Tutorial letter 101/3/2018

Mathematical Modelling

APM1514

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important information about your module and includes the assignment questions for both semesters.

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CONTENTS

1 INTRODUCTION

We wish to welcome you to the module APM1514 (Mathematical modelling), and hope that you will enjoy studying it. We shall do our best to make your study of this module successful.

This tutorial letter contains important information about how to study this module. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information about your study material. Please study this information carefully.
The Department of Despatch should have supplied to you the tutorial letter 101, the study guide and the work book (tutorial letter 102) shortly after your registration. The other tutorial letters will be sent to you throughout the semester. Follow the instructions in the brochure entitled *Study @ Unisa* if you have not received some of the material that should have been sent to you.

Note that if you have access to the Internet, you can view, download and print the study guide and all the tutorial letters for the modules for which you are registered on the University's online campus, myUnisa, at http://my.unisa.ac.za.

Take note that every tutorial letter you will receive is important and you should read them all immediately and carefully. Some information contained in these tutorial letters may be urgent, while others may, for example, contain examination information. So, it is wise to keep them all in a file!

2 PURPOSE OF AND OUTCOMES FOR THE MODULE

2.1 Purpose

In this module, you will be introduced to the basics of mathematical modelling, using difference equations, differential equations and systems of differential equations; as well as qualitative analysis of difference and differential equations. Students credited with this module will be able to use certain basic models; construct and analyse mathematical models; and do qualitative analysis of differential equations in one and two dimensions. You will be able to apply the skills and knowledge gained in this module to solve problems by using mathematical models

2.2 Outcomes

Qualifying students will be able to:

- solve separable differential equations,
- undertake qualitative analysis of an autonomous differential equation,
- undertake qualitative analysis of an autonomous system of two differential equations,
- apply standard mathematical models correctly,
- build and analyse mathematical models,
- solve problems by using a mathematical model.

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

The lecturers responsible for this module are as follows:

Dr. A. Kubeka
Tel: +2711 670 9157
Room no: 647
GJ Gerwel Building
Florida Campus
e-mail: kubekas@unisa.ac.za

Dr. J. Manale
Tel: (011) 471 2912
Room no: 646
GJ Gerwel Building
Florida Campus
e-mail: manaljm@unisa.ac.za

3.2 Department

The contact details of the department are as follows:

Department of Mathematical Sciences
Office: GJ Gerwel Building, Room 6-66
Telephone: +2711 670 9147
Fax: +2711 670 9171
E-mail: mathsciences@unisa.ac.za

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *Study @ Unisa* that you received with your study material. This brochure contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

4 RESOURCES

4.1 Prescribed books

There is no prescribed textbook for this module. All the material is contained in the study guide.

4.2 Recommended books

There are no recommended books for this module.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

4.4 Library services and resources information

For brief information, go to www.unisa.ac.za/brochures/studies

For detailed information, go to http://www.unisa.ac.za/library. For research support and services of personal librarians, click on ”Research support”.

The library has compiled a number of library guides:

- finding recommended reading in the print collection and e-reserves
  –http://libguides.unisa.ac.za/request/undergrad
- requesting material
  – http://libguides.unisa.ac.za/request/request
- postgraduate information services
  – http://libguides.unisa.ac.za/request/postgrad
- finding, obtaining and using library resources and tools to assist in doing research
  – http://libguides.unisa.ac.za/Research_Skills
- how to contact the library/finding us on social media/frequently asked questions
  – http://libguides.unisa.ac.za/ask

5 STUDENT SUPPORT SERVICES

For information on the various student support systems and services available at Unisa (e.g. student counseling, tutorial classes, language support), please consult the publication *Study @ Unisa* that you received with your study material.

5.1 Contact with Fellow Students

5.1.1 Study Groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. **Please consult the publication *Study @ Unisa* to find out how to obtain the addresses of students in your region.**

5.2 myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa – all through the computer and the internet.

Joining *myUnisa* will offer you the following benefits:

- You have access to the additional resources on this module.
- You will be able to immediately download all your study material from this site, in electronic format.
- You can use the discussion forum to communicate with your fellow students.
- You can contact your lecturer through the e-mail link of your *myUnisa* module page.

For this module, the lecturer will use announcements and FAQs (frequently asked questions) throughout the semester. You will also be able to access self assessment quizzes, which will help you know how well you understand the study material.

To go to the *myUnisa* website, start at the main Unisa website, http://www.unisa.ac.za, and then click on the "Login to *myUnisa*" link on the right-hand side of the screen. This should take you to the *myUnisa* website. You can also go there directly by typing in http://my.unisa.ac.za. On the website you will find general Unisa related information, plus a module site for each module you are registered for. Please consult the publication *Study @ Unisa* which you received with your study material for more information on *myUnisa*.

5.3 e-Tutors

Information on e-tutoring and face-to-face tutoring offerings at Unisa

Please be informed that, with effect from 2013, Unisa offers online tutorials (e-tutoring) to students registered for modules at NQF level 5 and 6, this means qualifying first year and second year modules. You are lucky since this module falls in this category.

Once you have been registered for this module, you will be allocated to a group of students with whom you will be interacting during the tuition period as well as an e-tutor who will be your tutorial facilitator. Thereafter you will receive an sms informing you about your group, the name of your e-tutor and instructions on how to log onto MyUnisa in order to receive further information on the e-tutoring process.

Online tutorials are conducted by qualified E-Tutors who are appointed by Unisa and are offered free of charge. All you need to be able to participate in e-tutoring is a computer with internet connection. If you live close to a Unisa regional Centre or a Telecentre contracted with Unisa, please feel free to visit any of these to access the internet. E-tutoring takes place on MyUnisa where you are expected to connect with other students in your allocated group. It is the role of the e-tutor to guide you through your study material during this interaction process. For your to get the most out of online tutoring, you need to participate in the online discussions that the e-tutor will be facilitating.

Moreover, there are modules which students have been found to repeatedly fail, these modules are allocated face-to-face tutors and tutorials for these modules take place at the Unisa regional centres. These tutorials are also offered free of charge, however, it is important for you to register at your nearest Unisa Regional Centre to secure attendance of these classes.

6 STUDY PLAN

The semester during which you study at UNISA consists of 15 weeks between the last day of registration and the beginning of the examination period, during which time you need to study and understand the contents of the module, complete and submit your assignments, and then prepare for the examination. Therefore it is important that you create a timetable for planning your studies for this module, and all the other modules you take this semester or year.
Please start studying as soon as you receive your study material. Note that if you are registered for Semester 1, then all your assignments need to be submitted by end of April and you will write your examination in May-June; and if you are registered for Semester 2, then your assignments need to be submitted by early October and you will write your examination in October-November.

6.1 Suggested time table

The following time tables for this module are provided as a starting point for your personal schedule.

SEMESTER 1	**Study units for preparing your assignments**	**From**	**To**
Assignment 1	Units 1, 2, 3, 4	Registration	3 March
Assignment 2	Units 5, 6, 7, 8	4 March	24 March
Assignment 3	Units 9, 10, 11	25 March	25 April
Exam	Prepare for the examination	26 April	Exam date

SEMESTER 2	**Study units for preparing your assignments**	**From**	**To**
Assignment 1	Units 1, 2, 3, 4	Registration	18 August
Assignment 2	Units 5, 6, 7, 8	19 August	8 Sept
Assignment 3	Units 9, 10, 11	9 Sept	10 Oct
Exam	Prepare for the examination	11 Oct	Exam date

6.2 How to study this module

6.2.1 An overview of the module

The outcomes of the module are listed in Section 2.2 of this tutorial letter. To pass this module, you must achieve these outcomes.

To do this, you will need to study and work through the material in the study guide, until you are able to understand and apply the concepts and principles involved. The study guide contains activities, which are there to help you ensure that you have mastered the material. Another way to find out how you are doing is through the assignments that you are supposed to submit throughout the semester. The lecturer will mark your work and give individual feedback to you.

There is also a workbook, which contains more exercises for you to practice your knowledge and skills on. The answers to all of the exercises are given at the back of the workbook, and many of them have fully worked solutions.

For even more help in case you need it, please join myUnisa — on the module web page at myUnisa, there will be more resources available. These will be explained on the web page.

The final decision on whether you have mastered the module outcomes well enough comes from your final mark for the module, which is calculated from your semester mark and the examination mark. (How exactly this is done is explained later on.)

Note that the examination date is fixed, and it is your duty to make sure that you are ready to write the examination when it comes! In mathematics, it is often very hard to catch up again with the work if you fall behind, since you need to understand previous material thoroughly before learning new things.

Although you do need to take responsibility for your studies, remember that you are not alone. Your lecturer is there to help you, and you can also contact your fellow students and use Unisa's student support systems. Details of all of these are listed elsewhere in this tutorial letter!

6.2.2 Guidelines for studying this module

Guidelines of what you should do while studying for this module are therefore as follows:

- There is quite a bit of work to be done in the 15 weeks of study time. Make a timetable for yourself, to make sure you know what amount of work you need to do by what time to keep up to date with the work.
- Work through the study guide. This includes doing the activities, and working on more examples from the workbook if you feel you need more practice.
- You will need to use a calculator for this module. Make sure you know how to use your calculator! You will be allowed to bring a non-programmable calculator to the examination.
- Submit the assignments by their due dates. The due dates of the assignments are chosen in such a way that you will need to work steadily through the semester. When you receive back your marked assignments, make note and take advantage of the lecturer's feedback on your work.
- Prepare well for the examination.

7 PRACTICAL WORK AND WORK-INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

Assessment is the process where the lecturer assesses your work by comparing it to the module outcomes and the related assessment criteria. The assessment in this module consists of formative assessment and summative assessment.

Formative assessment means assessment of your work while you are still studying. This is particularly important in distance learning since it might sometimes be the only way you can get feedback on how you are doing, while you can still benefit from it. In this module, formative assessment is through the assignments. The lecturer marks your work and gives you individual feedback on how you are doing, as well as suggestions for improvement. Make sure to take advantage of the lecturer's feedback! You will also receive model solutions to the assignments.

Summative assessment refers to the final mark you receive for this module. In this module, your final mark is calculated from you examination mark (which counts for 80%) and from your semester mark (which counts for 20%). The semester mark is determined by how well you did in your assignments. Details of how this works are given in the following.

The semester mark and the final mark

Your **final mark** will be calculated from your **semester mark** and the **examination mark.**

The **semester mark** is calculated from your assignment results (the percentages you receive for the assignments). The weights of the different assignments differ: Assignment 1 counts for 20%, Assignment 2 counts for 40% and Assignment 3 for 40% . That is, the semester mark is calculated as

$$\text{semester mark} = \frac{1}{100} * (20 \cdot A_1 + 40 \cdot A_2 \ + 40 \cdot A_3)$$

where A_1 to A_3 are the percentages you received in assignments 1 to 3, respectively. Assignments not submitted, or submitted late, will give you 0%.

- The **examination mark** is the percentage mark you get in the examination.

- The examination mark contributes 80% to the final mark, and the semester mark contributes 20%. That is, your **final mark** is calculated as

$$\text{final mark} = 0.8 * \text{(examination mark)} + 0.2 * \text{(semester mark)}.$$

You pass the module if your final mark is $\geq 50,$ and you pass it with distinction if your final mark is $\geq 75.$ There is also a subminimum rule, which says that you must get at least 40% in the examination to pass the module.

IMPORTANT: Please note that a poor semester mark could lower your final mark! It is therefore important that you try to complete all the assignments as well as your can – if your year mark is zero, you must get 63% in the exam to pass the module! Also, you must make sure that you submit all the assignments on time, since if we receive your assignment too late, we have to give you 0% for it.

8.1 Assessment plan

There are three assignments on this module.

- The first assignment is compulsory: You must submit Assignment 01 by its due date, otherwise you will not get examination admission.
- All three assignments will help you work through the study guide and give you an idea on which topics you understand correctly, and where you are struggling. Please do view these assignments as a chance for you to get feedback from your lecturer! Assignments 2 and 3 contribute a lot to the year mark, so you should make sure to complete them as well as you can.

In conclusion, you should complete all assignments as well as you can: To get admission to the examination; and because of the semester mark system which means that how well you do your assignments will also have a direct effect on the final mark you get for this module; and most importantly, because submitting the assignments gives you a chance to find out how well you have mastered the course contents, and for us to give you feedback on your progress!

8.2 Assignment numbers

8.2.1 General assignment numbers

The assignments are numbered 01 to 03. Please remember to give your assignment the correct number in the assignment cover. The assignment questions for Semester 1 and for Semester 2 are listed in Appendix A and Appendix B at the end of this tutorial letter.

8.2.2 Unique assignment numbers

Please note that each assignment has its unique six-digit assignment number which has to be written on the cover of your assignment or on the mark reading sheet upon submission. The unique numbers are given in the table in the next section of this tutorial letter; you will also find them in the heading of each set of assignment questions.

8.3 Assignment due dates

For each assignment there is a **FIXED CLOSING DATE**, which is the date by which the assignment **must reach** the university. The closing dates for submission of the assignments are given in the following table. We also give the contribution of each assignment to the semester mark.

SEMESTER 1			
Assignment no.	**Fixed closing date**	Semester mark %	Unique number
01	02 March 2018	20	876250
02	23 March 2018	40	806488
03	24 April 2018	40	878085
SEMESTER 2			
Assignment no.	**Fixed closing date**	Semester mark %	Unique number
01	17 August 2018	20	758703
02	07 September 2018	40	842562
03	03 October 2018	40	774918

8.4 Submission of assignments

Enquiries about assignments, such as whether they have been received by the university, what credit you obtained, when they were returned to you, etc., should be addressed to the Assignments section. For detailed information and requirements as far as assignments are concerned, see *Study @ Unisa*, which you received with your study package.

To submit an assignment **via myUnisa:**

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left–hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.

You can submit mathematics assignments in electronic format as PDF files, but please note that you must still use all the correct mathematical notation, and include all necessary graphs, diagrams, and so on, just as if you were submitting a hand-written assignment! You can use a word-processing program with an equation editor (e.g. MSWord) and convert it to PDF, or you can scan your hand-written assignment and submit it in PDF format; or you can use special mathematical typesetting programs such as LaTeX and submit the PDF output file.

Please note: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

8.5 The assignments

This tutorial letter 101 contains the assignments for both semesters, so select the semester you are enrolled for and do the set of assignments for that semester only. The assignments are in Appendix section of this tutorial letter.

8.6 Other Assessment methods

All the other assessment methods in this module are self-assessment. To find out whether you are on the right track, you can: Do the activities in the study guide and compare your answers with the feedback; and do exercises in the work book and compare your answers with the given ones.

8.7 The examination

Examination Admission
To be admitted to the examination you must submit the compulsory assignment, i.e. Assignment 01 by the due date. Note that admission therefore does not rest with the department and if you do not submit that particular assignment in time, we can do nothing to give you admission. Although you are most probably a part time student with many other responsibilities, work circumstances will not be taken into consideration for exemption from assignments or the eventual admission to the examination.

No concession will be made to students who do not qualify for the examination
CALCULATION OF FINAL MARK
Your final mark will be composed of 80% for your exam mark and 20% of your year mark.

Examination Period
This module is offered in a semester period of fifteen weeks. This means that

- if you are registered for the first semester, you will write the examination in May/June 2018 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in October/November 2018.
- if you are registered for the second semester, you will write the examination in October/November 2018 and should you fail and qualify for a supplementary examination, that supplementary examination will be written in May/June 2019.

The examination section will provide you with information regarding the examination in general, examination venues, examination dates and examination times. Eventually, your results will also be processed by them and sent to you.

Examination Paper
The examination consists of a two hour paper. You are allowed to use a non-programmable calculator in the examination. Should you have a final mark of less than 50%, it implies that you failed the module. However, should your results be within a specified percentage (from 40% to 49%), you will be given a second chance in the form of a supplementary examination. If you fail the examination with less than 40%, the year mark will not count to help you pass. Please note also that the year mark does not apply in the case of a supplementary examination. The final mark after a

supplementary examination is simply the mark you achieved in that examination, expressed as a percentage.

Previous Examination Papers
Previous examination papers are available to students on myUnisa under Additional Resources.

9 FREQUENTLY ASKED QUESTIONS

The Study @ Unisa brochure contains an A-Z guide of the most relevant study information. Please refer to this brochure for any other questions.

10 SOURCES CONSULTED

No books were consulted in preparing this tutorial letter.

11 IN CLOSING

We hope that you will enjoy this module and wish you all the best!

Your lecturer,

12 ADDENDUM

The questions for each assignment for Semester 1 follow. In each assignment, you can obtain a total of 100%, and the total percentage you can obtain from each question is also indicated in square brackets.

SEMESTER 1
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Based on Units 1, 2, 3, 4
Fixed closing date: 02 March 2018
Unique Assignment Number: 876250

1. A difference equation is given by $a_{n+1} = a_n (8 - 2a_n)$. Find all the equilibrium values of the difference equation. **[15]**

2. Consider a savings account, which operates as follows. Initially, the amount of R10 000 is deposited on the account. At beginning of each month, a deposit of R500 is added to the account. At the end of each month, interest is added at the rate of 2% per month to the amount of money which was on the account during that month. Let A_n denote the amount of money in the account at the end of month n, with A_0 denoting the original amount.

   (a) Write down the difference equation for A_n (that is, an expression for A_{n+1} in terms of A_n). Write also down the initial value, A_0.

   (b) Explain why we would not expect this system to have a positive equilibrium point. Prove also mathematically that there is no positive equilibrium point. **[25]**

3. Find the general solution $x(t)$ to the following differential equation. **[20]**

$$\frac{dx}{dt} = \frac{2t}{5x}$$

4. Solve the following initial value problem. **[20]**

$$\frac{dy}{dt} = 1 - 4y, \quad y(1) = 5.$$

5. Find the general solutions $x(t)$ to the following differential equations: **[20]**

   (a) $\dfrac{dx}{dt} = xt^2$

   (b) $\dfrac{dx}{dt} + 3x = 0$

Maximum marks: [100%]

SEMESTER 1

ASSIGNMENT 02
Based on Units 5, 6, 7, 4
Fixed closing date: 23 March 2018
Unique Assignment Number: 806488

1. Assume that the population of a town obeys Malthusian growth. Assume that the size of the population was 2000 in year 1900, and 50 000 in year 1950.

   (a) Find the value of the growth constant k.

   (b) How long does it take for the population to grow by 20%?

   (c) How big was the population in year 2000?

   (d) What was the rate of change of the population in year 2000?

   (e) Calculate the size of the population in year 2001, and from that the actual change in population during the one year, from 2000 to 2001.

   (f) Explain why the two values calculated in (d) and (e) above do not necessarily have to be the same. **[15]**

2. This questions deals with the decay of a radioactive substance. Assume that today there is 10 grams of the substance, while 1000 years ago there was 100 grams of it.

   (a) Find the half life of the substance (that is, the time it takes for the substance to decay from N_0 to $N_0/2$).

   (b) If there is 15 grams of the substance today, how much will there be 600 years from now?

   (c) If there is 15 grams of the substance today, what is the rate of change of the quantity today? What will be the rate of change 600 years from now? **[15]**

3. The populations of two countries, A and B, are assumed to grow according to the Malthusian model, Country A with a growth constant $k = 0.005$ and Country B with growth constant $k = 0.01$. Let $P_A(t)$ and $P_B(t)$ denote the sizes of the two populations at time t.

   (a) If the initial population sizes at some initial time $t = 0$ are 10 000 for country A, and 2 000 for Country B, when will the population sizes be the same?

   (b) Give an example of an initial population size for A and an initial population size for B, such that the populations are never going to be the same size for any time $t > 0$.

   (c) Prove that the ratio of the sizes of the populations,

   $$R(t) = \frac{P_A(t)}{P_B(t)},$$

   also obeys a Malthusian model. What is its growth constant? **[15]**

4. Consider the model
$$\frac{dP}{dt} = 2P - 4P^3.$$
Here, $P(t)$ denotes the size of a population at time t.

   (a) Is this a logistic model? Justify your answer!

   (b) Draw the phase line of the model.

   (c) Draw a sketch of the solution curve $P(t)$, if $P(0) = 2$. **[15]**

5. An object with the temperature of 200 degrees is placed in a tank of liquid. The temperature of the liquid is kept constant at 20 degrees. Assume that Newton's law of cooling applies, and assume that time t is measured in minutes.

   (a) If the temperature of the object was measured to be 80 degrees after 30 minutes, find the value of the coefficient k.

   (b) How long will it take for the object to cool down to 30 degrees? **[10]**

6. Which of the following statements are true, and which are false? Justify your answers!

   (a) In a logistic model, if the population is decreasing then $a < 0$.

   (b) In a logistic model, if the population is increasing then $a > b$.

   (c) In a logistic model, if $a = b$ then there will be just one equilibrium point. **[15]**

7. Let a system be given by
$$\frac{dx}{dt} = x\,(x - a)$$
where a is a real number.

   (a) How should a be chosen to make $x = 2$ an equilibrium point?

   (b) How should a be chosen to make $x = 0$ an unstable equilibrium point?

   (c) How should a be chosen to guarantee that any solution starting with $x_0 > 1$ will increase without bound?

   (d) Explain why, regardless of the value of a, no solution to this system can ever decrease without bound. **[15]**

Maximum marks: [100%]

SEMESTER 1

ASSIGNMENT 03
Based on Units 9, 10, 11
Fixed closing date: 24 April 2018
Unique Assignment Number: 878085

1. A model with harvesting is given by

$$\frac{dP}{dt} = 0.2P - 0.03P^2 - 50,$$

where $P(t)$ is the size of the population after time t. What should the initial population size P_0 be like, to guarantee that the population does not die out? Justify your answer! **[25]**

2. For each of the systems in (a) and (b) below, do the following: Draw the phase diagram of the system; list all the equilibrium points; determine the stability of the equilibrium points; and describe the outcome of the system from various initial points. You should consider all four quadrants of the xy-plane. (For full marks, all the following must be included, correct and clearly annotated in your phase diagram: The coordinate axes; all the isoclines; all the equilibrium points; the allowed directions of motion (both vertical and horizontal) in all the regions into which the isoclines divide the xy plane; direction of motion along isoclines, where applicable; examples of allowed trajectories in all regions and examples of trajectories crossing from a region to another, whenever such a crossing is possible.)

(a)

$$\begin{aligned}\frac{dx}{dt} &= x - y \\ \frac{dy}{dt} &= 2x\end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= -(x-1)\,y \\ \frac{dy}{dt} &= x^2\end{aligned}$$

[40]

3. Consider the system

$$\frac{dx}{dt} = 2xy - 3x + 3$$

$$\frac{dy}{dt} = xy - y$$

where x and y denote the sizes of two interacting populations.

(a) How does the x species and the y species, respectively, behave in the absence of the other species?

(b) Describe the type of interaction between the two species (e.g. competition, predator/prey, etc.)

(c) Draw the phase diagram and use it to predict the outcome of the system if initially $x_0 = 1$, $y_0 = 2$. **[35]**

Maximum marks: [100%]

The questions for each assignment for Semester 2 follow. In each assignment, you can obtain a total of 100%, and the total percentage you can obtain from each question is also indicated in square brackets.

SEMESTER 2
COMPULSORY ASSIGNMENT FOR EXAM ADMISSION

ASSIGNMENT 01
Based on Units 1, 2, 3, 4
Fixed closing date: 17 August 2018
Unique Assignment Number: 758703

1. A difference equation is given by

$$a_{n+1} = a_n^2.$$

   (a) If $a_0 = 2,$ find a_1, a_2, a_3 and $a_{4.}$
   (b) Find all the equilibrium values of the difference equation.
   (c) Write down an expression for a_1, a_2, a_3 and a_4 as a function of the initial value a_0.
   (d) From (c) above, find the general solution to the difference equation. **[20]**

2. Consider a house loan of R200 000, on which interest is charged at the rate of 1% per month at the end of the month; and a repayment of R3000 is made at the beginning of each month.

   Let A_n denote the amount of money still owing at the end of month n, with A_0 denoting the original loan amount.

   (a) Write down the difference equation for A_n (that is, an expression for A_{n+1} in terms of A_n).
   (b) Find the equilibrium point of the system.
   (c) Will the house loan will eventually be paid off or not? Justify your answer!
   (d) Does the difference equation in (a) change (and if it does, how), if we make each of the following changes:
      i. Make the initial loan amount $300000.$
      ii. Change the repayment to 1600 per month.
      iii. Change the interest rate to 2% per month. **[30]**

3. Find the general solutions $x\left(t\right)$ to the following differential equations:

   (a) $\dfrac{dx}{dt} = \dfrac{2x}{t}$

(b) $\dfrac{dx}{dt} + 2x - 1 = 0$ **[20]**

4. Solve the initial value problem. **[30]**

$$\frac{dy}{dt} = 2\left(4 - y\right), \qquad y\left(1\right) = 1.$$

Maximum marks: [100%]

SEMESTER 2

ASSIGNMENT 02
Based on Units 5, 6, 7, 4
Fixed closing date: 07 September 2018
Unique Assignment Number: 842562

1. The population of a town was 100 000 in year 2000 and 60 000 in year 2002. The population is assumed to obey the Malthusian model.

   (a) Find the growth constant $k.$

   (b) When will the population be 10 000$?$

   (c) According to the model, what will the population be in year 2010?

   (d) Write down an expression for $P(t)$, the size of the population after t years, if $t = 0$ in year 2010.

[15]

2. Assume that a radioactive substance decays with a constant of decay $k = 0.001$ when time is measured in years.

   (a) How long does it take for the substance to decay from 15 grams to 3 grams?

   (b) If 5% of an original quantity N_0 is present today, when was N_0 formed initially?

   (c) Explain why in (b) above, we do not need to know the exact value of N_0.

[10]

3. The populations of Country A and Country B both grow according to the Malthusian model, Country A with a doubling time 50 years and Country B with a doubling time 80 years. If the sizes of the populations were the same in year 2000, what was the ratio of the population of Country B to the population of Country A in 1950? What will the ratio be in 2050?

[15]

4. We are given the population readings:

$$\begin{aligned} P &= 120000 \text{ in } 1980, \\ P &= X \text{ in } 1990, \\ P &= 500000 \text{ in } 2000. \end{aligned}$$

   What should the value of X be, so that the population grows according to a Malthusian model?

   Justify your answer. **[15]**

5. Assume that $P(t)$ follows logistic growth, with initial value $P_0 = 100,\ a = 0.2$ and $b = 0.05.$

   (a) Find $P(1)$ and $P(5)$.

   (b) Find the rate of change of P at the times $t = 0$, $t = 1$ and $t = 5$. (Remember that the rate of change is the value of dP/dt!)

   (c) Draw the phase line of this population model.

   (d) Plot a sketch of the solution curve $P(t)$ when $P_0 = 100$, over the interval $0 \leq t \leq 6$. Use the information in your answers to (a), (b) and (c) to make the sketch as accurate as possible. **[15]**

6. For each of the following systems, draw a sketch of:

   (i) dx/dt as a function x,

   (ii) the phase line.

   Also, for each system, list all the equilibrium points and classify each of them as stable or unstable; and predict the outcome of the solution if the system starts at $x(0) = 0.5.$

   (a) $\dfrac{dx}{dt} = 2 - x$

   (b) $\dfrac{dx}{dt} = (x - 1)\, x^2$

   (c) $\dfrac{dx}{dt} = (x - 2)\, x\, (x + 1)$ **[15]**

7. A tank initially contains 200 litres of liquid A. Liquid B is pumped into the tank at the rate of 10 litres per minute. The mixture is stirred continuously and the tank is kept full at 200 litres at all times. The contents of the tank is pumped out at the same rate of 10 litres per minute. Let $X(t)$ denote the amount (in litres) of liquid B in the tank after t minutes.

   (a) Write down the differential equation for $X(t)$, and its initial value.

   (b) Solve the initial value problem in (a) to find the solution $X(t)$ for all $t.$

   (c) How long does it take until the tank contains the same amount of both liquid A and liquid B? **[15]**

Maximum marks: [100%]

SEMESTER 2

ASSIGNMENT 03
Based on Units 9, 10, 11
Fixed closing date: 03 October 2018
Unique Assignment Number: 774918

1. Assume that the population of a species grows according to the model

$$\frac{dP}{dt} = 4(P-2)\,(200-P)\,.$$

   (a) Draw the phase line of the system.

   (b) Explain how the population behaves, for the following initial values:

      i. $P\,(0) = 1;$

      ii. $P\,(0) = 150.$

   (c) Are there any initial population sizes in this model for which the population grows without bound? Justify your answer. **[25]**

2. For each of the systems in (a) and (b) below, do the following: Draw the phase diagram of the system; list all the equilibrium points; determine the stability of the equilibrium points; and describe the outcome of the system from various initial points. You should consider all four quadrants of the xy-plane. (For full marks, all the following must be included, correct and clearly annotated in your phase diagram: The coordinate axes; all the isoclines; all the equilibrium points; the allowed directions of motion (both vertical and horizontal) in all the regions into which the isoclines divide the xy plane; direction of motion along isoclines, where applicable; examples of allowed trajectories in all regions and examples of trajectories crossing from a region to another, whenever such a crossing is possible.)

   (a) $\dfrac{dx}{dt} = x^2, \quad \dfrac{dy}{dt} = -y$

   (b) $\dfrac{dx}{dt} = x\,(y-1)\,, \quad \dfrac{dy}{dt} = 2y$ **[40]**

3. Assume that a model describing two competing species in a closed environment is given by the following system of two differential equations:

$$\begin{aligned}\frac{dx}{dt} &= \left(1-x^2-2y\right)x \\ \frac{dy}{dt} &= (1-2x-y)\,y\end{aligned}$$

(a) Draw the phase diagram of the system.

(b) Is coexistence of the two species possible? Is coexistence likely? Justify your answer!

(c) Explain what the outcome in the system will be if the initial values are $x(0) = 3$, $y(0) = 2$.

[35]

Maximum marks: [100%]