Tutorial letter 101/3/2017

Quantitative Modelling 1
DSC1520

Semesters 1 and 2

Department of Decision Sciences

This is a fully online module.
Information available on myUnisa at DSC1520-2017-S1 or DSC1520-2017-S2.
Communication through myLife e-mail account.
Contents

1 Introduction and welcome 4
2 Lecturer and contact details 4
   2.1 Lecturer ................................................................. 4
   2.2 Department .............................................................. 4
   2.3 University ............................................................... 5
3 Student support system 5
   3.1 E-tutors ................................................................. 5
   3.2 Face-to-face tutors ................................................... 5
4 Online services 5
   4.1 myUnisa ................................................................. 6
   4.2 myLife ................................................................. 6
5 Study material 6
   5.1 Printed study material .............................................. 6
   5.2 Where to buy the textbook ........................................ 6
6 Overview of module 7
   6.1 Purpose and outcomes ............................................. 7
   6.2 Syllabus ............................................................... 7
7 Calculator 7
8 Study approach 7
   8.1 Study time ............................................................ 7
   8.2 Study method ........................................................ 7
9 Assessment 8
   9.1 Compulsory assignments ......................................... 8
   9.2 Examination .......................................................... 8
   9.3 Semester and examination marks ............................... 8
   9.4 Assignment due dates and unique numbers .................. 8
   9.5 Submission of assignments ..................................... 8
10 Semester 1 – Compulsory assignments 9
    10.1 Assignment 01 (S1) ................................................ 9
11 Semester 2 – Compulsory assignments

11.1 Assignment 01 (S2) ......................................................... 23
11.2 Assignment 02 (S2) ......................................................... 29
11.3 Assignment 03 (S2) ......................................................... 32
1 Introduction and welcome

It is a pleasure to welcome you to this module, *Quantitative Modelling 1*.
We hope that you will find this module interesting and enjoyable and that you will complete it successfully.
This is a fully online module. You therefore need to go online to see your study material and read about the module. The myUnisa website is at [http://my.unisa.ac.za/](http://my.unisa.ac.za/) where you need to login with your student number and password. If you do not have a password yet, go to *Claim UNISA login* (at the left) to register.
Once logged in, you will see the module site DSC1520-17-S1 or DSC1520-17-S2 at the top, depending on the semester you are registered for. If you don’t see it, check under *More Sites*.
This tutorial letter (Tutorial letter 101) and the online study guide (MO001) are supplied in printed format so you can study from it even when you cannot go online.

2 Lecturer and contact details

You are welcome to contact your lecturer(s), the Department of Decision Sciences or the university when you encounter study related problems during the semester.

Provide your student number and module code whenever you contact us.

2.1 Lecturer

You will find the name of the lecturer responsible for this module on the DSC1520 site on myUnisa. We suggest that you write the name and contact details of the lecturer in the space below so it will be available whenever you need to contact him/her.

Your lecturer will assist you with problems you may have with the study material (not administrative matters). You may contact him/her by means of e-mail, telephone or fax. You may also visit him/her at the office, but only by appointment.

2.2 Department

You can reach the Department of Decision Sciences in one of the following ways:

E-mail:  qm@unisa.ac.za  
Tel:  012 433-4684  
Fax:  012 429-4898
2.3 University

We assume that you are acquainted with the contents of the brochure *my Studies @ Unisa*. This helpful document is available on the myUnisa homepage — simply click on *Undergraduate & honours*. You may consult the online version here or open the printable PDF version at the bottom of the list on the left. Consult this brochure if you need to contact the university about matters not related to the content of this module.

3 Student support system

Consult *my Studies @ Unisa* for information on the various student support systems and services available at Unisa. For example, student counselling, tutorial classes and language support.

Unisa also provides student support through e-tutors and face-to-face tutors. These tutors are qualified experts in the subjects for which they have been appointed.

3.1 E-tutors

E-tutoring entails the delivery of teaching and learning online via the internet. The e-tutors for DSC1520 undertake to support students in their groups to be successful in this module.

Students are grouped to e-tutors after the registration process has ended. You will receive a myLife e-mail notification when you have been allocated to an e-tutor. You can contact and communicate with your e-tutor through the e-tutor site on myUnisa named DSC1520-17-S1-E1 or DSC1520-17-S1-E2 (semester 1) or DSC1520-17-S2-E1 or DSC1520-17-S2-E2 (semester 2).

3.2 Face-to-face tutors

For face-to-face contact sessions with a tutor, you need to go to the Unisa Regional Learning Centre (RLC) nearest to you to enrol. In such face-to-face sessions students meet with their tutors in a classroom setup at the RLCs.

The RLC will supply you with the dates and times of the sessions for the modules you have enrolled for. Note that tutor classes are not presented at all RLCs, since there must be a certain minimum number of students enrolled at an RLC for a face-to-face tutor to be appointed there.

4 Online services

This module is presented fully online and you need to have access to a computer that is linked to the internet. If you do not have your own computer with internet access, consult *my Studies @ Unisa* for guidelines on how and where to get access. (Under *Prepare for your study success*, go to *Connect online*.)

As a registered Unisa student you have free access to myUnisa (Unisa’s online campus) and you receive a free myLife email address.
4.1 myUnisa

myUnisa is a platform where you can communicate electronically with your lecturers, other students and the administrative departments of Unisa.

This is also where you will submit assignments, view results, join online discussion forums, etc.

4.2 myLife

Announcements, marked assignments and other forms of communication from Unisa will be sent to your myLife e-mail address.

If you regularly use another e-mail account, we suggest that you configure your myLife account to forward e-mails to this account.

5 Study material

The study material for this module consists of the following:


3. A module online study guide, MO001, intended to guide you through the textbook.

4. Additional tutorial letters that will be posted on myUnisa (under Additional Resources) during the semester.

Important:
The textbook is the basis of the study material for this module. Purchase the textbook as soon as possible.
It is not possible to pass this module without the textbook.

5.1 Printed study material

Printed copies of Tutorial letter 101 and MO001 will be sent to all students.

These are exactly the same as the electronic versions available on myUnisa. If you haven’t received the printed version, please do not waste valuable time by waiting for it to arrive before starting with your studies. Download it from myUnisa and print it for yourself.

5.2 Where to buy the textbook

To find out where you can buy the textbook, consult the list of booksellers and their addresses that is available on the myUnisa homepage under Official Booksellers.

If you cannot obtain the textbook from these booksellers, please contact the Processing Section at the library as soon as possible by sending an e-mail to rospresc@unisa.ac.za.
6 Overview of module

6.1 Purpose and outcomes

The purpose, specific outcomes and assessment criteria of this module are available on myUnisa, under Additional resources.

6.2 Syllabus

The study material is subdivided into five study units, namely

<table>
<thead>
<tr>
<th>Study unit</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear functions</td>
</tr>
<tr>
<td>2</td>
<td>Simultaneous linear functions</td>
</tr>
<tr>
<td>3</td>
<td>Nonlinear functions</td>
</tr>
<tr>
<td>4</td>
<td>Differentiation</td>
</tr>
<tr>
<td>5</td>
<td>Integration</td>
</tr>
</tbody>
</table>

Details regarding the contents of each study unit can be found in the table of contents of MO001.

7 Calculator

You may use any (programmable) scientific or financial pocket calculator for this module. Although we say “programmable”, you will not need the programmable features of such a calculator in this module.

8 Study approach

8.1 Study time

A semester is quite short and it is essential that you plan your study program carefully. Use the assignment due dates as a guideline to manage your time in such a way that you master the study material, submit the assignments on time and have enough time to prepare for the examination.

You will have to work consistently throughout the semester if you wish to be successful in this module.

8.2 Study method

We suggest that you approach the study material as follows:

- Work through each study unit in the study guide (MO001) together with the textbook.
- Follow the instructions carefully and do the activities on your own (with pen on paper).
- Contact your lecturer (or e-tutor) if you need help with the study material, preferably before you carry on with a new study unit.
9 Assessment

This module is assessed by means of three compulsory assignments (formative assessment) and a written examination (summative assessment).

9.1 Compulsory assignments

To benefit fully from our formative tuition and assessment, you should complete and submit the three compulsory assignments before or on the due dates. Assignments 01 and 03 consist of multiple-choice questions (MCQ) and Assignment 02 is a written assignment.

9.2 Examination

To gain admission to the examination, you must submit the compulsory assignments before or on the due dates.
The examination at the end of the semester will consist of a two hour MCQ paper.

9.3 Semester and examination marks

The assignments all count towards your semester mark, which contributes 20% to your final mark for the module. Assignments 01 and 03 (MCQ) each contributes 30% to the semester mark, while Assignment 02 (written) contributes 40%.
The written examination contributes 80% to the final mark. A subminimum of 40% applies to the examination. This means that your year mark will not be taken into account if your examination mark is less than 40%. In this case your examination mark will also be your final mark.

9.4 Assignment due dates and unique numbers

The assignment due dates and unique numbers for the first and second semesters are as follows:

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Type</th>
<th>Dates and unique numbers</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First semester</td>
<td>Second semester</td>
</tr>
<tr>
<td>01</td>
<td>MCQ</td>
<td>6 March 2017</td>
<td>655837</td>
</tr>
<tr>
<td>02</td>
<td>Written</td>
<td>31 March 2017</td>
<td>633730</td>
</tr>
<tr>
<td>03</td>
<td>MCQ</td>
<td>21 April 2017</td>
<td>713047</td>
</tr>
</tbody>
</table>

9.5 Submission of assignments

You must submit all assignments electronically through myUnisa. See my Studies @ Unisa for instructions to submit MCQ assignments.

To submit a written assignment electronically, you can either write it by hand and scan it in PDF format, or you can type it in MSWord (or a similar word processing package) and then save it as a PDF before submitting it.
10 Semester 1 – Compulsory assignments

10.1 Assignment 01 (S1)

This MCQ assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 March 2017</td>
<td>655837</td>
</tr>
</tbody>
</table>

Instructions
▷ Work through Study units 1 and 2 in MO001 before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

Question 1
A line cuts the x-axis at 4 and goes through the point (2; 4). The equation of the line is

[1] \( y = 4 \).

[2] \( y = 4 - 2x \).

[3] \( y = -2x + 8 \).

[4] \( y = \frac{1}{2}x + 3 \).


Question 2
A market is defined by the following demand and supply functions:

\[
P_d = 50 - 3Q \quad \text{and} \quad P_s = 14 + 1.5Q.
\]

Determine the equilibrium price, \( P_0 \), and quantity, \( Q_0 \).

[1] \( P_0 = 26; Q_0 = 30 \)

[2] \( P_0 = 30; Q_0 = 8 \)

[3] \( P_0 = 8; Q_0 = 26 \)

[4] \( P_0 = 26; Q_0 = 8 \)

[5] None of the above.
Questions 3 and 4 are based on the following information:
The demand function of a commodity is given by

$$P = 215 - 5Q,$$

where $P$ and $Q$ are the price and quantity, respectively.

**Question 3**
Determine the point price elasticity of demand if $P = 15$.

[1] $\frac{3}{8}$
[2] $-\frac{8}{15}$
[3] $\frac{3}{20}$
[4] $-\frac{3}{20}$
[5] None of the above.

**Question 4**
Which one of the following statements is true?

[1] Since $|\varepsilon_d| = \frac{3}{8} < 1$, demand is elastic, meaning that the percentage change in demand is less than the percentage change in price.

[2] Since $|\varepsilon_d| = -\frac{8}{15} > 1$, demand is elastic, meaning that the percentage change in demand is greater than the percentage change in price.

[3] Since $|\varepsilon_d| = \frac{3}{20} < 1$, demand is inelastic, meaning that the percentage change in demand is less than the percentage change in price.

[4] Since $|\varepsilon_d| = -\frac{3}{20} > 1$, demand is inelastic, meaning that the percentage change in demand is greater than the percentage change in price.

[5] None of the above.

**Question 5**
Find the point of intersection $(x; y)$ of the lines

$$4x + 3y = 11$$
and
$$2x + y = 5.$$ 

[1] $(-2; 9)$
[2] $(2; 1)$
[3] $(0,4; 5,4)$
[4] $(4; -\frac{5}{3})$
Questions 6 and 7 are based on the following information:
The demand and supply functions of a product are

\[ P_d = 255 - 4Q \quad \text{and} \quad P_s = 25 + 7.5Q, \]

where \( P \) is price and \( Q \) is quantity.

**Question 6**
The price and the consumer surplus at equilibrium are

1. \( P = 20; \ CS = 20125. \)
2. \( P = 24; \ CS = 1152. \)
3. \( P = 159; \ CS = 18364.50. \)
4. \( P = 175; \ CS = 800. \)
5. None of the above.

**Question 7**
If tax of R11,50 is imposed on the product, then the equilibrium price and quantity are

1. \( P = 6986.25; \ Q = 69. \)
2. \( P = 182.50; \ Q = 21. \)
3. \( P = 179.00; \ Q = 19. \)
4. \( P = 167.50; \ Q = 19. \)
5. None of the above.

**Question 8**
Lighting Warehouse produces \( q \) solar lamps per week at a fixed cost of R10 000. Each lamp costs R150 to produce and is sold for R350. Their total cost and profit functions are

1. \( TC = 150q + 10000; \ \pi = 200q - 10000. \)
2. \( TC = 350q; \ \pi = 350q - 10000. \)
3. \( TC = 150q - 10000; \ \pi = (350 - 150)q + 10000. \)
4. \( TC = 150q; \ \pi = 250q + 10000. \)
5. None of the above.
**Question 9**

Consider the following set of inequalities:

\[
\begin{align*}
    x + y & \leq 13 \quad (1) \\
    2x - y & \leq 8 \quad (2) \\
    -2x + 3y & \leq 12 \quad (3) \\
    x; y & \geq 0. \quad (4)
\end{align*}
\]

The correct representation of the feasible region is

Question 10
A small iron works manufactures two types of gates. The time requirements (in hours) for each stage of production, the limitations on the available labour hours and the selling price per unit are given in the following table:

<table>
<thead>
<tr>
<th>Gate type</th>
<th>Stages of production</th>
<th>Selling price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Welding</td>
<td>Finishing</td>
</tr>
<tr>
<td>Security</td>
<td>4,5</td>
<td>1,0</td>
</tr>
<tr>
<td>Ornamental</td>
<td>2,0</td>
<td>2,0</td>
</tr>
<tr>
<td>Hours available</td>
<td>900</td>
<td>400</td>
</tr>
</tbody>
</table>

If the number of security and ornamental gates to be manufactured are denoted by $x$ and $y$, respectively, then the objective function and the constrains are

1. Maximise $TR = (7200 + 665)(x + y)$; $4.5x + 2y \leq 900$; $x + 2y \leq 400$; $x, y \leq 0$.
2. Maximise $TR = 7200x + 665y$; $4.5x + 2y \leq 900$; $x + 2y \leq 400$; $x, y \geq 0$.
3. Minimise $TC = 900x + 400y$; $4.5x + 2y \geq 900$; $x + 2y \leq 400$; $x, y \geq 0$.
4. Maximise $\pi = 7200x + 665y$; $4.5x + 2y \leq 900$; $x + 2y \leq 400$.
5. None of the above.

Question 11
The supply function of a certain product is $P = 80 + 5Q$, where $P$ is the price and $Q$ is the number of units produced. Determine the producer surplus if the market price is R200.

1. 960
2. 4440
3. 2400
4. 3360
5. None of the above.

Question 12
It costs R2,50 to print a paperback book and it costs R1,00 to bind each book. Printing and binding set-up costs are R1,000. These paperback books are sold to retailers for R4,75 each. Find the number of paperback books that must be printed, bound and sold to break even.

1. -800
2. 286
3. 800
4. 1,008,25
5. None of the above.
Question 13
Consider the following graph showing the demand and supply functions of a certain good. Which one of the statements below is true?

![Graph showing demand and supply functions]

1. Point A represents the break-even point.
2. Line 1 represents a supply function and line 2 represents a demand function.
3. Point B represents the minimum price consumers will pay.
4. The area of triangle ABC represents consumer surplus.
5. None of the above.

Question 14
Residents at a retirement village have the option to eat lunch at a central lounge. When the price per lunch is R45, there is a demand for 80 lunches. It is known that for each R1 increase in price, demand decreases by 2 lunches. Find the demand function with quantity, \( Q \), as a function of price, \( P \).

1. \( Q = 170 - 2P \)
2. \( Q = 35 - 2P \)
3. \( Q = 130 - \frac{1}{2}P \)
4. \( Q = 35 + 2P \)
5. None of the above.

Question 15
Suppose you have a supply function\[ P = 30 + 0.5Q. \]

Determine the arc price elasticity of supply if the price increases from R10 to R20. What does this mean?

1. \( \varepsilon_s = -1 \); while \( 10 \leq P \leq 20 \), supply decreases by 1% for each 1% increase in price.
2. \( \varepsilon_s = 1 \); while \( 10 \leq P \leq 20 \), supply increases by 1% for each 1% increase in price.
3. \( \varepsilon_s = -0.25 \); while \( 10 \leq P \leq 20 \), supply decreases by 25% for each 1% increase in price.
4. \( \varepsilon_s = 0.25 \); for \( P = 10 \) and \( P = 20 \), supply increases by 0.25% for each 1% increase in price.
5. none of the above.
10.2 Assignment 02 (S1)

This written assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 March 2017</td>
<td>633730</td>
</tr>
</tbody>
</table>

Instructions

▷ Work through Study unit 3 in MO001 before attempting this assignment.
▷ Answer all the questions.
▷ Show all steps you followed to find the answers (including graphs). Marks will be given for these and not only for the answer.
▷ Submit your answers in PDF format on myUnisa.
▷ You may use Excel or Maxima for assistance when you need to draw graphs and/or solve equations.

Only a selection of questions will be marked, but you will receive solutions to all the questions.

Question 1

Consider the following table of \((x; y)\) pairs:

\[
\begin{array}{c|c|c|c|c|c|c}
 x & 0 & 5 & 10 & 15 & 20 & 25 & 30 \\
 y & 0 & 200 & 300 & 300 & 200 & 0 & -300 \\
\end{array}
\]

(a) Have these pairs been generated by a linear or non-linear function? Justify your answer by providing a rough graph.

(b) What is this kind of curve called?

Question 2

Solve the equation

\[
\frac{50}{x + 2} = 2x + 10.
\]

Question 3

Simplify the following expressions:

(a) \[
\frac{x^2 + 7x + 12}{x + 3}
\]

(b) \[
\sqrt{\frac{x^2y^3}{x^3}}
\]

(c) \[
\frac{(3^{-1}a^4b^{-3})^{-2}}{(6a^2b^{-1}c^{-2})^2}
\]
Question 4
Use the rules of logarithms to solve the following equations:

(a) \( \ln(x - 1) + \ln 3 = \ln x \)
(b) \( 3 \ln(5x) - 2 \ln x = 7 \) (round to two decimals)

Question 5
Consider the quadratic function

\[ f(x) = 2x^2 + 6x - 20. \]

(a) Find the coordinates of the turning point of \( f \).
(b) Find the roots of \( f \).
(c) Draw a rough graph of the function \( f \), clearly indicating the labels of the axes, the roots and the turning point.

Question 6
Consider the cubic function

\[ f(x) = x^3 - 3x + 2. \]

(a) Draw a graph of \( f \) for \( -3 \leq x \leq 3 \).
(b) From the graph, estimate the roots of \( f \) and the coordinates of the turning points of \( f \).

Question 7
Consider the following demand and total cost functions of a commodity:

\[ Q = 6000 - 30P \quad \text{and} \quad TC = 5000 + 20Q, \]

where \( P \) is the price and \( Q \) the quantity.

(a) Find the total revenue function \( (TR) \) in terms of \( Q \).
(b) Determine the company’s profit function.
Question 8
The demand function of a certain product is given by

\[ P = 100 - 20Q, \]

where \( p \) is the price per unit and \( Q \) is the number of units produced (in thousands).
The daily fixed costs amount to R40 and the production cost per unit is R10.

(a) Write down the total revenue \((TR)\), total cost \((TC)\) and profit \((\pi)\) functions.

(b) For what values of \( Q \) does the firm break even?

(c) How many units should be produced to maximise profit and what is the optimum price for the product?

(d) Draw a graph of the \( TR \) and \( TC \) functions, indicating the break-even points and the area where profit is made.

Question 9
The percentage of teenagers who own a new game \( t \) weeks after it has been launched, is modelled by

\[ F(t) = 100 - 95e^{-0.15t}. \]

(a) Find the percentage of teenagers who own the game

(i) when it is first launched;
(ii) after 20 weeks;
(iii) in the long run.

(b) After how many weeks do 50\% of teenagers own the game?

Question 10
The demand and supply functions of a certain product are given by

\[ P_d = 25 - Q \quad \text{and} \quad P_s = Q^2 + 2Q + 7, \]

where \( P \) is the price per unit and \( Q \) the number of units produced.

(a) Determine the equilibrium price and quantity for the product.

(b) Draw a graph of the demand and supply functions for \(-7 \leq Q \leq 5\), clearly indicating the equilibrium point.
10.3 Assignment 03 (S1)

This MCQ assignment is compulsory.

<table>
<thead>
<tr>
<th>Due date</th>
<th>Unique number</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 April 2017</td>
<td>713047</td>
</tr>
</tbody>
</table>

Instructions
- Work through Study units 4 and 5 in MO001 before attempting this assignment.
- Answer all the questions.
- Submit your answers electronically through myUnisa.

Question 1
Differentiate the function \( f(x) = \frac{x - 4x^2}{x^3} \).

1. \( f'(x) = \frac{1 - 8x}{3x^2} \)
2. \( f'(x) = 3x^2 - 24x \)
3. \( f'(x) = -\frac{2}{x^2} + \frac{4}{x} \)
4. \( f'(x) = \frac{2}{x^2} - \frac{4}{x} \)
5. None of the above.

Question 2
The derivative of \( f(x) = 12xe^{6x} \) is

1. \( 84e^{6x} \)
2. \( 12e^{6x} \)
3. \( 12e^{6x}(1 + 6x) \)
4. \( 36e^{6x}(1 + x) \)
5. None of the above.

Question 3
The derivative of \( f(x) = \ln(4x^3 + 8x^2) \) is

1. \( \frac{4(3x^2 + 4x)}{\ln(4x^3 + 8x^2)} \)
2. \( 4 \ln(4x^3 + 8x^2)(3x^2 + 4x) \)
3. \( \frac{4x^2}{x^2 + 8x} \)
4. \( \frac{3x^2 + 4x}{8x + 2x} \)
5. None of the above.
**Question 4**
Differentiating the function \( f(x) = 3e^{4x^2-3x} + 4x^2 - 3x + 6 \) results in

1. \( 3e^{4x^2-3x} + 8x - 3 \).
2. \( (8x-3)(3e^{4x^2-3x} + 1) \).
3. \( 24xe^{4x^2-3x} + 8x - 3 \).
4. \( e^{4x^2-3x} + 8x \).
5. None of the above.

**Question 5**
Integrating the function \( f(x) = \frac{2+4}{x} \) results in

1. \( 1 - \frac{4}{x} + c \).
2. \( -\frac{1}{x^2} - \frac{8}{x^3} \).
3. \( \ln x - \frac{2}{x} + c \).
4. \( -\ln x + \frac{4}{x} + c \).
5. None of the above.

**Question 6**
Integrate the function \( f(x) = \frac{4}{(3x+1)^2} \).

1. \( -\frac{36}{(3x+1)^3} + c \).
2. \( \frac{2}{3(3x+1)^2} + c \).
3. \( -\frac{2}{(3x+1)^2} + c \).
4. \( \frac{2}{3(3x+1)^2} + c \).
5. None of the above.

**Question 7**
Evaluate the definite integral \( \int_0^1 18e^{3x+1} \, dx \) (round to an integer).

1. 314
2. 322
3. 934
4. 965
5. None of the above.
Question 8
The total revenue function of a firm is given by

\[ TR = 2x^3 - \frac{x^2}{2} + 10x + 15, \]

where \( x \) is the number of units sold. What is the marginal revenue when five units are sold?

[1] 20
[2] 450
[3] 302
[4] 491
[5] None of the above.

Question 9
A firm’s production function is given by

\[ Q = 6L^2 - 0.2L^3, \]

where \( Q \) denotes the number of units produced and \( L \) the number of labourers. Find the size of the workforce that maximises output.

[1] \( L = 0 \)
[2] \( L = 20 \)
[3] \( L = 60 \)
[4] \( L = 120 \)
[5] None of the above.

Question 10
Find the stationary points of the function

\[ f(x) = 2x^3 - 3x^2 - 12x + 4 \]

and for each determine whether it is a maximum or minimum point.

[1] \( x = -2 \) (minimum); \( x = 1 \) (maximum)
[2] \( x = -0.9 \) (maximum); \( x = 1.5 \) (minimum)
[3] \( x = 0.9 \) (maximum); \( x = -1.5 \) (minimum)
[4] \( x = 2 \) (minimum); \( x = -1 \) (maximum)
[5] None of the above.
Question 11
The supply function of a good is given by
\[ P_s = Q^2 - 2Q + 12. \]
Determine the producer surplus if ten units are supplied \((Q = 10)\) at price \(P\).

1. 366.67
2. 566.67
3. 686.67
4. 902.00
5. None of the above.

Question 12
If the demand and supply functions of a certain product are given by
\[ P_d = 74 - Q^2 \quad \text{and} \quad P_s = 10 + 6Q, \]
where \(P\) is price and \(Q\) is quantity (rounded to an integer), the consumer surplus at equilibrium is

1. \(CS = 83.33\)
2. \(CS = 112.44\)
3. \(CS = 128.33\)
4. \(CS = 325.40\)
5. none of the above.

Questions 13 and 14 are based on the following information:
The demand and total cost functions for a monopolist are
\[ P = 120 - 5Q \quad \text{and} \quad TC = \frac{4}{30} Q^3 - 2Q^2 + 18Q + 60 \]
respectively, where \(P\) is price and \(Q\) quantity.

Question 13
The total revenue and profit functions are given by

1. \(TR = 120 - 5Q; \ \pi = -\frac{1}{30} Q^3 + 2Q^2 - 23Q + 60.\)
2. \(TR = 120 - 5Q; \ \pi = -\frac{4}{30} Q^3 - 2Q^2 + 13Q + 200.\)
3. \(TR = 120Q - 5Q^2; \ \pi = -\frac{4}{30} Q^3 - 3Q^2 + 102Q - 60.\)
4. \(TR = 120Q - 5Q^2; \ \pi = -\frac{4}{30} Q^3 - 7Q^2 + 138Q + 60.\)
5. none of the above.
Question 14
How many units should be produced to maximise profit?

[1] 4
[2] 10
[3] 11
[4] 20
[5] None of the above.

Question 15
The demand function of a good is given by

\[ P = 44e^{-0.7Q}, \]

where \( P \) is price and \( Q \) is quantity. Find the price elasticity of demand at \( Q = 2 \) and say whether demand is elastic or inelastic at this level of output (justify your answer).

[1] \( \varepsilon_d = -0.016 \); demand inelastic since \( |\varepsilon_d| = 0.016 < 1 \)
[2] \( \varepsilon_d = -0.714 \); demand inelastic since \( |\varepsilon_d| = 0.714 < 1 \)
[3] \( \varepsilon_d = -2.8 \); demand elastic since \( |\varepsilon_d| = 2.8 > 1 \)
[4] \( \varepsilon_d = -15.5 \); demand inelastic since \( |\varepsilon_d| = -15.5 < 1 \)
[5] None of the above.
11 Semester 2 – Compulsory assignments

11.1 Assignment 01 (S2)

This MCQ assignment is compulsory.

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Instructions

▷ Work through *Study units 1 and 2* in MO001 before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

**Question 1**

Rewriting the equation

\[ V = \frac{5t + 1}{t - 1} \]

with \( t \) in terms of \( V \), gives

[1] \( t = \frac{V}{5} \).
[2] \( t = \frac{V + 1}{V - 5} \).
[3] \( t = \frac{V - 1}{V - 5} \).
[4] \( t = \frac{V + 1}{V - 5} \).
[5] None of the above.

**Question 2**

The demand function of a product is given as

\[ P = 58 - 0.4Q, \]

where \( P \) and \( Q \) are the price and quantity, respectively. Determine the consumer surplus of the product if the market price \( P = 10 \).

[1] 1296
[2] 1566
[3] 2880
[4] 3480
[5] None of the above.
Questions 3 and 4 are based on the following information:
The demand and supply functions for free-range chickens are

\[ P_d = 50 - 0.6Q \quad \text{and} \quad P_s = 20 + 0.4Q, \]

where \( P \) is the price and \( Q \) the quantity.

**Question 3**
The equilibrium price and quantity are

[1] \( P = 8.00; \ Q = 70. \)
[2] \( P = 32.00; \ Q = 30. \)
[3] \( P = 48.50; \ Q = 3. \)
[4] \( P = 68.00; \ Q = 30. \)

**Question 4**
If free-range chickens are subsidised by R4, the equilibrium price and quantity are

[1] \( P = 17.20; \ Q = 3. \)
[2] \( P = 29.60; \ Q = 34. \)
[3] \( P = 34.40; \ Q = 26. \)
[4] \( P = 84.00; \ Q = 170. \)

**Question 5**
If the demand function of a commodity is

\[ Q = 80 - 2.5P, \]

where \( P \) and \( Q \) are price and quantity respectively, determine the price elasticity of demand when the price is R20. Indicate whether demand is elastic or inelastic at this price and provide justification for your answer.

[1] \( \varepsilon_d = -1.7; \) because \( | -1.7| = 1.7 > 1, \) demand is elastic
[2] \( \varepsilon_d = -0.6; \) because \( | -0.6| = -0.6 < 1, \) demand is elastic
[3] \( \varepsilon_d = 0.6; \) because \( |0.6| = 0.6 < 1, \) demand is inelastic
[4] \( \varepsilon_d = 1.7; \) because \( |1.7| = 1.7 > 1, \) demand is inelastic
[5] None of the above.
**Question 6**

Which one of the following statements is true?

Price elasticity of demand measures

[1] changes in a product’s price.

**Question 7**

Solve the following system of linear equations:

\[
\begin{align*}
  x + 2y - z &= 5 \\
  2x - y + z &= 2 \\
   y + z &= 2
\end{align*}
\]

The sum of the values of \(x\), \(y\) and \(z\) of the solution is

[1] 0.

**Question 8**

A furniture company manufactures dining room tables and chairs. The company has 150 hours of assembly time available per week and workers must spend at least 100 hours on finishing per week. A table requires 540 minutes for assembly and 180 minutes for finishing. Each table is sold for R4000 and each chair for R1500. If \(x\) is the number of tables and \(y\) the number of chairs produced per week, the constraints and the objective function of the company is

[1] \(2,5x + y \geq 1500; 9x + 3y \leq 4000; x, y \geq 0; \pi = 150x + 100y.\)
[2] \(9x + 2,5y \geq 150; 3x + y \leq 100; x, y \geq 0; \pi = 4000x + 1500y.\)
[3] \(9x + 2,5y \leq 150; 3x + y \geq 100; x, y \geq 0; \pi = 4000x + 1500y.\)
[4] \(540x + 150y \geq 150; 180x + 60y \leq 100; \pi = 4x + 1,5y.\)
**Question 9**

Consider the following set of inequalities:

\[
\begin{align*}
y & \geq 5 - 2.5x \quad (1) \\
y & \leq 3 - x \quad (2) \\
x; y & \geq 0. \quad (3)
\end{align*}
\]

The correct graphical representation of this set of inequalities is given by

**Question 10**
The supply function of a certain product is

\[ P = 50 + 2Q, \]

where \( P \) is the price and \( Q \) is the number of units produced. Find the producer surplus if the market price \( P = 230 \).

[1] 3400
[2] 8100
[3] 12600
[4] 20700
[5] None of the above.

**Questions 11 and 12 are based on the following information:**

Consider the following graph showing the demand and supply functions for a certain product:

![Graph showing demand and supply functions](image)

**Question 11**
Which line represents the demand function and which represents the supply function?

[1] Line BD represents the supply function and there is no demand line.
[2] Both lines (1) and (2) represent supply functions and there is no demand line.
[3] Line (1) represents the demand function and line (2) the supply function.
[4] Line (1) represents the supply function and line (2) the demand function.
[5] None of the above.
Question 12
Which of the following statements is true?

[1] Point D is the break-even point.
[2] Point C represents the price that no consumers will accept.
[3] The area of triangle ABD represents the consumer surplus at equilibrium.
[4] The area of triangle ACD represents the producer surplus at equilibrium.
[5] None of the above.

Question 13
The toy factory produces $X$ cast-iron toy aeroplanes per month at a fixed cost of R25 000. Each aeroplane costs R15 to produce and it is sold for R45 each. The total cost and profit functions are

[1] $TC = 15X + 25 000; \; \pi = 30X - 25 000.$
[2] $TC = 45X + 25 000; \; \pi = 30X - 25 000.$
[3] $TC = 45X; \; \pi = 60X - 25 000.$
[4] $TC = 25 000 + 15X; \; \pi = 30X + 25 000.$

Question 14
It is known that pancake sales at an athletics meeting between a number of schools follow the following pattern: when the price is R3,00 per pancake, 3 000 pancakes are sold during the day and when the price is R2,50 each, 5 000 are sold. Find the demand function with quantity, $Q$, as a function of price, $P$.

[1] $Q = -4P + 15$
[2] $Q = -4000P + 15000$
[3] $Q = -1000P + 6000$
[4] $Q = 4000P - 9000$
[5] None of the above.

Question 15
Ndumi produces costume jewellery. It costs her R30 to produce a bracelet and her fixed costs for the bracelets are R900. If she sells the bracelets for R60 each, how many should she produce and sell to break even?

[1] 30
[2] 10
[3] 30
[4] 450
[5] None of the above.
11.2 Assignment 02 (S2)

This written assignment is compulsory.

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Instructions

▷ Work through *Study until 3* in MO001 before attempting this assignment.
▷ Answer all the questions.
▷ Show your steps to find the answers. Marks will be given for these and not only for the answer.
▷ Submit your answers in PDF format on myUnisa.
▷ Always provide your answer in a full sentence with the appropriate units.
▷ You may use Excel or Maxima for assistance when you need to draw graphs and/or solve equations.

Only a selection of questions will be marked, but you will receive solutions to all the questions.

Question 1

Consider the following table of \((x; y)\) pairs:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0.25</td>
<td>0.50</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Have these pairs been generated by a linear or non-linear function? Justify your answer by providing a rough graph.

(b) What is this kind of curve called?

Question 2

(a) Simplify the expression

\[ \frac{-x^2 + 6x - 5}{x - 5} \]

(b) Solve the equation

\[ 5 + x = 4x^2 - 4 + x. \]
Question 3
Use the rules for indices in the following questions:

1. Simplify the expression
\[
\frac{e^x(1 + e^{1-x})}{e}.
\]

2. Solve the equation
\[
3^x3^{x+1} = \sqrt{9}.
\]

Question 4
Use the rules of logarithms in the following questions:

(a) Simplify the expression
\[
8 \ln 2 + 3 \ln 8 - 2 \ln 3.
\]
(Write it as a single term containing \(\ln\) before calculating the value. Round the answer to two decimals.)

(b) Solve the equation
\[
2 \ln x - \ln(x + 2) = 0.
\]

Question 5
Consider the quadratic function
\[
f(x) = -3x^2 + 15x - 12.
\]

(a) Find the coordinates of the turning point of \(f\).

(b) Find the roots of \(f\).

(c) Sketch a rough graph of the function \(f\).

Question 6
The demand and supply functions of a certain product are given by
\[
P_d = \frac{56}{Q+2} \quad \text{and} \quad P_s = 10 + 2Q,
\]
where \(P\) is price and \(Q\) is quantity.

1. Find the equilibrium price and quantity for the product.

2. Draw a graph of the demand and supply functions for \(-0.5 \leq Q \leq 3\), clearly indicating the equilibrium point.
**Question 7**
The demand function of a commodity is

\[ Q = 80 - 2P, \]

where \( Q \) is quantity (in hundreds) and \( P \) is price. Fixed costs are R100 and it costs R15 to produce each unit.

(a) Write down the total revenue \((TR)\), total cost \((TC)\) and Profit \((\pi)\) functions.
(b) How many units should be produced to maximise profit?
(c) Find the optimum price for the product and the maximum profit that can be made.

**Question 8**
A firm has a total cost function

\[ TC = \sqrt{4 + 6Q}, \]

with \( Q \) the number of units produced (in thousands).

(a) If units are sold for R2 each, find the firm’s profit function.
(b) Determine the firm’s break-even point.
(c) Draw a graph of the \( TR \) and \( TC \) functions for \(-1 \leq Q \leq 5\) indicating the break-even point and the areas where profit and loss is made.

**Question 9**
The number of people who have contracted a contagious disease \( t \) days after an epidemic has started, is approximated by the exponential function

\[ Q(t) = \frac{5000}{2 + 1.249e^{-0.3t}}. \]

(a) How many people contracted the disease initially?
(b) Approximately how many people have contracted the disease after 15 days?
(c) After how many days, rounded to an integer, will 1000 people have contracted the disease?

**Question 10**
The roots of a cubic function \( f \) are \(-3, 3 \) and \( 4 \).

(a) Find the function \( f \).
(b) Draw a graph of \( f \), clearly indicating the labels of the axes and the roots.
(c) Provide an estimate of the coordinates of the maximum and minimum points of \( f \) (rounded to two decimals).
11.3 Assignment 03 (S2)

This MCQ assignment is compulsory.

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Instructions

▷ Work through Study units 4 and 5 in MO001 before attempting this assignment.
▷ Answer all the questions.
▷ Submit your answers electronically through myUnisa.

**Question 1**
The derivative of \( f(x) = \frac{2}{x} - 2\sqrt{x} + 3 \) is

1. \(-\frac{2}{x^2} - \frac{1}{\sqrt{x}}\)
2. \(-x^2 - \frac{2}{\sqrt{x}} + 3\).
3. \(2x^{-2} + \frac{2}{\sqrt{x}}\)
4. \(-2x^{-2} + x^{-\frac{3}{2}} + 3\).
5. none of the above.

**Question 2**
The slope of the function \( f(x) = \frac{1}{4}x^3 - 3x^2 + 2x + 8 \) at \( x = 3 \) is

1. \(-16\).
2. \(-7\).
3. \(-4\).
4. \(7\).
5. none of the above.

**Question 3**
The derivative of \( f(x) = \frac{x}{x+10} \) is

1. \(\frac{1}{1}\)
2. \(-\frac{1}{(x-10)x}\).
3. \(\frac{x+1}{x+10}\)
4. \(\frac{1}{x+10} - \frac{x}{(x+10)^2}\).
5. none of the above.
Question 4

The derivative of \( f(x) = 2 \ln(x^3 - 8x) + e^{-3x^2} \) is

1. \( \frac{2}{x^3 - 8x} + e^{-3x^2} \).
2. \( \frac{6x^2 - 16}{x^3 - 8x} + \frac{6x}{e^{-x^2}} \).
3. \( \frac{2(3x^2 - 8)}{x^3 - 8x} - \frac{6x}{e^{3x^2}} \).
4. \( \frac{3x^2 - 8}{x^3 - 8x} + xe^{-3x^2} \).
5. none of the above.

Question 5

Integrating the function \( f(x) = \frac{x + \sqrt{x}}{x} \) results in

1. \( x + 2 \sqrt{x} + C \).
2. \( -\sqrt{x} + C \).
3. \( x - \sqrt{x} + C \).
4. \( \frac{x^2 + 4x^3}{x^2} + C \).
5. none of the above.

Question 6

The integral \( \int \sqrt{3x + 4} \, dx \) is equal to

1. \( \frac{-2}{3\sqrt{3x + 4}} + C \).
2. \( \frac{2}{3} \sqrt{(3x + 4)^3} + C \).
3. \( \frac{2}{3} \sqrt{27x^3} + 2x + C \).
4. \( \frac{2}{3} \sqrt{(3x + 4)^3} + C \).
5. none of the above.

Question 7

Evaluate the definite integral \( \int_{0}^{5} 40e^{-0.3t} \, dt \). (Round to one decimal.)

1. \(-103.6\)
2. \(-31.1\)
3. \(29.7\)
4. \(103.6\)
5. None of the above.
Question 8
The marginal cost function for a good is given by

\[ MC = -Q^2 + 80Q. \]

Find the total cost function if fixed costs are 500.

[1] \(-\frac{Q^3}{3} + 40Q^2 + 500\)
[2] \(\frac{Q^3}{3} + 40Q^2 + C\)
[3] \(-\frac{Q^3}{3} + 80Q^2 + C\)
[4] \(Q^3 + 80Q^2 + 500\)
[5] None of the above.

Question 9
The average cost function of a good is given by

\[ AC = \frac{200}{Q} + 2Q - 36, \]

where \(Q\) is quantity produced. The level of output that minimises average cost is


Question 10
A firm’s production function is given by

\[ Q = 700Le^{-0.02L}, \]

where \(Q\) denotes the number of units produced and \(L\) the number of labourers. Find the size of the workforce that maximises output.

[1] \(L = -50.\)
[2] \(L = 50.\)
[3] \(L = 328.\)
[4] \(L = 700.\)
**Question 11**

Find the values of $x$ for which the function

$$f(x) = x^3 - 12x + 6$$

has turning points. Determine for each of the points whether it is a minimum or maximum point.

1. $x = -3.46$ (maximum); $x = 3.46$ (minimum)
2. $x = 0$ (minimum); $x = 6$ (maximum)
3. $x = -3$ (minimum); $x = 3$ (maximum)
4. $x = -2$ (maximum); $x = 2$ (minimum)
5. None of the above.

**Questions 12 and 13 are based on the following information:**

The demand function for a good is given by

$$P = -2.5Q + 500,$$

where $P$ is price and $Q$ quantity.

**Question 12**

The price and quantity for which total revenue is a maximum, are

1. $P = 0; Q = 200.$
2. $P = 100; Q = 160.$
3. $P = 250; Q = 100.$
4. $P = 300; Q = 80.$
5. none of the above.

**Question 13**

Determine the price elasticity of demand at maximum total revenue. What does this value mean?

1. $\varepsilon = -1; \ 1\% \ price \ increase \ causes \ 1\% \ decrease \ in \ demand \ - \ no \ effect \ on \ total \ revenue$
2. $\varepsilon = -0.4; \ 1\% \ price \ increase \ causes \ TR \ to \ decrease \ by \ 0.4\%$
3. $\varepsilon = 0.4; \ 1\% \ price \ increase \ causes \ demand \ to \ increase \ by \ 0.4\%$
4. $\varepsilon = 1; \ change \ in \ price \ will \ have \ no \ effect \ on \ demand$
5. None of the above.
Questions 14 and 15 are based on the following information:
The demand and supply functions for a certain product are given by
\[ P_d = 35 - Q^2 \text{ and } P_s = 3 + Q^2, \]
with \( P \) representing price and \( Q \) quantity.

**Question 14**
At equilibrium price and quantity are

1. \( P_0 = 4; \ Q_0 = 19. \)
2. \( P_0 = 19; \ Q_0 = 4. \)
3. \( P_0 = 22; \ Q_0 = 4.4. \)
4. \( P_0 = 259; \ Q_0 = 16. \)
5. none of the above.

**Question 15**
Find the consumer surplus at equilibrium.

1. \( CS = -4.949.33 \)
2. \( CS = -1.697.33 \)
3. \( CS = 142.21 \)
4. \( CS = 42.67 \)
5. None of the above.