

Question 4

a)

(i) If $z_1 = 1 + \sqrt{3}i$ then $|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = 2$ and

$$z_1 = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$\left(\frac{1}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 = 1$ thus the point $\left(\frac{1}{2}; \frac{\sqrt{3}}{2} \right)$ lies on the circumference of a circle of radius 1 thus it follows that there exists a θ such that

$$\cos \theta = \frac{1}{2} \text{ and } \sin \theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

The polar form is given by

$$z_1 = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$