

Question 4

c)

$$z_1 = 1 + i\sqrt{3}$$

Represent in polar form:

$$|z_1| = \sqrt{1^2 + 3} = 2$$

$$z_1 = \frac{|z_1|}{|z_1|}(1 + i\sqrt{3}) = 2 \left( \frac{1}{2} + i\frac{\sqrt{3}}{2} \right)$$

$$\cos \theta = \frac{1}{2} \text{ and } \sin \theta = \frac{\sqrt{3}}{2}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\theta = \frac{\pi}{3}$$

$$\text{Polar form: } z_1 = 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z_2 = \sqrt{3} + i$$

Represent in polar form:

$$|z_2| = \sqrt{1^2 + 3} = 2$$

$$z_2 = \frac{|z_2|}{|z_2|}(\sqrt{3} + i) = 2 \left( \frac{\sqrt{3}}{2} + i\frac{1}{2} \right)$$

$$\cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

$$\text{Polar form: } z_2 = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$\begin{aligned} z_1 z_2 &= 2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \cdot 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 2^2 \left( \cos \left( \frac{\pi}{3} + \frac{\pi}{6} \right) + i \sin \left( \frac{\pi}{3} + \frac{\pi}{6} \right) \right) \\ &= 4 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \end{aligned}$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{2 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} \\ &= \cos \left( \frac{\pi}{3} - \frac{\pi}{6} \right) + i \sin \left( \frac{\pi}{3} - \frac{\pi}{6} \right) \\ &= \cos \left( \frac{\pi}{6} \right) + i \sin \left( \frac{\pi}{6} \right) \end{aligned}$$