May/June 2012 Examination paper

Question 1

a) If

$$A = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

Check whether or not $AB = BA$.

solution:

$$AB = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -1.1 + 0.3 & -1.2 + 3.0 \\ 2.1 + 3.3 & 2.2 + 3.0 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 11 & 4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 6 \\ -3 & 0 \end{pmatrix}$$

Thus is follows that $AB \neq BA$

b) Reduce the following matrix to reduced row-echelon form

$$\begin{pmatrix} 0 & 3 & 9 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{pmatrix}$$

$1/3R_1 - - - - > R_1$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{pmatrix}$$

$-2R_1 + R_2 - - - - > R_2$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & -10 \\ 0 & 0 & 3 \end{pmatrix}$$

$-1/10R_2 - - - - - > R_2$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$1/3R_3 - - - - > R_3$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$-R_1 + R_3 - - - - - - - > R_3$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$-3R_2 + R_1 - - - - > R_1$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

c) Solve the system

$$\begin{aligned} 3x + 4y + z &= 1 \\ 2x + 3y &= 0 \\ 4x + 3y - z &= -2 \end{aligned}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 3 & 4 & 1 & 1 \\ 2 & 3 & 0 & 0 \\ 4 & 3 & -1 & -2 \end{array}\right]$$

$R_2 < - - - > R_1$

$$\left[\begin{array}{ccc|c} 2 & 3 & 0 & 0 \\ 3 & 4 & 1 & 1 \\ 4 & 3 & -1 & -2 \end{array}\right]$$

$1/2R_1 - - - - > R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 0 \\ 3 & 4 & 1 & 1 \\ 4 & 3 & -1 & -2 \end{array}\right]$$

$-3R_1 + R_2 - - - -- > R_2$

$-4R_1 + R_3 - - - - > R_3$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 1 \\ 0 & -3 & -1 & -2 \end{array}\right]$$

$-2R_2 - -- > R_2$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & -3 & -1 & -2 \end{array}\right]$$

$3R_2 + R_3 - - - - - - > R_3$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -7 & -8 \end{array}\right]$$

$-1/7R_3 - - - - - > R_3$

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & \frac{8}{7} \end{array}\right]$$

Thus
$z = \frac{8}{7}$

$y - 2z = -2 \quad \therefore y = \frac{2}{7}$

$x + \frac{3}{2}y = 0 \quad \therefore x = -\frac{3}{7}$

d) Let

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

Using row operations, find the A^{-1} and verify it ie show that $AA^{-1} = I = A^{-1}A$

solution:
$\left[\begin{array}{c|c} A & I \end{array}\right]$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right]$$

$1/2R_1 - - - - - > R_1$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 + R2 - - - - - > R2$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 \end{array}\right]$$

$2R_2 - - - - - > R_2$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

$-1/2R_2 + R_1 - - - - - > R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

Thus we get

$$A^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$$

To verify the A^{-1} is the inverse of matrix A we perform the product

$$AA^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$A^{-1}A = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$