Tutorial Letter 102/2/2015

Quantitative Modelling
DSC1520

Semester 2

Department of Decision Sciences

Important Information:

This tutorial letter contains information on the examination and a previous examination paper with model solutions.

Bar code

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Contents

1 Letter to students

Dear Student

This tutorial letter contains information on the examination, additional exercises and solutions, as well as a previous examination paper with model solutions. Try to work through all the evaluation exercises, assignments, and the previous examination paper when you prepare for the examination. It is also a good idea to work through the assignments of the second semester.

Please contact me via email, fax, telephone or appointment if you need help regarding the study material. My contact details are:

Office: Club One Building, Room 4-37, Hazelwood

Tel : +27 12 433-4602 Email: **mabemgv@unisa.ac.za**

It is better to sort out your problems before the examination than to repeat the module next semester.

Finally, well done you have made it up to here! We wish you everything of the best with the last hurdle, the examination. I hope that you have enjoyed the module. It was a pleasure assisting you in this module. Best wishes for the future.

Kind regards

Ms Victoria Mabe-Madisa (Lecturer)

2 Examination Information

1. Make sure that you know **WHERE** and **WHEN** you are writing DSC1520. Consult myUnisa if you are not sure of the date and time you are writing the examination.

2. The examination is a **two hour** paper.

3. The examination paper consists of 30 multiple-choice questions, giving a subtotal of 100 marks. These questions have to be answered on a mark-reading sheet.

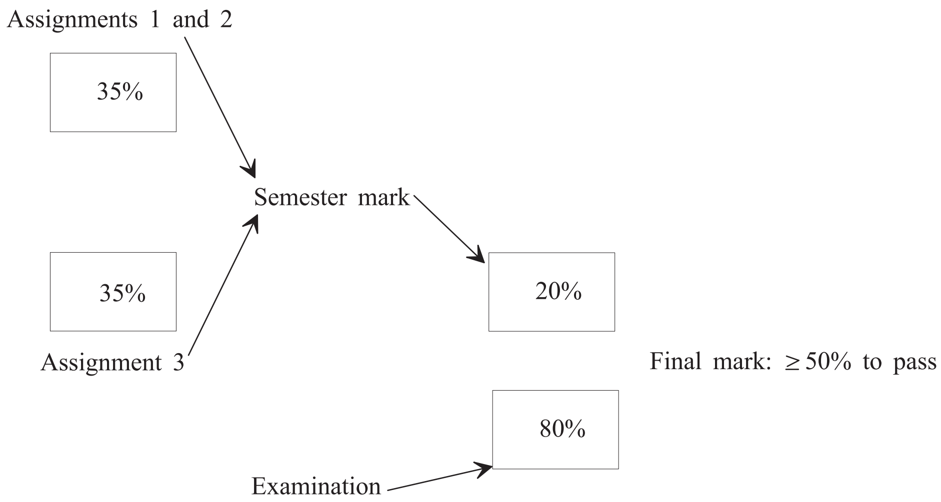
4. Read the questions carefully before answering. See what is asked and then answer the question accordingly. Use the rough work paper supplied for your calculations.

5. Do not panic if you cannot answer a question. Go to the next question and return later.

6. You must take your calculator with you to the examination hall. Make sure it is in working order. A programmable calculator is permitted.

7. Remember to write your student number and module code on the multiple-choice answer sheet.

8. Composition of your final mark for the module:



9. The questions in the examination paper are similar to the problems in the assignments (first and second semester), and the previous examination papers.

10. **TO PASS, YOU NEED AN AGGREGATED TOTAL OF 50% FROM THE EXAMINATION AND COMPULSORY ASSIGNMENTS.**

3 Sample examination paper

Answer ALL the questions on the mark-reading sheet supplied. Carefully follow the instructions for completing the mark-reading sheet.

Also pay attention to the following example of a question:

$$3 + 2 \times -1 + 4 \div 2 =$$

[1] 7.

[2] 1.

[3] 3.

[4] 4.

[5] none of the above.

The correct answer is [3]. Only one option (indicated as [1] [2] [3] [4] [5]) per question is correct. If you mark more than one option, you will not receive any marks for the question. Marks WILL NOT be subtracted for incorrect answers.

Question 1

Simplify the following expression:

$$1 + \frac{36}{45} \times \frac{5}{12} \div \frac{2}{3}.$$

[1] 2

[2] $\frac{11}{9}$

[3] $\frac{9}{8}$

[4] $1\frac{1}{2}$

[5] None of the above.

Question 2

Simplify the following expression:

$$\ln\left(\sqrt{e^{2x^2}}\right)$$

[1] x^2

[2] $2x$

[3] $\ln e^x$

[4] e^x

[5] None of the above.

Question 3

The price of a microwave oven in 2015 is R3 315. This price is actually 35% lower than the price in 2014. What was the price in 2014?

[1] R2 154,75

[2] R5 100,00

[3] R4 475,25

[4] R9 471,43

[5] None of the above.

Question 4

Simplify the following expression:

$$\sqrt{\frac{4x^2}{y^{-4}}}$$

[1] $16x^4y^8$

[2] $\frac{2x^2}{y^{-4}}$

[3] $2xy^2$

[4] $\sqrt{\frac{4x^2}{y^{-4}}}$

[5] None of the above.

Question 5

$\log_{20}\left(\frac{410}{1\,234}\right)$ approximated to four decimal places is equal to

[1] 0,0423.

[2] 0,0049.

[3] 1,1019.

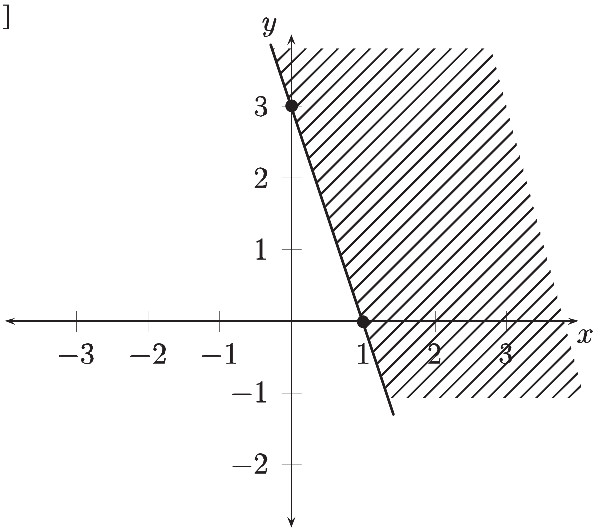
[4] 0,3678.

[5] none of the above.

Question 6

The inequality $y \geq 3 - 3x$ can graphically be represented as

[1]



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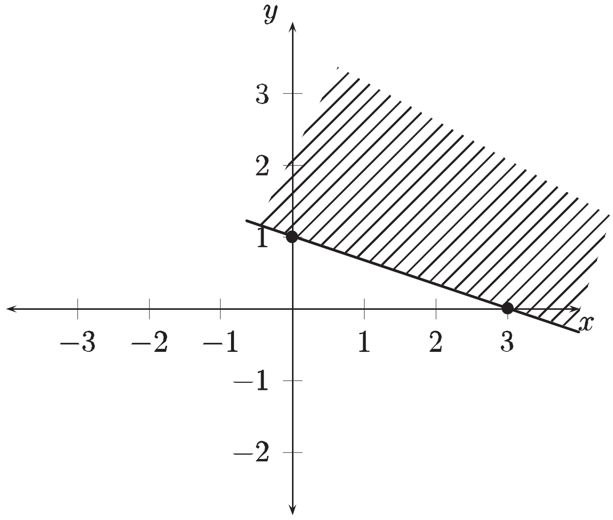

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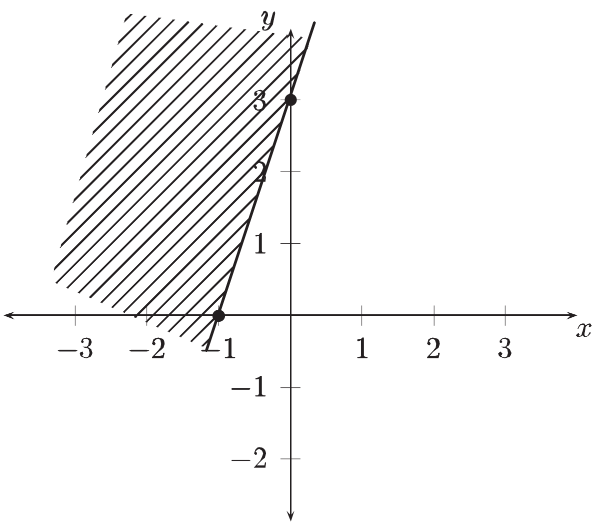

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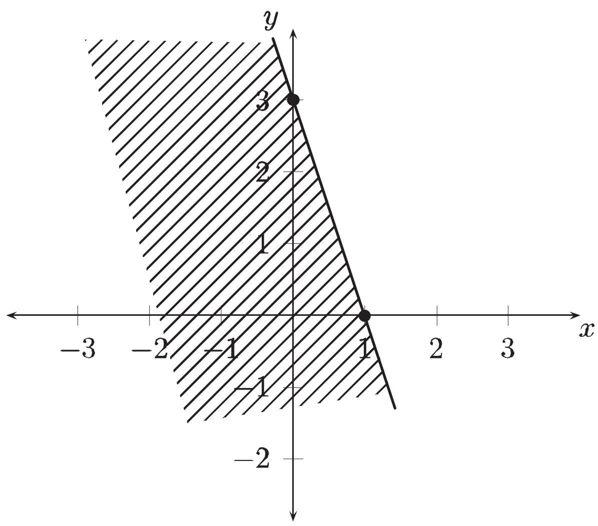


[5] none of the above.

Question 7

Find the slope of the line represented by the equation $0 = 6 + 3x - 2y$.

[1] $\frac{2}{3}$
[2] $\frac{3}{2}$
[3] 3
[4] 2
[5] None of the above.

Question 8

Suppose the cost of producing 10 units of a product is R40 and the cost of manufacturing 20 units is R70. If the cost C is linearly related to output Q (units produced), the cost of producing 35 items, is

[1] R115,00.
[2] R121,67.
[3] R113,33.
[4] R65,00.
[5] none of the above.

Question 9

The lines $2y + 4x + 8 = 8x + 1$ and $y = zx - 4$ are parallel. What is the value of z?

[1] 2
[2] $-3{,}5$
[3] -4
[4] -8
[5] None of the above.

Question 10

The regular price of a cellphone is R1 288,40 and the current sale price is R988,20. By what percentage was the price reduced? Choose the option closest to the correct answer.

[1] 76,70%
[2] 30,38%
[3] 69,62%
[4] 23,30%
[5] None of the above.

Question 11

If the demand function is $P = 40 - Q$, where P and Q are the price and quantity respectively, give an expression for the price elasticity of demand in terms of P only.

[1] $\frac{P-40}{2P}$

[2] $\frac{P}{20-\frac{1}{2}P}$

[3] $\frac{P}{20-P}$

[4] $\frac{P}{P-40}$

[5] None of the above.

Question 12

Given the demand function $P = 60 - 0{,}2Q$. What is the arc price elasticity of demand when price decreases from R50 to R40?

[1] $-\frac{1}{3}$

[2] $\frac{1}{3}$

[3] -3

[4] 3

[5] None of the above.

Question 13

Solve the following system of linear equations:

$$\begin{aligned} x + \ \ y + z &= 8 \\ x - 3y \qquad &= 0 \\ 5y - z &= 10 \end{aligned}$$

The sum of the values of x, y and z of the solution is

[1] 8.

[2] 4.

[3] -4.

[4] 2.

[5] none of the above.

Question 14

In the following market:

$$\begin{aligned} \text{Demand function: } & Q = \ \ 50 - 0{,}1P \\ \text{Supply function: } & Q = -10 + 0{,}1P \end{aligned}$$

where P and Q are the price and quantity respectively. Calculate the equilibrium price and quantity.

[1] $P = 300$; $Q = 20$

[2] $P = 200$; $Q = 30$

[3] $P = 20$; $Q = 300$

[4] $P = 30$; $Q = 200$

[5] None of the above.

Question 15

The cost y (in rands) to produce x bicycles is

$$y = 240x + 720.$$

How many bicycles have been manufactured if the cost is R30 000?

[1] 120

[2] 122

[3] 123

[4] 125

[5] None of the above.

Question 16

The consumer surplus for the demand function $P = 60 - 4Q$ when the market price $P = 16$, is

[1] 242.

[2] 484.

[3] 88.

[4] 32.

[5] none of the above.

Question 17

After training, a new employee will be able to assemble

$$Q(t) = 50 - 30e^{-0{,}05t}$$

units of a product per day, where t is the number of months after an employee has started working at the factory. Approximately how many months after an employee has started working at the factory will he/she be able to assemble 40 units of the product per day?

[1] 20

[2] 50

[3] 10

[4] 22

[5] None of the above.

Question 18

You are baking for a street bazaar and are given 18 kg of flour, 36 eggs and 10 kg of sugar. You are planning to bake two types of cakes. Cake 1 uses 1,8 kg of flour, 3 eggs and 0,4 kg of sugar per unit. Cake 2 uses 0,75 kg flour, 2 eggs and 0,6 kg sugar per unit. If x is the number of units of cake 1 and y the number of units of cake 2, choose the system of linear inequalities that describes the appropriate constraints.

[1] $1{,}8x + 0{,}75y \leq 18;\ 3x + 2y \leq 36;\ 0{,}4x + 0{,}6y \leq 10;\ x, y \geq 0$

[2] $1{,}8x + 0{,}75y \leq 18;\ 2x + 3y \leq 36;\ 0{,}4x + 0{,}6y \leq 10;\ x, y \geq 0$

[3] $0{,}75x + 1{,}8y \leq 18;\ 3x + 2y \leq 36;\ 0{,}4x + 0{,}6y \leq 10;\ x, y \geq 0$

[4] $1{,}8x + 0{,}75y \leq 18;\ 3x + 2y \leq 36;\ 0{,}6x + 0{,}4y \leq 10;\ x, y \geq 0$

[5] None of the above.

Question 19

The roots of the function

$$y = x^2 + x - 6$$

are

[1] $x = 2$ and $x = -3$.

[2] $x = 3$ and $y = 2{,}5$.

[3] $x = -0{,}5$ and $x = -6{,}25$.

[4] $x = -2$ and $x = 3$.

[5] none of the above.

Question 20

The demand function of a firm is $Q = 150 - 0{,}5P$, where P and Q represent the price and quantity respectively. At what value of P is marginal revenue equal to zero?

[1] 150
[2] 75
[3] 113
[4] 0
[5] None of the above.

Question 21

The fixed cost of offering daily driving lessons for a driving school is R1 250. The variable cost is given as R50 for each lesson given. Choose the linear equation that represents the total cost of the driving school per day.

[1] Cost $= 50x + 1\ 250$
[2] Cost $= 1\ 300x$
[3] Cost $= 1\ 200x$
[4] Cost $= 1\ 250x + 50$
[5] None of the above.

Question 22

Find the coordinates of the point of intersection of the lines

$$\begin{aligned} 2x + y - 5 &= 0 \\ 3x - 2y - 4 &= 0 \end{aligned}$$

[1] $x = 3;\ y = 1$
[2] $x = 1;\ y = 2$
[3] $x = 2;\ y = 1$
[4] $x = 1;\ y = 3$
[5] None of the above.

Question 23

A company's profit function (in hundred thousands of rand) can be represented by the function

$$y = -x^2 + 6x + 7$$

where x is the number of units produced. What is the value of the company's maximum profit?

[1] 7
[2] 16
[3] -1
[4] 3
[5] None of the above.

Question 24

Find the values of x for which the function $f(x) = x^3 + 3x^2$ has a maximum or a minimum value.

[1] $x = 0$; $x = -2$
[2] $x = 0$; $x = 2$
[3] $x = 0$; $x = -6$
[4] $x = -3$; $x = -6$
[5] None of the above.

Question 25

If the total cost is given by

$$TC = 2Q^3 - Q^2 + 80Q + 150,$$

what is the marginal cost when $Q = 10$?

[1] 80
[2] 118
[3] 660
[4] 2 850
[5] None of the above.

Question 26

Find the derivative of the function

$$f(x) = x^2 + 5x + \sqrt{x^3}.$$

[1] $f'(x) = 2x + 5$

[2] $f'(x) = 5x + 2 + \frac{3}{2}x^{\frac{1}{2}}$

[3] $f'(x) = 5 + 2x + \frac{1}{2}\sqrt{x}$

[4] $f'(x) = 2x + 5 + \frac{3}{2}\sqrt{x}$

[5] None of the above.

Question 27

Evaluate the following integral:

$$\int (x^2 + 2x + x^{\frac{1}{2}})dx.$$

[1] $x^3 + 2x^2 + \frac{2}{3}x^{\frac{3}{2}} + c$

[2] $\frac{x^3}{2} + \frac{x^2}{2} + \frac{2}{3}x^{\frac{3}{2}} + c$

[3] $\frac{x^3}{3} + x^2 + \frac{2}{3}x^{\frac{3}{2}} + c$

[4] $3x^3 + 4x^2 + \frac{2}{3}x^{\frac{3}{2}} + c$

[5] None of the above.

Question 28

Evaluate the following definite integral:

$$\int_{-1}^{2} (-4x + 6)dx.$$

[1] -10

[2] 6

[3] 12

[4] -6

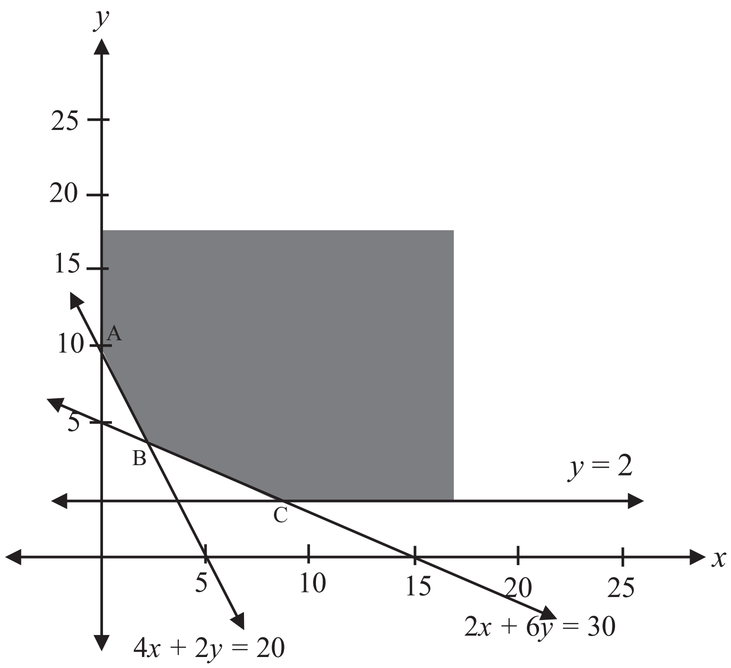
[5] None of the above.

Question 29

Consider the following set of inequalities:

$$\begin{array}{rcll} 2x + 6y & \geq & 30 & (1) \\ 4x + 2y & \geq & 20 & (2) \\ y & \geq & 2 & (3) \\ x, y & \geq & 0. & \end{array}$$

In the graph below the inequalities are drawn and the feasible region is shaded in grey.



Determine the minimum value of the objective cost function

$$Z = 18x + 12y.$$

The minimum cost is equal to

[1] 120

[2] 102

[3] 186

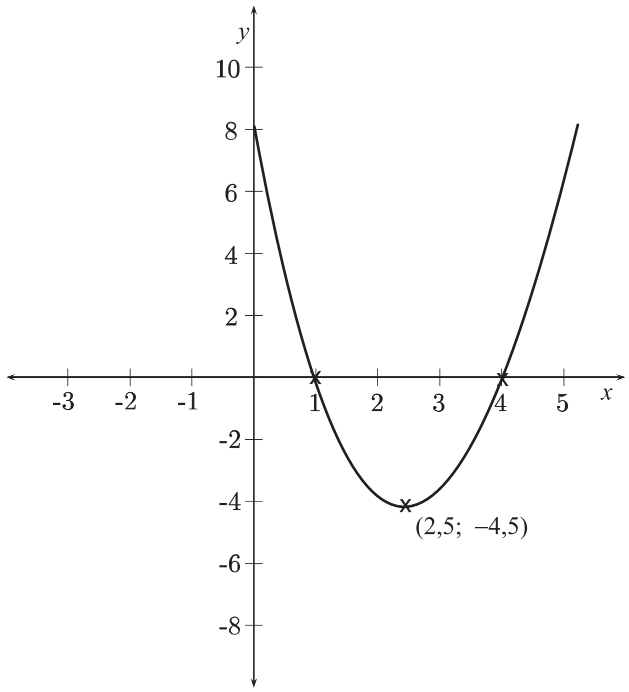
[4] 96

[5] None of the above.

Question 30

Which graph represents the function $2y = -4x^2 + 20x - 16$?

[1]



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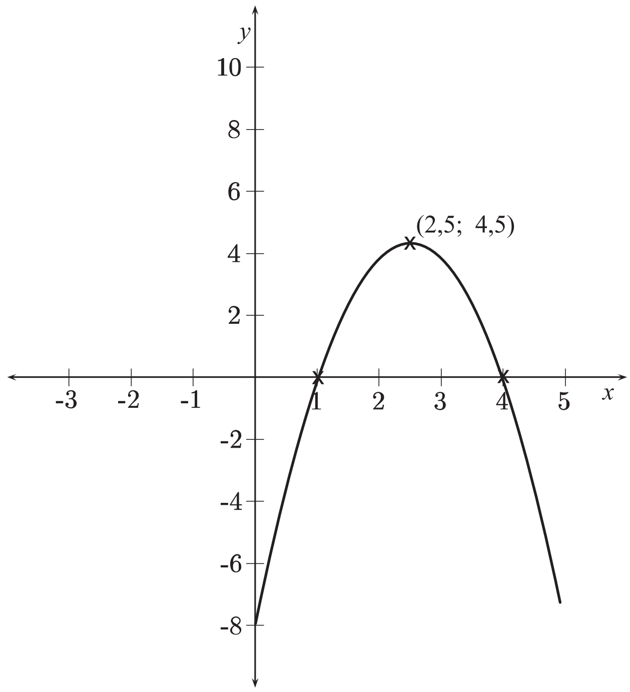

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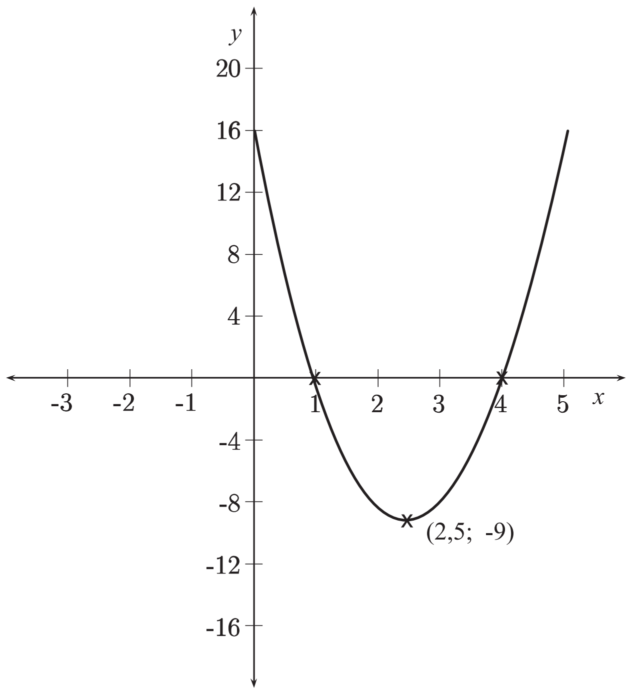
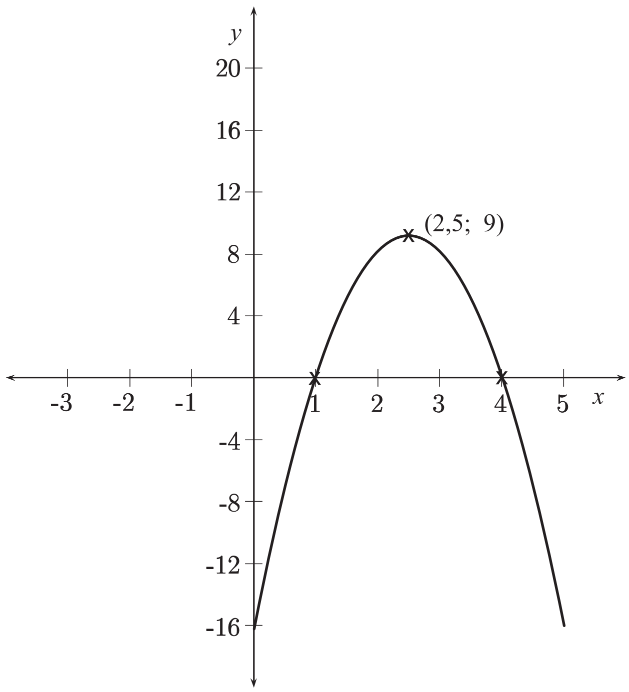


[4]



[5] None of the above

4 Solutions to the examination paper

Question 1

Simplifying gives

$$\begin{aligned} 1+\tfrac{36}{45}\times\tfrac{5}{12}\div\tfrac{2}{3} &= 1+\tfrac{1}{3}\times\tfrac{3}{2} \\ &= 1+\tfrac{1}{2} \\ &= 1\tfrac{1}{2}. \end{aligned}$$

[Option 4]

Question 2

Simplifying gives

$$\begin{aligned} \ln\left(\sqrt{e^{2x^2}}\right) &= \ln\left(e^{2x^2}\right)^{\frac{1}{2}} \\ &= \ln e^{x^2} \\ &= x^2\ln e \quad [\ln e = 1] \\ &= x^2. \end{aligned}$$

[Option 1]

Question 3

In 2015 the price is 35% less than the price in 2014, that is

$$\text{the price in 2015} = \text{the price in 2014} - [0{,}35\times\text{the price in 2014}]\,.$$

If we denote the price in 2014 by x, then we get

$$3\;315 = x - 0{,}35x.$$

Solving for x gives

$$\begin{aligned} 3\;315 &= (1-0{,}35)\,x \\ 3\;315 &= 0{,}65x \\ x &= \tfrac{3\;315}{0{,}65} \\ &= 5\;100. \end{aligned}$$

The price of a microwave oven in 2014 was therefore R5 100.

[Option 2]

Question 4

Simplifying the expression gives

$$\begin{aligned}
\sqrt{\tfrac{4x^2}{y^{-4}}} &= \sqrt{4x^2} \times \sqrt{y^4} \\
&= (2^2x^2)^{\frac{1}{2}} \times (y^4)^{\frac{1}{2}} \\
&= 2x \times y^2 \\
&= 2xy^2.
\end{aligned}$$

[Option 3]

Question 5

The only logs your calculator can handle are log to the base 10 and ln, which is log to the base e. We therefore need to change the base 20 to a base of either 10 or e.

Therefore,

$$\begin{aligned}
\log_{20}\left(\tfrac{410}{1\,234}\right) &= \frac{\ln\left(\frac{410}{1\,234}\right)}{\ln 20} \\
&= \frac{\ln 410 - \ln 1\,234}{\ln 20} \\
&= \frac{-1{,}10186}{2{,}99573} && \text{Rounded to 5 decimal places} \\
&\approx -0{,}3678. && \text{Rounded to 4 decimal places}
\end{aligned}$$

The value of $\log_{20}\left(\frac{410}{1\,234}\right)$ to 4 decimal places is equal to $-0{,}3678$.

[Option 4]

Question 6

To draw the line $y = 3 - 3x$, we need two points on the line. Therefore, select any two values for y or x and find the coordinates. To simplify the calculations, we choose $x = 0$ and find

$$y = 3 - 3(0),$$

giving the coordinate (0; 3). We also choose $y = 0$ to find

$$0 = 3 - 3x$$

resulting in $x = 1$. The second coordinate is therefore (1; 0). We can now plot the two calculated points and draw the line.

To find the area covered by the inequality select the point $(0; 0)$ and substitute it into the inequality, that is $0 \geq\ 3 - 3(0) = 3$, which is false. The area that makes the inequality true is therefore the area that doesn't include (0; 0) (above the line).

[Option 1]

Question 7

To determine the slope of the given line

$$0 = 6 + 3x - 2y,$$

we need to change the equation so that y is the subject of the equation, that is

$$y = 3 + \frac{3}{2}x.$$

The slope is therefore $\frac{3}{2}$.

[Option 2]

Question 8

We first need to find the line representing cost. We are given two points on this line, namely, $(Q_1; C_1) = (10; 40)$ and $(Q_2; C_2) = (20; 70)$. To determine the equation of the line $C = mQ + c$ we need to determine the slope m and y-intercept c of the line.

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 40}{20 - 10} = \frac{30}{10} = 3,$$

giving

$$C = 3Q + c.$$

To determine the y-intercept c of the line, we use one of the given points $(Q_1; C_1)$ and $(Q_2; C_2)$, say $(10; 40)$, to find

$$40 = 30 + c$$

which results in

$$c = 10.$$

The equation of the line is $C = 3Q + 10$ and the cost of manufacturing 35 items is

$$\begin{aligned} C &= 3(35) + 10 \\ &= 115, \end{aligned}$$

or R115,00.

[Option 1]

Question 9

Parallel lines have the same slope, therefore calculating the slope of the first line will give the slope of the second line.

$$\begin{aligned} 2y + 4x + 8 &= 8x + 1 \\ 2y &= 4x - 7 \\ y &= 2x - \tfrac{7}{2} \end{aligned}$$

The slope is therefore 2, which is the value of z.

[Option 1]

Question 10

The regular price is R1 288,40 and we need to find by what % the current sale price of R988,20 is of it. The price is reduced by (1 288,40 − 988,20) = 300,20, and the percentage reduction is

$$\frac{300{,}20}{1288{,}40} \times 100 = 23{,}30\%.$$

[Option 4]

Question 11

The given demand function is

$$P = 40 - Q.$$

According to the textbook the price elasticity of demand is given by $\varepsilon_d = -\frac{1}{b} \times \frac{P}{Q}$, with a and b the values of the demand function $P = a - bQ$, that is $a = 40$ and $b = 1$.

We also need to write $P = 40 - Q$ with Q as the subject, that is

$$Q = 40 - P.$$

We now substitute b and Q into the formula for elasticity of demand to find

$$\begin{aligned} \varepsilon_d &= -\tfrac{1}{1} \times \tfrac{P}{40-P} \\ &= -\tfrac{P}{40-P} \\ &= \tfrac{P}{P-40}. \end{aligned}$$

Alternatively:

We can rewrite the general demand function $P = a - bQ$ as

$$-bQ = P - a.$$

The formula for elasticity of demand can be written as

$$\varepsilon_d = -\frac{1}{b}\frac{P}{Q} = \frac{P}{-bQ} = \frac{P}{P-a}. \text{ (See page 89, equation 2.14).}$$

Substituting $a = 40$ into this gives

$$\varepsilon_d = \frac{P}{P-40}\,.$$

[Option 4]

Question 12

The arc price elasticity of a demand function $P = a - bQ$ between two prices P_1 and P_2 is given by

$$\text{arc price elasticity of demand} = -\frac{1}{b} \times \frac{P_1 + P_2}{Q_1 + Q_2},$$

with b the slope of the demand function, P_1, P_2 the given prices and Q_1, Q_2 the quantities demanded. From the demand function we find that $b = 0{,}2 = \frac{1}{5}$. When we make Q the subject, we find

$$0{,}2Q = 60 - P$$

resulting in

$$Q = 300 - 5P.$$

Now, by substituting $P_1 = 50$ and $P_2 = 40$ into this equation, we find

$Q_1 = 300 - 5 \times 50 = 5$ and $Q_2 = 300 - 5 \times 40 = 100$. Therefore,

$$\begin{aligned}
\text{arc price elasticity of demand} &= -\tfrac{1}{b} \times \tfrac{P_1+P_2}{Q_1+Q_2} \\
&= -\tfrac{1}{0{,}2} \times \tfrac{50+40}{50+100} \\
&= -5 \times \tfrac{90}{150} \\
&= -3.
\end{aligned}$$

[Option 3]

Question 13

We need to solve the following system of equations:

$$\begin{aligned}
x + y + z &= 8 \qquad (1) \\
x - 3y &= 0 \qquad (2) \\
5y - z &= 10 \qquad (3)
\end{aligned}$$

Make x the subject of equation (2) and z the subject of in equation (3):

$$\begin{aligned}
x &= 3y \qquad (4) \\
z &= -10 + 5y \qquad (5)
\end{aligned}$$

Substitute equations (4) and (5) into equation (1):

$$\begin{aligned}
(3y) + y + (-10 + 5y) &= 8 \\
9y &= 8 + 10 \\
y &= \tfrac{18}{9} \\
&= 2.
\end{aligned}$$

Substitute $y = 2$ into equation (4):

$$\begin{aligned} x &= 3y \\ &= 3 \times 2 \\ &= 6 \end{aligned}$$

and into equation (5):

$$\begin{aligned} z &= -10 + 5y \\ &= -10 + 5(2) \\ &= -10 + 10 = 0 \end{aligned}$$

Therefore, $x = 6$; $y = 2$ and $z = 0$. The sum is 8.

[Option 1]

Question 14

Equilibrium is at the price and quantity where the demand and supply functions are equal. This is at the point where the demand and supply functions intersect.

Therefore, we need to determine the value of P and Q for which $P_d = P_s$ or $Q_d = Q_s$. It is given that $Q_d = 50 - 0{,}1P$ and $Q_s = -10 + 0{,}1P$. Thus,

$$\begin{aligned} Q_d &= Q_s \\ 50 - 0{,}1P &= -10 + 0{,}1P \\ -0{,}2P &= -60 \\ P &= \tfrac{-60}{-0{,}2} \\ &= 300. \end{aligned}$$

To calculate the quantity at equilibrium, we substitute the value of P into the demand or supply function and calculate Q. Say we use the demand function, then

$$\begin{aligned} Q &= 50 - 0{,}1(300) \\ &= 50 - 30 \\ &= 20. \end{aligned}$$

The equilibrium price is equal to 300 and the quantity is 20.

[Option 1]

Question 15

The cost to produce x bicycles is given as $y = 240x + 720$. We have to find the number of bicycles produced if the cost is R30 000. Therefore,

$$\begin{aligned} 30\ 000 &= 240x + 720 \\ x &= \tfrac{30\,000-720}{240} \\ &= 122. \end{aligned}$$

[Option 2]

Question 16

We need to find the consumer surplus for the demand function

$$P = 60 - 4Q$$

when the market price $P = 16$.

To find the consumer surplus CS if $P = 16$, we need to graph the given demand function $P = 60-4Q$. Writing the demand function with Q as the subject gives

$$\begin{aligned} 16 &= 60 - 4Q \\ -4Q &= P - 60 \\ Q &= 15 - \tfrac{P}{4}. \end{aligned}$$

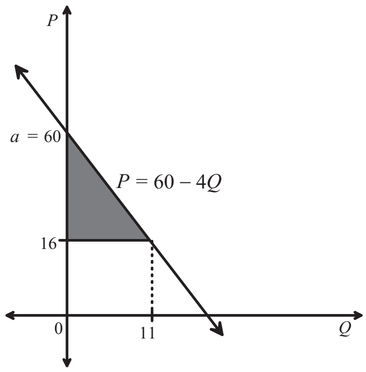
So, if $P = 16$,

$$Q = 11,$$

giving one point on the line as (11; 16).

We know that the y-axis intercept of the demand function $P = a - bQ$ is given by $a = 60$. Therefore another point is (0; 60).

Next we draw the demand function and shade the area above the line, $P = 16$.



The consumer surplus is the area of the shaded triangle of the sketch, that is

$$\begin{aligned} CS &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 11 \times (60 - 16) \\ &= \tfrac{11 \times 44}{2} \\ &= 242. \end{aligned}$$

The consumer surplus is equal to 242 if the price P is equal to 16.

[Option 1]

Question 17

We need to determine the number of months, t, after which an employee will be able to assemble 40 units of a product per day. Thus,

$$\begin{aligned} 40 &= 50 - 30e^{-0{,}05t} \\ 30e^{-0{,}05t} &= 10 \end{aligned}$$

giving

$$e^{-0{,}05t} = \frac{1}{3}.$$

Taking ln on both sides and solving for t results in

$$\begin{aligned} -0{,}05t \ln e &= \ln \tfrac{1}{3} \qquad [\ln\ e = 1] \\ t &= \tfrac{-1{,}0986}{-0{,}05} \\ &= 21{,}9722 \approx 22. \end{aligned}$$

After approximately 22 months an employee will be able to assemble 40 units of a product per day.

[Option 4]

Question 18

It is given that x = number of unit of cake 1 and y = number of units of cake 2.

To help us with the formulation we summarise the information given in the following table:

Items with restrictions	**Cake 1 (x)**	**Cake 2 (y)**	**Capacity/ requirements**
Flour	1,8	0,75	18 kg
Eggs	3	2	36
Sugar	0,4	0,6	10 kg
Number of units of cakes			Never negative

Using the table, the constraints are as follows:

$$\begin{aligned} 1{,}8x + 0{,}75y &\leq 18\,\text{(flour)} \\ 3x + 2y &\leq 36\,\text{(eggs)} \\ 0{,}4x + 0{,}6y &\leq 10\,\text{(sugar)} \\ x;\, y &\geq 0 \end{aligned}$$

[Option 1]

Question 19

To find the roots of the function $x^2 + x - 6 = 0$, which are the values of x when $y = 0$, we use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Comparing the given equation

$$y = x^2 + x - 6$$

with the general form of a quadratic function

$$y = ax^2 + bx + c,$$

we conclude that $a = 1$, $b = 1$ and $c = -6$. Substituting a, b and c into the quadratic formula gives

$$\begin{aligned} x &= \tfrac{-1 \pm \sqrt{(1)^2 - 4(1)(-6)}}{2(1)} & & \\ &= \tfrac{-1+5}{2} & \text{or} \quad x &= \tfrac{-1-5}{2} \\ &= 2 & &= -3 \end{aligned}$$

Alternatively, by factorising we get

$$y = (x + 3)(x - 2).$$

If $y = 0$,

$$x + 3 = 0 \quad \text{giving} \quad x = -3$$

or

$$x - 2 = 0 \quad \text{giving} \quad x = 2.$$

The roots of the function $x^2 + x - 6 = 0$ are therefore -3 and 2.

[Option 1]

Question 20

Revenue is price × demand or $R = P \times Q$

It is given that price is P and demand is $Q = 150 - 0{,}5P$.

Therefore, substitute $Q = 150 - 0{,}5P$ into the formula for R to get

$$\begin{aligned} R &= P \times (150 - 0{,}5P) \\ &= 150P - 0{,}5P^2. \end{aligned}$$

To determine the marginal revenue (MR) we need to differentiate the revenue function, that is

$$\begin{aligned} MR &= \tfrac{dR}{dP} \\ &= \tfrac{d}{dP}(150P - 0{,}5P^2) \\ &= 150 - (2 \times 0{,}5)P \\ &= 150 - P. \end{aligned}$$

We need to find P when $MR = 0$, that is

$$0 = 150 - P$$

or

$$P = 150.$$

The value of P is equal to 150 if the marginal revenue is equal to 0.

[Option 1]

Question 21

We need to find the linear equation for the total cost. Therefore, if x is the number of lessons per day, we have fixed cost $= 1\,250$ and variable cost $= 50x$.

The total cost function is therefore

$$\text{Cost} = \text{fixed cost} + \text{variable cost} = 1\,250 + 50x.$$

[Option 1]

Question 22

Eliminate one variable by adding or subtracting one equation or multiple of an equation from another equation.

The equations of the two lines are

$$2x + y - 5 = 0 \quad \text{or} \quad 2x + y = 5 \qquad (1)$$

and

$$3x - 2y - 4 = 0 \quad \text{or} \quad 3x - 2y = 4 \qquad (2)$$

Multiplying equation (1) by 2 gives $4x + 2y = 10 \quad (3)$.

Equation (2) + equation (3) gives

$$\begin{aligned} 3x - 2y + 4x + 2y &= 4 + 10 \\ 7x &= 14 \end{aligned}$$

resulting in

$$x = 2.$$

Substitute $x = 2$ into either equation (1) or (2) and solve for y. If we, for instance, substitute $x = 2$ into equation (1) we get

$$\begin{aligned} 2(2) + y &= 5 \\ y &= 1. \end{aligned}$$

The two lines intersect in the point $(x; y) = (2; 1)$.

[Option 3]

Question 23

We need to determine the number of units of a product that maximises the profit function. To find the turning point of the profit function we need to find the derivative of the function and set it equal to zero.

Differentiating the function

$$y = -x^2 + 6x + 7$$

and setting it equal to zero gives

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -2x + 6 = 0$$

resulting in

$$x = 3.$$

They therefore need to produce 3 units to maximise profit.

Next, we substitute the value of x into the equation to find the maximum profit. Thus,

$$\begin{aligned} y &= -(3)^2 + 6(3) + 7 \\ &= -9 + 18 + 7 \\ &= 16. \end{aligned}$$

Therefore, the value of the company's maximum profit is 16 hundred thousand rand.

[Option 2]

Question 24

We need to find the maximum or minimum value of the function

$$f(x) = x^3 + 3x^2.$$

The maximum or minimum value of the function is found where the derivative of the function is equal to zero.

Differentiating the function $f(x)$, using the power rule of differentiation, namely $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$, gives

$$\begin{aligned} f'(x) &= 3x^2 + 3 \times 2x \\ &= 3x^2 + 6x. \end{aligned}$$

To find the maximum or minimum value we set $f'(x) = 0$, that is

$$\begin{aligned} 3x^2 + 6x &= 0 \\ 3x(x + 2) &= 0 \end{aligned}$$

giving

$$x = 0 \qquad \text{or} \qquad x = -2.$$

[Option 1]

Question 25

Marginal cost is defined as

$$MC = \frac{\mathrm{d}TC}{\mathrm{d}Q}$$

We need to find the marginal cost if

$$TC = 2Q^3 - Q^2 + 80Q + 150.$$

Differentiating gives

$$MC = 6Q^2 - 2Q + 80.$$

Substituting the value of Q into the differentiated function gives

$$\begin{aligned} MC(10) &= 6(10)^2 - 2(10) + 80 \\ &= 660. \end{aligned}$$

The marginal cost when $Q = 10$ is therefore 660.

[Option 3]

Question 26

To apply the power rule, we need to simplify the given function as follows:

$$\begin{aligned} f(x) &= x^2 + 5x + \sqrt{x^2} \\ &= x^2 + 5x + (x^3)^{\frac{1}{2}} \\ &= x^2 + 5x + x^{\frac{3}{2}}. \end{aligned}$$

Now we differentiate the simplified expression, and find

$$\begin{aligned} f'(x) &= 2x + 5 + \tfrac{3}{2}x^{\frac{1}{2}} \\ &= 2x + 5 + \tfrac{3}{2}\sqrt{x}. \end{aligned}$$

[Option 4]

Question 27

To evaluate the integral, we use the power rule of integration, namely $\int x^n = \frac{x^{n+1}}{n+1} + c$ when $n \neq -1$. Therefore,

$$\begin{aligned} \int (x^2 + 2x + x^{\frac{1}{2}})\mathrm{d}x &= \tfrac{x^{2+1}}{2+1} + \tfrac{2x^{1+1}}{1+1} + \tfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c \\ &= \tfrac{x^3}{3} + x^2 + \tfrac{2}{3}x^{\frac{3}{2}} + c. \end{aligned}$$

[Option 3]

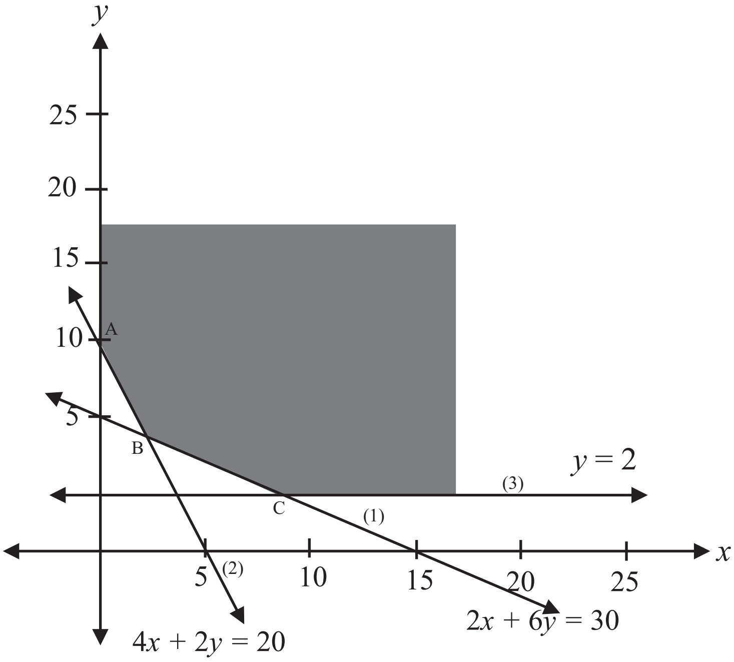
Question 28

Evaluating the definite integral, gives

$$\begin{aligned} \int_{-1}^{2} (-4x + 6)\mathrm{d}x &= \tfrac{-4x^2}{2} + \tfrac{6x}{1}\Big|_{-1}^{2} \\ &= -2x^2 + 6x\Big|_{-1}^{2} \\ &= -2(2)^2 + 6(2) - (-2(-1)^2 + 6(-1)) \\ &= -8 + 12 + 2 + 6 \\ &= 12. \end{aligned}$$

[Option 3]

Question 29



The corner points of the feasible region are the points A, B, C.

Point A is the point where the line (2) cuts the y-axis, that is at (0; 10).

Point B is at the intersection of lines (1) and (2), that can be written with y as the subject as

$$y = -\frac{1}{3}x + 5 \qquad (1)$$

and

$$y = -2x + 10 \qquad (2)$$

Setting these equal gives

$$\begin{aligned} -2x + 10 &= -\tfrac{1}{3}x + 5 \\ -2x + \tfrac{1}{3}x &= 5 - 10 \\ \tfrac{-6+1}{3}x &= -5 \\ x &= \tfrac{3\times 5}{5} = 3 \\ x &= 3. \end{aligned}$$

Substituting $x = 3$ into equation (2) gives

$$\begin{aligned} &= 10 - 2(3) \\ &= 4. \end{aligned}$$

Point B is therefore at (3; 4).

Point C is the point where lines (3) and (1) intersect.

Substituting the value $y = 2$ into equation (1) gives

$$\begin{aligned} 2x + 6(2) &= 30 \\ x &= \tfrac{18}{2} \\ &= 9. \end{aligned}$$

Point C is at (2; 9).

Corner points of feasible region	**Value of $Z = 18x + 12y$**
A: $x = 0$; $y = 10$	$Z = 18(0) + 12(10) = 120$
B: $x = 3$; $y = 4$	$Z = 18(3) + 12(4) = 102$ ←**Minimum**
C: $x = 9$; $y = 2$	$Z = 18(9) + 12(2) = 186$

The minimum of Z is at point B where $x = 3$, $y = 4$ with $Z = 102$.

[Option 2]

Question 30

Given the function

$$2y = -4x^2 + 20x - 16,$$

and making y the subject gives

$$y = -2x^2 + 10x - 8.$$

The roots of the function are at $y = 0$, that is

$$(x - 4)(x - 1) = 0$$

resulting in

$$x = 4 \quad \text{and} \quad x = 1.$$

Comparing the function with the standard form of the quadratic function

$$y = ax^2 + bx + c,$$

we can conclude that $a = -2, b = 10$ and $c = -8$ giving the axis of symmetry to be

$$\begin{aligned} x &= \tfrac{-b}{2a} \\ &= \tfrac{-10}{-4} \\ &= 2\tfrac{1}{2}. \end{aligned}$$

Alternatively, we can differentiate the function and set the derivative equal to zero, that is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -4x + 10 = 0$$

from this we find

$$x = \frac{10}{4} = 2{,}5$$

and

$$y = 4{,}5.$$

The turning point is at point (2,5; 4,5). We also notice that $a < 0$, so the graph has a maximum value and cuts y-axis at -8.

[Option 2]