By plotting \( v_1 \) and \( v_2 \) on the same axes, using the same scale, over one cycle, obtain expressions for (a) \( v_1 + v_2 \)  (b) \( v_1 - v_2 \)

[Diagram with labeled curves showing \( v_1 \), \( v_2 \), and \( v_1 \pm v_2 \) against time.]
(a) From the sketched graphs above,  \( v_1 + v_2 = 12.6\sin(\omega t - 0.32) \)

(b) From the sketched graphs above,  \( v_1 - v_2 = 4.4\sin(\omega t + 2) \)

2. Repeat Problem 1 by calculation.

(a) The relative positions of  \( v_1 \)  and  \( v_2 \)  at time  \( t = 0 \)  are shown as phasors in diagram (i).

The phasor diagram is shown in diagram (ii). Using the cosine rule,

\[
(ac)^2 = 5^2 + 8^2 - 2(5)(8)\cos 150^\circ
\]

from which,  \( ac = 12.58 \)

Using the sine rule,

\[
\frac{8}{\sin \phi} = \frac{12.58}{\sin 150^\circ}
\]

from which,  \( \sin \phi = \frac{8\sin 150^\circ}{12.58} = 0.317965 \)

and  \( \phi = \sin^{-1} 0.317965 = 18.54^\circ \)  or  \( 0.324 \) radians

Hence,  \( v_1 + v_2 = 12.58\sin(\omega t - 0.324) \)

(b) The relative positions of  \( v_1 \)  and  \( v_2 \)  at time  \( t = 0 \)  are shown as phasors in diagram (iii).

The phasor diagram is shown in diagram (iv). Using the cosine rule,

\[
(ac)^2 = 5^2 + 8^2 - 2(5)(8)\cos 30^\circ
\]

from which,  \( ac = 4.44 \)

© John Bird Published by Taylor and Francis
Using the sine rule, \( \frac{8}{\sin \phi} = \frac{4.44}{\sin 30^\circ} \) from which, \( \sin \phi = \frac{8 \sin 30^\circ}{4.44} = 0.90090 \)

and \( \phi = \sin^{-1} 0.90090 = 64.28^\circ \) or \( 180^\circ - 64.28^\circ = 115.72^\circ \)

From the phasor diagram, \( \phi = 115.72^\circ \) or 2.02 radians

Hence, \( v_1 - v_2 = 4.44 \sin(\omega t + 2.02) \)

3. Construct a phasor diagram to represent \( i_1 + i_2 \) where \( i_1 = 12 \sin \omega t \) and \( i_2 = 15 \sin(\omega t + \pi/3) \). By measurement, or by calculation, find a sinusoidal expression to represent \( i_1 + i_2 \)

The phasor diagram is shown below.

By drawing the diagram to scale and measuring, \( i_R = 23.5 \) and \( \phi = 34^\circ \) or 0.59 rad

By calculation, using the cosine rule,

\[
(i_R)^2 = 12^2 + 15^2 - 2(12)(15) \cos 120^\circ
\]

from which, \( i_R = 23.43 \)

Using the sine rule, \( \frac{15}{\sin \phi} = \frac{23.43}{\sin 120^\circ} \) from which, \( \sin \phi = \frac{15 \sin 120^\circ}{23.43} = 0.55443 \)

and \( \phi = \sin^{-1} 0.55443 = 33.67^\circ \) or 0.588 rad

Hence, \( i_1 + i_2 = 23.43 \sin(\omega t + 0.588) \)
4. Determine, either by plotting graphs and adding ordinates at intervals, or by calculation, the following periodic function in the form \( v = V_m \sin(\omega t \pm \phi) \)

\[
10 \sin \omega t + 4 \sin \left( \omega t + \frac{\pi}{4} \right)
\]

The following is determined by calculation.

The relative positions of \( v_1 \) and \( v_2 \) at time \( t = 0 \) are shown as phasors in diagram (i).

![Diagram (i)](image)

The phasor diagram is shown in diagram (ii). Using the cosine rule,

\[
(ac)^2 = 10^2 + 4^2 - 2(10)(4)\cos 135^\circ
\]

from which, \( ac = 13.14 \)

Using the sine rule, \( \frac{4}{\sin \phi} = \frac{13.14}{\sin 135^\circ} \) from which, \( \sin \phi = \frac{4 \sin 135^\circ}{13.14} = 0.2153 \)

and \( \phi = \sin^{-1} 0.2153 = 12.43^\circ \) or \( 0.217 \) rad

Hence,

\[
10 \sin \omega t + 4 \sin \left( \omega t + \frac{\pi}{4} \right) = 13.14 \sin \left( \omega t + 0.217 \right)
\]

5. Determine, either by plotting graphs and adding ordinates at intervals, or by calculation, the following periodic function in the form \( v = V_m \sin(\omega t \pm \phi) \)

\[
80 \sin \left( \omega t + \frac{\pi}{3} \right) + 50 \sin \left( \omega t - \frac{\pi}{6} \right)
\]

The following is determined by calculation.

The relative positions of \( v_1 \) and \( v_2 \) at time \( t = 0 \) are shown as phasors in diagram (iii).
The phasor diagram is shown in diagram (iv). Since abc is a right angled triangle, Pythagoras’
theorem is used.

\[ ac = \sqrt{50^2 + 80^2} = 94.34 \]

and \[ \phi = \tan^{-1}\left(\frac{50}{80}\right) = 32^\circ \]

Hence, in diagram (iv), \( \alpha = 60^\circ - 32^\circ = 28^\circ \) or \( 0.489 \) rad.

Thus,

\[ 80\sin\left(\omega t + \frac{\pi}{3}\right) + 50\sin\left(\omega t - \frac{\pi}{6}\right) = 94.34\sin\left(\omega t + 0.489\right) \]

6. Determine, either by plotting graphs and adding ordinates at intervals, or by calculation, the
following periodic function in the form \( v = V_m \sin(\omega t \pm \phi) \)

\[ 100\sin\omega t - 70\sin\left(\omega t - \frac{\pi}{3}\right) \]

The following is determined by calculation.

The relative positions of \( v_1 \) and \( v_2 \) at time \( t = 0 \) are shown as phasors in diagram (v). Since the
waveform of maximum value 70 is being subtracted it phasor is reversed as shown.

The phasor diagram is shown in diagram (vii).
Using the cosine rule,

\[(ac)^2 = 100^2 + 70^2 - 2(100)(70)\cos 60^\circ\]

from which, \(ac = 88.88\)

Using the sine rule, \(\frac{88.88}{\sin 60^\circ} = \frac{70}{\sin \phi}\) from which, \(\sin \phi = \frac{70 \sin 60^\circ}{88.88} = 0.68206\)

and \(\phi = \sin^{-1} 0.68206 = 43^\circ\) or \(0.751\) rad

Hence, \(100\sin \omega t - 70\sin \left(\omega t - \frac{\pi}{3}\right) = 88.88\sin \left(\omega t + 0.751\right)\)

7. The voltage drops across two components when connected in series across an a.c. supply are \(v_1 = 150 \sin 314.2t\) and \(v_2 = 90 \sin (314.2t - \pi/5)\) volts respectively. Determine (a) the voltage of the supply, in trigonometric form, (b) the r.m.s. value of the supply voltage, and (c) the frequency of the supply.

Cosine and sine rules or horizontal and vertical components could be used to solve this problem; however, an alternative is to use complex numbers, as shown below.

(a) **Supply voltage**, \(v = v_1 + v_2 = 150 \sin 314.2t + 90 \sin (314.2t - \pi/5)\)

\[= 150 \angle 0^\circ + 90 \angle -36^\circ\]

\[= (150 + j0) + (72.81 - j52.90)\]

\[= 222.81 - j52.90\]
= 229° - 13.36° = 229° - 0.233 rad

= 229 sin(314.2t - 0.233) V

(b) **R.m.s value of supply** = 0.707 × 229 = 161.9 V

(c) \(\omega = 314.2 = 2\pi f\) from which, **frequency**, \(f = \frac{314.2}{2\pi} = 50\) Hz

8. If the supply to a circuit is 25 sin 628.3t volts and the voltage drop across one of the components is 18 sin (628.3t - 0.52) volts, calculate (a) the voltage drop across the remainder of the circuit, (b) the supply frequency, and (c) the periodic time of the supply.

(a) Voltage, \(v_2 = v - v_1 = 25\sin 628.3t - 18\sin(628.3t - 0.52)\)

\[= 25\angle0 - 18\angle-0.52\text{ rad using complex numbers}\]

\[= (25 + j0) - (15.621 - j8.944)\]

\[= 9.379 + j8.944\]

\[= 12.96\angle0.76\text{ rad}\]

\[= 12.96\sin(628.3t + 0.762) V\]

(b) \(\omega = 628.3 = 2\pi f\) from which, **supply frequency**, \(f = \frac{628.3}{2\pi} = 100\) Hz

(c) **Periodic time**, \(T = \frac{1}{f} = \frac{1}{100} = 0.01\text{ s or } 10\text{ ms}\)
9. The voltages across three components in a series circuit when connected across an a.c. supply are:

\[ v_1 = 30\sin\left(300\pi t - \frac{\pi}{6}\right) \text{ volts,} \quad v_2 = 40\sin\left(300\pi t + \frac{\pi}{4}\right) \text{ volts and} \]

\[ v_3 = 50\sin\left(300\pi t + \frac{\pi}{3}\right) \text{ volts.} \]

Calculate (a) the supply voltage, in sinusoidal form, (b) the frequency of the supply, (c) the periodic time, and (d) the r.m.s. value of the supply.

(a) **Supply voltage**, \( v = v_1 + v_2 + v_3 = 30\sin\left(300\pi t - \frac{\pi}{6}\right) + 40\sin\left(300\pi t + \frac{\pi}{4}\right) + 50\sin\left(300\pi t + \frac{\pi}{3}\right) \)

\[ = 30\angle -30^\circ + 40\angle 45^\circ + 50\angle 60^\circ \text{ using complex numbers} \]

\[ = 79.265 + j56.586 \]

\[ = 97.39\angle 35.52^\circ \text{ V or } 97.39\angle 0.620 \text{ V} \]

\[ = 97.39\sin\left(300\pi t + 0.620\right) \text{ V} \]

(b) \( \omega = 300\pi = 2\pi f \) from which, **supply frequency**, \( f = \frac{300\pi}{2\pi} = 150 \text{ Hz} \)

(c) **Periodic time**, \( T = \frac{1}{f} = \frac{1}{150} = 0.0667 \text{ s or } 6.67 \text{ ms} \)

(d) **R.m.s value of supply** = \( 0.707 \times 97.39 = 68.85 \text{ V} \)