Simple interest

For all your simple interest calculations, you must remember your formulas:

- **I = PRT**
- **S = P + I**
- **S = P(1 + RT)**

where:

- I = simple interest amount
- P = present value
- R = simple interest rate
- T = term

Step to follow:

- Read the question carefully and note that the given information as you go
- Plug the given info into the formula and solve for the unknown variable, noting that you may need to manipulate the equation as required
- Interest rate is typically given as a percentage. Always divide this by 100 in your calculation
- Similarly, when you are solving for "R", always remember to multiply your final answer by 100 to get to the interest rate

NB note:

- The units used for the interest rate have to be the same as the units used for the term. In other words, if the term is months and the interest is charged per annum you have to adjust to a common unit before you can go any further with your calculation
- Refer to example 7.1 in your study guide where you have a simple interest rate per annum but the term of the loan is 90 day. In this case you need to convert the 90 days into a yearly equivalent. Given that there are 365 days in a year, a 90 day period is the same as $\frac{90}{365}$ (i.e. the term is 90 days out of a total of 365 days). If in the same example, the term of the loan was a quarter of a year then ask yourself: how many quarters are there in year….the answer is "4", and so this would be equivalent to $\frac{1}{4}$ (i.e. the term is one out of a total of 4 quarters)

Following these guidelines, you should be able to answer questions 1, 2 and 3 of the assignment

Let's go through some examples:

<u>Example 1 – solving for the interest amount</u>
Martha invested R1 500 for 21 months. The applicable simple interest rate is 9% per year. Determine the amount of interest that Martha will receive in 21 months' time

Solution:
P = 1 500
T = 21 months (= $\frac{21}{12}$ months since there are 12 months in a year)

R = 9% per annum
You need to solve for I

- **I = PRT**
- **I = (1 500)*(** $\frac{9}{100}$)*($\frac{21}{12}$) = 236.25

Therefore Martha will receive R236.25 in interest at the end of the period

Note how the term of 21 months is "treated" (i.e. diving by 12 since there are 12 months in a year). When the period is quoted in terms of months, you would do the same whether the period is less than or greater than a year…as is the case in Q3 of the assignment.

<u>Example 2 – solving for the principal amount</u>
Sibongile must pay Jabu an amount of R3 500 in 6 years' time. A 12.5% simple interest rate is applicable. Determine the amount that Sibongile borrowed from Jabu 6 years ago

Solution:
S = 3 500
T = 6 years
R = 12.5% per annum

You need to solve for P

- **S = P(1 + RT)**
- **3 500 = P(1 +** $\frac{12.5}{100}$ *** 6)**
- **3 500 = P(1.75)**
- $\frac{3\ 500}{1.75}$ **= P**
- **2 000 = P**

Therefore Sibongile borrowed R2 000 from Jabu six years ago

<u>Example 3 – solving for the interest rate</u>
Temba borrowed R12 500 from Thabo. He must pay him R20 000 six years from now. Determine the applicable simple interest rate at which Temba borrowed the money

Solution:
P = 12 500
S = 20 000
T = 6 years

You need to solve for R

- **S = P(1 + RT)**
- **20 000 = 12 500(1 +R*6)**
- $\frac{20\ 000}{12\ 500}$ **= 1 + R*6**
- **1.6 = 1 +R*6**
- **1.6 - 1 = R*6**
- **0.6 = R*6**

- $\frac{0.6}{6}$ **= R**
- **0.1 = R**

REMEMBER to multiply this by 100 to give R = 10%

Therefore the applicable simple interest rate is 10% per annum

NB note – solving for the interest amount is not the same thing as solving for the interest rate. Always read the question carefully, and use the appropriate formula!!

This note is relevant for Q4 of the assignment, even though it is a compound interest question!

<u>Compound interest</u>

The trick to these calculations is working out the effective interest rate per annum. Once this known, then you can simply plug the given information into the formula and solve for the required variable.

- **S = P(1 + R)^T**

R in the equation is the effective interest rate per annum

Once you get an interest that is compounded in over anything other than "once per year", then the first step is to find the equivalent of R

Refer to the first question of exercise 7.3 in the study guide:
"Determine the interest earned if R1 000 is invested for one year at 8% per annum. But the interest rate is compounded biannually"

So instead of R (which is an interest rate per annum that is compounded once a year), we have a rate of interest that is compounded every six months

Ask yourself: how many "six months" are there in a year…the answer should be "2"

So this means that the effective interest rate for a six month period is $\frac{8\%}{2}$
Because there are two opportunities to earn interest over the year, your term is "extended" from 1 year to 2 periods (this is effectively your new "T")

So remember this:

- If interest is compounded annually, that is the effective interest rate that is paid at the end of each year.
- If interest is compounded semi-annually (half-yearly), the quoted interest rate is divided by 2 in order to get the effective interest rate paid after every 6 months.
- If interest is compounded quarterly (4 quarters in a year), the quoted interest rate is divided by 4 in order to get the effective interest rate paid after every 3 months.

- If interest is compounded monthly (12 months in a year), the quoted interest rate is divided by 12 in order to get the effective interest rate paid after every month.

In each of these cases, you need to extend the term accordingly!

Let's look at an example:
Suppose you have 5 numbers of the LOTTO Powerball and your share of the winnings amounts to R200 000. You decide to invest this for your child's education, who will be going to university in 6 years. The fund you chose offers interest at a rate of 12% per annum compounded quarterly. What will the value of your investment be at this time?

- At what rate is the compounding happening?.....4 times a year (compounded quarterly)
- What is the effective interest rate per period (i.e. per quarter)?.....$\frac{12\%}{4} = \frac{0.12}{4} =$ 0.03 = 3%
- Over the 6 year period, how many times will the interest be due/paid?......4 periods in a year for 6 years gives you a total of 24 periods
- So you have $S = P(1 + 3\%)^{24}$ = R406 558.82…..you know what "P" is so you can work this out on your calculator

You should now be able to do question 5, 6 and 7 of the assignment

Note: the variable you are solving for question 7 is in the exponent…refer to study unit 4.4 if you have forgotten how to solve for these.

<u>Time Value of Money</u>

As we progress into the Mathematics of Finance study unit you will note that later sections build on the knowledge of the sections before it. It is therefore essential that you have a good understanding of compound interest before attempting the exercises on the time value of money.

Basic Formula is $\mathbf{S = P(1+R)^T}$

Interpretation of the equation is very important. To find the future value at the end of the period (S), we need to accumulate the principal amount at the given rate for the applicable period. In other words the **accumulating factor is $(1 + R)^T$**

If we manipulate to make P the subject of the formula, then we get: $\mathbf{P} = \frac{S}{(1+R)^T}$

In other words, to find the present value (the original principal amount), we need to discount the accumulated value S, to the present time. And therefore $\frac{1}{(1+R)^T}$ is the **discounting factor**.

In mathematical terms, accumulating and discounting are inverse functions, and hence the **discounting factor =** $\frac{1}{\text{accumulating factor}}$

NB Tip

- Drawing a timeline will be very useful. Placing your numbers correctly on the timeline from the given information is a critical first step. If drawn properly, this will also show you the point to which you need to accumulate or discount.
- When drawing your timeline remember to use time periods that match the compounding period – for ease of reference

Please go through the **examples in the study guide** as well as the exercises in the **workbook (worksheet 3 on page 151)**.
Further to this, the concept learnt here will be used in the annuities section which follows, so make sure that you are comfortable with this concepts before you move on.

<u>Annuities</u>

An annuity is a set of **regular payments** made or received over a specified term.

- Ordinary annuity - payments are made in arrears
- Annuity due – payments are in advance
- Annuity certain – payments are for a fixed term
- Perpetuity – payments go on forever (no fixed term)

These distinctions will be important for your assignment. Note for instance that **Q8** of assignment 3 speaks to a R2 000 payment made **at the end** of the year. This is an ordinary annuity.

Make sure that you read the question carefully and use the correct formula in your assignment and in your exam!!

For **continuous compounding**, the accumulated value can be found by:
$S = Pe^{RT}$
You should be able to manipulate this equation to find any of the other variables.

<u>NB note for annuities:</u>

- Term and interest rate are no longer represented by R and T respectively
- n = number of periods
- i = interest rate
- R = periodic payment

Let's look at some examples:

Example 4 – solving for the future value
Determine the accumulated amount of an annuity with quarterly payments of R1 200, at an interest rate of 16% per annum compounded quarterly, made over a period of 10 years.

Solution:

The formula for future value is $S = R\left[\frac{(1+i)^n - 1}{i}\right]$

- Period over which payments are made = quarterly
- Period over which interest is compounded = quarterly

It is important that these are the same!

So we know the following:

- R = 1 200 (periodic payment)
- i = $\frac{0.16}{4}$ = 0.04 (effective interest rate per quarter)
- n = 10*4 = 40 quarters (10 years, 4 quarters per year)
- S = (1 200)$*$ ($\frac{(1.04)^{40}-1}{0.04}$)
- S = 114 030.62

Example 5 – solving for the term
Mokgadi needs R55 000 for her dream holiday to Mauritius to celebrate her 30th birthday in a few years' time? She plans to save for this by making annual deposits of R9 500 into an investment account that earns interest at 7.34% interest per year. If Mokgadi's birthday is in exactly 4 years time, will she have saved enough for her goal?

Solution – you need to find out how long it will take Mokgadi to accumulate R55 000, and then compare this to the time frame that she has

S = 55 000

R = 9 500

i = 7.34%

note that interest is compounded once a year and the investment payments are also made once a year…so no adjustments necessary

55 000 = (9 500)*($\frac{1.0734^{n}-1}{0.0734}$)

$\frac{55\,000}{9\,500} = \frac{1.0734^n - 1}{0.0734}$

$5.789474 = \frac{1.0734^n - 1}{0.0734}$

$(5.789474)*(0.0734) = 1.0734^n - 1$

$0.424947 + 1 = 1.0734^n$

$1.424947 = 1.0734^n$

The variable that you wish to solve for is in the exponent. We therefore what you learnt in study unit 4 on log and exponential functions!

Taking the natural log of both sides gives:

$\ln(1.424947) = n * \ln(1.0734)$
$0.35413 = n * (0.070831)$
$\frac{0.35413}{0.070831} = n$
$4.9997 = n$

It will therefore take Mokgadi close to 5 years to accumulate the required R55 000. Given that her birthday is in exactly 4 years' time, she will not fulfil this goal

<u>Example 6 – solving for the periodic payment</u>
Senzeni owes Petrus an amount of R120 000. This debt must be paid off in 6 years' time. An interest rate of 13.6% per annum compounded quarterly is applicable. Determine Senzeni's quarterly payments

Solution:

The formula for present value is

$$\begin{aligned} P &= Ra_{\overline{n}|i} \\ &= R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \end{aligned}$$

So we know the following:

- P = 120 000 (loan amount)
- $i = \frac{0.136}{4}$ = 0.034 (effective interest rate per quarter)
- n = 6*4 = 24 quarters (6 years, 4 quarters per year)

So, $(1+1)^n = (1.034)^{24} = 2.230966$

Plugging into the formula: $120\,000 = R\left(\frac{2.230966 - 1}{(0.034)*(2.230966)}\right) = R\left(\frac{1.230966}{0.075853}\right) = R(16.22834)$

$\frac{120\ 000}{16.22834}$ = R

7 394.47 = R

You should now be able to do up to Q13 of the assignment

Hint for Q8

- draw a timeline to represent all the information you have been provided
- break the question up into 2 parts
  - the first part is a regular annuity over a period of 8 years. Find the future value
  - the second part an investment of a lumpsum (there are no regular payments) over a 36 year period

Amortisation

Key concepts to keep in mind:

- every instalment represents a repayment of both capital and interest
- the interest portion within each instalment reduces with time
- the balance outstanding at any given point is the present value of all future instalments at that point in time

Working out the present value of a loan as required in Q14 and Q16 is similar to solving annuity type question as done above

If you need a loan to buy a house or a car and you have a deposit, then the amount of money to be borrowed equals the value of the house/car less the deposit that you put down. Keep this in mind!

Example 6 – amortization schedule
Draw up an amortization schedule for a loan of R12 000 with interest at 10% per annum compounded yearly over a term of five years.

P = 12 000
i = 0.1
n = 5
You need to solve for R (annual payments, in this instance)

12 000 = R $\left(\frac{1.1^5-1}{(0.1)*(1.1)^5}\right)$
12 000 = R $\left(\frac{0.061051}{0.161051}\right)$

12 000 = R(3.790787)

$\frac{12\ 000}{3.790787}$ = R

3 165.57 = R

This is similar to what is being asked of you in Q15 and Q18 of the assignment

Every instalment covers a portion of capital and a portion of interest, as stated above. You now need to determine how much interest is due in the first year:
I = PRT……where T = 1 (don't consider the whole term of the loan, since you are interested in the interest due in the first year)
I = (12 000)*(0.1)*(1) = 1 200

Therefore, out of the yearly instalment of R3 165.57, R1 200 of that will go to interest payment and the rest will go towards the principal repayment. Your debt after this first instalment will therefore be reduced by R1 965.57 (3 165.57 – 1 200)

At the start of the second year, therefore, the amount outstanding (the new loan value) is R10 034.43

Remember that interest is due on the amount outstanding. Therefore as you pay off the loan over time the balance outstanding also reduces. Since your instalment is constant, this means that more of less of the instalment goes to cover interest and more goes to paying off the principal debt. This can be seen from the table clearly

- in the first year the interest due was R1200 (10% of R12 000), whereas the interest due in the second year wasR1 003.44 (10% of R10 034.43)
- in the second year the outstanding loan balance is R10 034 (R12 000 less R1 965.57), since some capital has been repaid. The interest amount is therefore reduced (10% of a smaller amount), and therefore more of the instalment is sued to repay the principal debt
- …and so on

	Outstanding balance	Periodic Instalment	Interest due in that period	Principal repaid after instalment
At the start of year 1	**12 000** (original loan amount)	**3 165.57** (constant throughout term)	1 200	1 965.57 (difference between instalment and interest amount)
At the start of year 2	10 034.43	**3 165.57** (constant throughout term)	1 003.44 (10% of R10 034.43)	2 162.13 (difference between instalment and interest amount)
At the start of year 3	7 872.30	**3 165.57** (constant throughout term)	787.23 (10% of R7 872.30)	2 378.34 (difference between instalment and interest amount)
At the start of year 4	5 493.96	**3 165.57** (constant throughout term)	549.40 (10% of R5 493.96)	2 616.17 (difference between instalment and interest amount)
At the start	2 877.79	**3 165.57**	287.78	2 877.79

of year 5		(constant throughout term)	(10% of R2 877.79)	(difference between instalment and interest amount)

From the table you are able to read off the balance outstanding after any number of installments….similar to what is asked in Q17
You can also read off the total amount of interest paid…similar to what is asked in Q20

These questions can also be answered by using your calculator, without having to draw up an amortization schedule. Please consult the calculator manual and ensure that you are comfortable with these calculations