Teaching numeracy to adults

ABT1518

University of South Africa, Pretoria

Printed and published by the
University of South Africa
Muckleneuk, Pretoria

ABT1518/1/2011–2013

98621777

InDesign

ABET-Style

Contents

Introduction

Welcome to this course on **Teaching numeracy to adults**, one of the modules in both the Higher Certificate and the Diploma in Adult Basic Education and Training. This module aims to familiarise you with the key concepts and knowledge relating to adult learning. We hope that you will enjoy each of the five units in this module and that you will be learning things that you want to know about why and how adults learn. We hope also that you will gain knowledge and experience which will help you in your work as an educator, trainer and developer.

This module is an introductory one at level 5 of the National Qualifications Framework (NQF) and is meant to outline adult learning information that is of practical application to an entry level adult basic education and training practitioner.

In this Introduction we introduce you to the contents of the module on **Teaching numeracy to adults**. In it you will find basic information about

- the aims of the module
- the learning outcomes of the module
- the units in the module
- the study guide material and readings
- sources of additional information
- tutorial support
- assignments and assessment
- study expectations

AIMS OF THE MODULE

This module aims to enable you to gain introductory knowledge, skills and applied competence in facilitating numeracy learning with adult learners. You will be able to teach numeracy at ABET levels 1 and 2, including the selecting and implementing of appropriate learning and teaching strategies and applying principles of outcomes-based education.

LEARNING OUTCOMES

The statements of learning outcomes provided below tell you what **results** are expected from your study of this module. They tell you what content you are expected to know as well as what you should be able to do or demonstrate. Of course, these learning outcomes rest on the assumption that you will read and study this study guide, engage in the recommended study activities, and complete all the assignments.

By the end of the module you should be able to do the following (both in spoken and appropriate written form):

- demonstrate subject knowledge of Mathematics at ABET levels 1 and 2
- apply the requirements of the unit standards for numeracy learners at ABET levels 1 and 2
- help learners to identify and develop their own knowledge of numeracy
- apply the concept of the common developmental path of mathematical concepts and skills to teaching numeracy as required by the ABET levels 1 and 2 unit standards (eg methods of calculation and number concept of whole numbers and fractions, data handling, probability, spatial concepts and skills, measurement)
- explain and use the rationale for problem-solving tasks in numeracy teaching
- identify typical misconceptions of mathematical concepts and causes, and apply prevention strategies

THE MODULE UNITS

The **Teaching numeracy to adults** module consists of the following topic areas that are covered in five study units of printed material:

Unit	Title	Topics covered
1	The basics of ABET levels 1 and 2 Mathematics	What do we mean by numeracy and mathematics? Numeracy and mathematics in ABET The fear of mathematics Basic mathematics
2	Understanding the requirements of the unit standards	The units standards for ABET levels 1 and 2 mathematics Using standards to plan teaching Outcomes and performance in ABET mathematics
3	What do the learners know already and how can it be used?	Using learners' prior knowledge in mathematics Helping learners recognise what they already know about numbers
4	Developing math-ematical concepts and skills	Finding a common developmental path for mathematical concepts and skills The structured development of numeracy concept Some ideas for teaching content areas of mathematics that are crucial and that many learners find difficult

Unit	Title	Topics covered
5	Teaching problem solving and error avoiding in beginner mathematics.	What is problem solving in mathematical learning? Using problem solving to extend concepts and calculations in mathematics Problems involving addition, subtraction, division and multiplication of whole numbers and fractions Evaluating numeracy materials and courses in relation to the range and mix of problem types in terms of purpose and likely challenges and difficulties for learners Typical misconceptions in ABET level mathematics, their causes and remedies

THE STUDY GUIDE MATERIAL AND READINGS

The instruction given in this module is done in two ways:

- through this study guide with its units of printed course materials and readings which you are expected to read and study
- through interactions with a tutorial group (if you can attend one)

Units

The printed course material for the **Teaching numeracy to adults** module is divided up into six **units**. Each unit covers topics related to one of the learning outcomes.

What is in each unit?

Each unit contains

- aims
- learning outcomes
- content material
- activities
- readings
- further reading

Aims

These provide a **general** statement on what you will learn in the unit and what material has been provided to help you to do this.

Learning outcomes

These are **specific** statements about what you will be able to do when you have worked through the unit and engaged in other course activities related to the unit.

Content material

This is the material you will read and think about. Other supporting materials, mainly readings, may also be used.

Activities

Included in the study material will be a number of activities. These tasks should help you check your own understanding of the material. The activities will include questions, exercises, self-tests and ideas to think and write about.

Readings

At the end of some of the units you may find a reading or readings.

Further reading

A list of further readings will be provided near the end of each unit. These readings can be found in textbooks, books, journal articles and other publications as well as on the Internet. You will have to find these readings yourself. In nearly all cases the books and journals are available in the University of South Africa library and some are available at the regional centres of the University.

How much time should I spend on each unit?

How long do I need to study this module and each unit of this module? This module is rated at 12 credits. This means that it is assumed that you will spend about 120 hours of study on this module. Usually this means that you must plan to spend time

- reading the materials
- engaging in activities as you read and attending tutorials
- writing assignments, and preparing for and writing the examination

We recommend that you study for the following number of hours:

- reading through this introduction and the six units – 40 hours (about 6 hours per module)
- doing activities and attending tutorials – 40 hours
- writing assignments (this includes preparation, reading, writing and careful editing) – 20 hours
- preparing for and writing the examination – 20 hours

SOURCES OF ADDITIONAL INFORMATION

Most of the basic information you need for the *Exploring Introduction to adult education* course is either presented in this study guide or in the class sessions or is available in the tutorial letter you received with it.

But what about information you need that is not found in the above? What other information do you need?

Further reading

Some information can be obtained from the recommended books or journal articles listed in the "Further reading" section near the end of some units. These you can borrow from the University library.

The Internet

Another primary source of information is the Internet or World Wide Web. Computer facilities are available at the University where you may use this computer-based resource.

The other students studying this module

Studying by yourself can be a lonely task. A rich source of support, information and experience is your fellow students (whether given informally, by your setting up a study group, or by joining a Unisa tutorial group).

Support from the module coordinator

You can make use of the support given by the module coordinator. You are welcome to make appointments to see the coordinator and you can also communicate with him or her by letter, telephone, fax or email (details are given in the tutorial letter).

ASSIGNMENTS AND ASSESSMENT

What is going to be assessed?

The assessment in this module will be based on the **assignments** and the **examination**.

Assignments

You will be asked to complete **two compulsory assignments** during the course. Each written assignment has a due date and must be submitted by that date. The first assignment allows you to gain entrance to the examination. Completion of the **first assignment** is a requirement for entry to the final examination. Please note that if this assignment has not been submitted you will not be allowed to write the examination. The **second assignment** contributes 10% of you final mark.

Examination

This will be written at one of the recognised Unisa examination centres. The examination will last two hours.

Things to remember about assessment

In thinking about assessment, remember that what should be assessed is **your demonstration** that you have achieved the **learning outcomes** of this course. To do this you need to

- show that you have **knowledge** about teaching numeracy to adults (which you demonstrate by writing in appropriate ways in assignments and examination answers)
- demonstrate that you have the **skills** to think and plan how you would use your knowledge of teaching numeracy in practical ways in your adult education and training activities
- display **attitudes** that indicate that the knowledge you have of how adults can become numerate is meaningful to you as an educator of adults and that the way you use your skills will be effective when working with them (which is displayed in the way you demonstrate your knowledge and skills)

Some things that will **not** help you demonstrate your achievement of the learning outcomes are the following:

Writing what you have learnt by **rote** (this means learning words, texts or facts off by heart without really understanding their meaning). You will not do very well in your assessments if you simply write what is in the study material.

Not making use of your own experience. We are very interested in your experiences, ideas, feelings and activities as an adult learner yourself. You will do well in your assessments if you combine what you have learnt from the study material with your own well-thought-out ideas. You will do well if you can show us that you can use what you have learnt in your work and in the activities.

STUDY EXPECTATIONS

To be truly successful in this module you will need to spend a considerable number of study hours reading and writing. The module was written on the assumption that you have a school Grade 12 level competence in the language of instruction and in reading and writing skills. It is further assumed that you can learn from predominantly written material and that you can find, analyse and evaluate information relevant to the learning programme. Lastly, you are expected to spend time reading and studying the course material and readings provided carefully and to do the assignments and prepare for the examinations.

Unit 1

The basics of ABET levels 1 and 2 mathematics

1 INTRODUCTION

Numeracy is an important part of most adults' lives. To gain confidence in their ability to work with numbers is an important gain for adults – it helps them to manage their personal and work lives better. Adults also have varying degrees of numeracy. Some adults may be illiterate but can work effectively with large sums of money. Others may be highly literate but cannot do basic addition.

The same variation is found among educators. Some educators are highly numerate and may have studied mathematics to a high level. Others are very weak in numeracy. The learners' difficulties in learning numeracy are not helped by educators by teachers who themselves are not really numerate and know little more than the learners. Ignorance of basic maths, particularly of decimals and fractions, is often embarrassing to teachers.

2 AIMS OF THE UNIT

This unit aims to remind educators of the basics of numeracy and mathematics. It outlines the basic subject knowledge that educators need to be able to teach ABET level 1 and 2 mathematics.

LEARNING OUTCOMES

By the end of this unit you should be able to demonstrate subject knowledge of Mathematics at ABET levels 1 and 2.

You will demonstrate this by being able to

- work with whole numbers and fractions and put them in writing using words and mathematical signs and conventions
- represent three dimensional objects correctly in two dimensions using drawings and diagrams
- associate different views of three dimensional objects correctly with the two-dimensional representations
- identify shapes based on their properties
- measure and calculate linear and area measurements appropriately in macro and micro contexts (macro contexts being situations where the object to be measured cannot be accessed from a fixed position, eg the area of a house, whereas micro contexts can be accessed from a fixed position, eg the area of a house on a floor plan)
- explain concepts and methods of data handling
- explain concepts of probability
- explain the development of number systems

WHAT DO WE MEAN BY NUMERACY AND MATHEMATICS?

In this area of specialisation we sometimes use the word “numeracy”, and at other times the word “mathematics”. When you look at courses on developing mathematical skills for adults, you will notice that they are sometimes called “mathematics” courses, and sometimes “numeracy” courses.

When the word “numeracy” is used to describe a course, you will usually find that the course deals with numbers only (eg writing numbers, counting, addition, subtraction, multiplication and division). But if you think about what adult learners need to know in order to satisfy the unit standards at ABET levels 1, 2, 3 and 4, you can see that it will not be enough for adult learners to simply know how to work with numbers, nor will it be enough for them to be able to count, add, subtract, multiply and divide – they will have to know other things as well. For example, they will have to know how to measure lengths, weights and capacities, how to calculate areas, and how to work with statistics.

In the study guide for this module we have used both the terms “numeracy” and “mathematics” because we have tried to deal with **all** the areas that adult learners will need to know about in order to satisfy the unit standards for ABET levels 1, 2, 3 and 4 (although in this module we concentrate only on ABET levels 1 and 2 mathematics).

Numeracy is the skill to work with numbers and mathematics. To be truly numerate a person has to be able to work confidently and logically with basic numbers, measuring, orders of magnitude, geometry, algebra, probability and statistics. This requires understanding of the number system, being able to use a number of mathematical techniques, and an ability to solve numerical problems in different contexts, including everyday tasks. Numeracy also includes understanding the various ways in which numerical data are gathered by counting and measuring, and then presented in graphs, diagrams, charts and tables.

At its simplest level **numeracy** is a mastery of the basic symbols and processes of arithmetic dealing with numbers, addition, subtraction, simple multiplication and division, simple weights and measures, counting money and telling the time.

At a more complex level **mathematics** deals with the more difficult problems related to numbers, quantity, structure, space or change. It uses numbers and symbols and looks at patterns. Mathematics deals with these difficult problems by using careful logical reasoning and a range of abstract symbols. It is the use of these abstract symbols and the rigorous logical reasoning that makes it hard for many people to do mathematics, because they cannot see its connection to their everyday concrete reality and context. But it is precisely because mathematics is not tied to everyday reality that makes it such a powerful scientific tool. Mathematics includes such things as algebra, geometry and applied mathematics.

NUMERACY AND MATHEMATICS IN ABET

In all modern societies, numeracy and basic mathematics are considered vital skills and in South Africa the government and other key stakeholders believe that developing adult learners' skills in mathematics is of great and fundamental importance.

A learner cannot gain a Certificate in General Education and Training without achieving certain mathematical credits.

THE FEAR OF MATHEMATICS

The fact that mathematics is seen as fundamental in formal ABET qualifications, and is given the same weight as communication, may make many ABET practitioners panic. Many educators get very worried when they think that they will have to teach mathematics. Some of the things people say are:

> "I can't teach mathematics – I don't know any mathematics myself so how can I teach it?"
>
> "I'll never be able to help adults with mathematics because I don't understand mathematics well enough to teach someone else."
>
> "Oh no, I hate mathematics and I can't do it."
>
> "I'm really stupid when it comes to mathematics so I'll never be able to teach it."
>
> "Only really clever people can learn mathematics."

ACTIVITY 1

■ ■ ■

Think about the following two questions as they relate to learners and to you:

(1) Why is it that many people don't like dealing with numeracy and mathematics?
(2) Why do many people become tense when they are doing numerical work?

■ ■ ■

In this module we hope to show you how you can overcome doubts and fears when teaching numeracy to adults.

We will first of all point out

- that people who say they can't do mathematics are actually already using mathematics in their everyday lives, even though they are not calling these everyday activities "mathematics"
- that there are at least two different ways of using numeracy and mathematics – one we use in ordinary life, and one we use in our formal learning environments, for example at education classes or at work

We will show you how you can overcome some of your fears and worries about mathematics. We will start in this unit by helping you to develop a better understanding of numbers, and explaining some key mathematical concepts to you.

The key to mastering mathematics is to understand these basic numeracy and mathematical concepts. It is therefore important that you as the teacher should be comfortable with these concepts and that you should have a variety of methods available to teach the concepts to your learners.

If you understand the basic mathematical concepts, then you will be able to apply the general teaching and facilitating skills you have learnt in this certificate programme to teach the specific content areas in numeracy and mathematics.

BASIC MATHEMATICS

Calculation with whole numbers and fractions

A **whole number** is also called an **integer**. Each number consists of one or more **digits**. For example, the whole number 43 is an integer consisting of two digits. Some numbers are **odd** (they cannot be divided by two) and some are **even** (they can be divided by two).

There are four kinds of operations you have to be able to do to be numerate:

adding

subtracting

multiplying

dividing

All the other things we do with numbers – fractions, decimals, ratios, and so on – are really variations on these four operations.

Addition +

We **add** one number to another in the operation of **addition,** the result of which is the **sum**.

Because we do a lot of addition in numeracy it makes sense to learn some basic sums off by heart. These are often called number **bonds**. You should be able to add or subtract any two digits instantly.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Subtraction –

We **subtract** one number from another in the operation of **subtraction,** the result of which is the **difference**.

Because we do a lot of subtraction in numeracy it makes sense to learn by heart some basic subtraction operations using any two digits.

Multiplication x

We **multiply** one number by another in the operation of **multiplication,** the result of which is the **product**.

Multiplication is essentially a process of repeated addition.

Because we do a lot of multiplication in numeracy it makes sense to learn some products of multiplication off by heart. These are the multiplication tables and should be learnt for the numbers 2 to 12.

Division ÷

We **divide** one number by another in the operation of **division,** the result of which is the **quotient**. The number that is doing the division is the **divisor**.

Division is essentially a process of repeated subtraction.

The = sign

The = is the sign that the number that follows is the result of addition, subtraction, multiplication or division.

Estimation

To be truly numerate you need to be able to estimate. To estimate is to make a rough calculation to get the approximate value of something. To be able to make these guesses about numbers is very important in preventing us from making mistakes in calculation. By making a sensible guess at the answer before we do the actual calculation we can reduce the number of errors we make.

For example:

47 + 54 is about 50 + 50, so the answer should be about 100 (it is actually 101). If, for the answer above, you had got 91 or 911 you would then immediately know that you had made a mistake.

This skill of estimation is particularly important when using calculators. Without it you would often not know whether the answer is correct or not.

Rounding off

Most estimation uses a process called **rounding off**; rounding off makes the numbers easier to use. Most rounding off takes the number to the nearest ten (or hundred or thousand, whichever is most relevant).

For example, using the example we used to explain estimation:

47 is rounded up to 50

54 is rounded down to 50.

Layout

When doing calculations the way you lay out the figures on the page can make for easier for more difficult calculations. Numbers should be written clearly (particularly when they may be easily confused – like 1 and 7). Digits should be in the correct column – tens, hundreds, thousands and so on – that is, the place values of digits should be shown in the layout of the numbers.

Horizontal and vertical addition

Because much numerical information that we read is found in tables, one must get used to the idea of both vertical addition and horizontal addition:

Take this example:

	Men	**Women**	**Totals**
Nurses	1	8	9
Teachers	8	12	20
Social workers	4	6	10
Totals	**13**	**26**	***39***

Fractions

A fraction of something is a part of it. Fractions, therefore, are parts of whole numbers.

We are all familiar with parts of things. We all have experience in particular of parts of food – half a loaf of bread, a third of a bar of chocolate, a cake sliced into many separate pieces, a quarter of a cup of milk, and so on.

Taking the example of three quarters ($\frac{3}{4}$), the 3 is the numerator and the 4 is the denominator. There are three parts of a whole that is made up of four parts. It is a **fraction** or part of the whole number 4.

Some fractions where the numerator is bigger than the denominator are called **improper fractions**, such as $\frac{16}{4}$. In practical terms one can imagine how such a fraction comes into being. Say one had four loaves of bread and you cut each of them into four parts, you would now have 16 quarter loaves.

You can also get a **mixed number**, that is, a whole number with a fraction, for example $2\frac{1}{3}$

To add or subtract fractions you must first make all the fractions have the same **lowest common denominator**.

Thus for example: if we want to add $\frac{2}{3}$ and $\frac{1}{5}$ we have to find a denominator which both thirds and fifths can divide into. The lowest number for this is 15, so it is the lowest common denominator.

$\frac{2}{3}$ becomes $\frac{10}{15}$

$\frac{1}{5}$ becomes $\frac{3}{15}$

We can then add up the fractions $\frac{13}{15}$

When we add or subtract mixed numbers, the numbers must be changed to improper fractions. So, for example, with $2\frac{1}{2} + 1\frac{3}{4}$

$2\frac{1}{2}$ becomes $\frac{5}{2}$ becomes $\frac{10}{4}$

$1\frac{3}{4}$ becomes $\frac{7}{4}$ becomes $\frac{7}{4}$

We can then add up the fractions, which will come to $\frac{17}{4}$.

And turn it back to an improper fraction, which is $4\frac{1}{4}$.

When we multiply or divide fractions we first change the mixed numbers to improper fractions, then cancel where it is possible and then multiply the denominators and then the denominators.

Percentages

Percentages are fractions of a hundred.

Other fractions can be expressed as percentages, thus for example ½ is equivalent to $\frac{50}{100}$ thus 50%.

Decimal fractions

Decimals are fractions that are based on units of ten.

To write a decimal we use a decimal point (.) or comma (,).

So, for example, if we convert the mixed number 2¼ into a decimal fraction, we get 2.25 (we say 2 point two five or two comma two five).

If we convert ⅔ into a decimal fraction we get 0.6666666 and it goes on indefinitely (when this happens we say that the 6 is **recurring**).

A decimal can be to any number of **decimal places**.

Thus 0.1 is one tenth, 0.01 is one hundredth and 0.001 is one thousandth and so on. Calculate with whole numbers and fractions and put in writing using words and mathematical signs and conventions.

Where a decimal fraction goes on for several decimal places we can decide to limit it to only one or two or three places. We say that such a decimal fraction is **correct to** (the number) **decimal places**.

Ratio and proportion

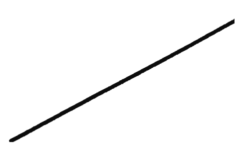
This is a concept closely linked to division. Ratio describes the relationship between things which you are combining or sharing. For example, when you bake a cake or mix cement, you combine the ingredients in a certain proportion in relation to the other ingredients. Thus, for each bag of cement you would use two bags of sand; for two cups of flour you would use one egg.

Identifying lines and shapes

This line is horizontal.

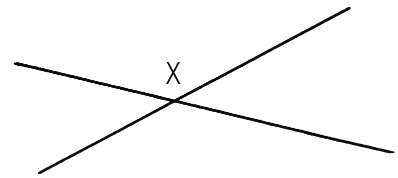
This line is vertical.

This line is oblique.

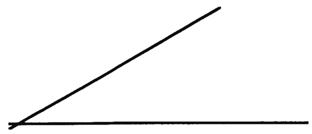


These lines are parallel.

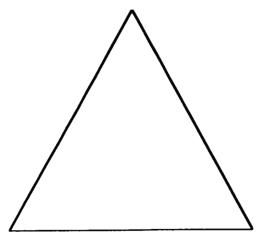
These lines cross or **intersect** at point x.



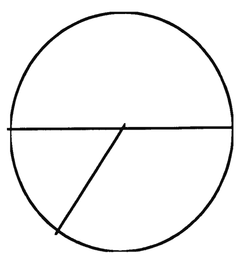
The space between where two lines join is an **angle**.



A three-sided figure is a **triangle**.



The top point is the **apex**, the bottom line the **base**.



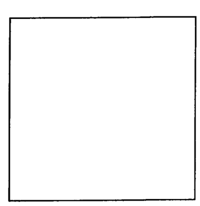
This is a **circle**.

The distance around the circle is the **circumference**.

The line drawn from one side of the circle to the other and passing through the centre point of the circle is the **diameter**.

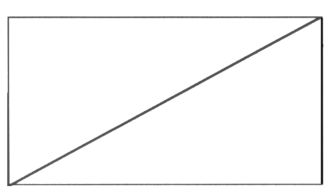
Each half of the circle is called a **semi-circle**.

The line drawn from the centre of the circle to the circumference is called the **radius**.

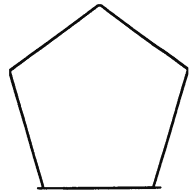


This is a **square** with four equal sides.

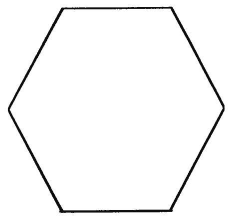
This is a **rectangle**.



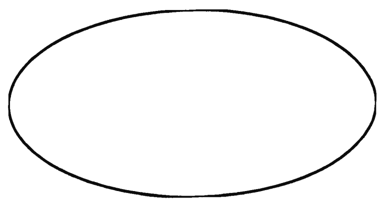
The line joining two opposite corners is a **diagonal**.



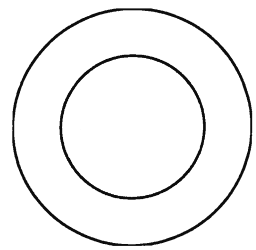
A five-sided **pentagon**



A six-sided **hexagon**



An **ellipse**

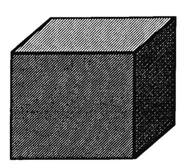


Two **concentric** circles

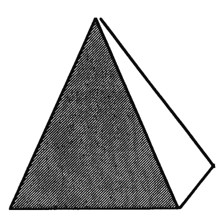
REPRESENTING THREE DIMENSIONAL OBJECTS IN TWO DIMENSIONS

Learners need to be able to represent three dimensional objects correctly in two dimensions through drawings and diagrams. They also have to be able to recognise when a two dimensional drawing is representing a particular view of a three dimensional object.

Cylinders are based on the shape at the end (in this case a circle.)



Cubes are based on square shapes.



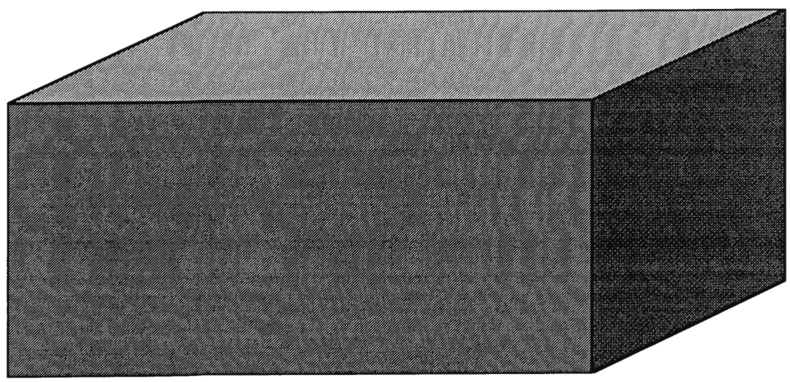
Pyramids are based on rectangles

IDENTIFICATION OF SHAPES IS BASED ON THEIR PROPERTIES

Shapes and two-dimensional representations of three-dimensional figures should be identified on the basis of their properties. Thus, for example, if a shape has three sides one knows it has to be a triangle.

MEASURING LINES AND AREAS

One must be able to measure and calculate linear and area measurements on both the small scale (like drawings on this page) and large scale (such as measuring a room so as to fit a carpet in it).



This is a rectangular solid and has **length**, **width** and **height**.

Length multiplied by width gives one the **area**.

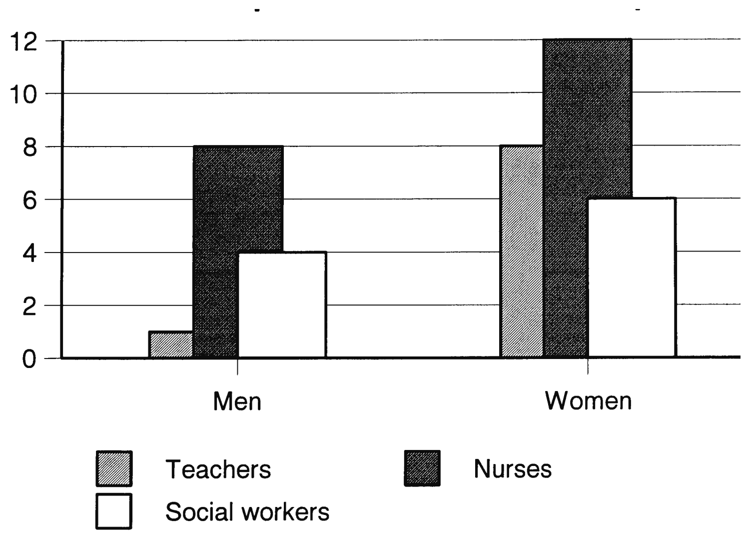
Length multiplied by width and height gives one the **volume** or **capacity**.

DATA HANDLING

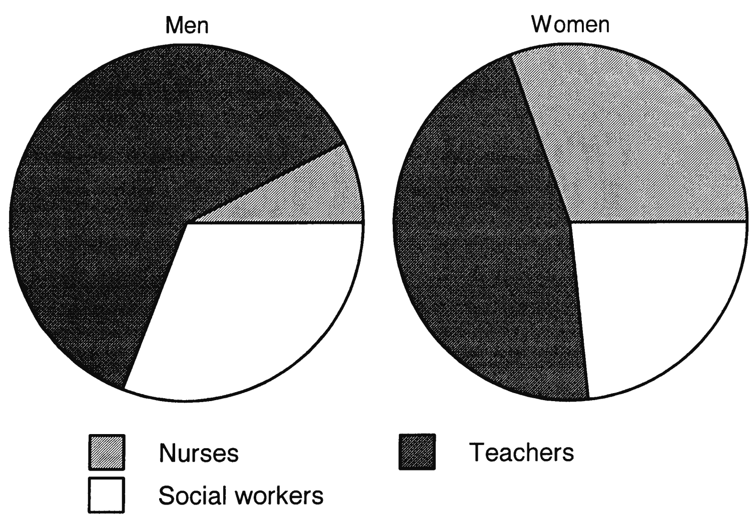
This refers mainly to ways in which we can use number data in meaningful ways. This is usually done by means of charts and graphs of various kinds.

Learners need to become familiar with simple graphs such as **bar graphs** (histograms) and **pie charts**.

Participants in workshop



Participants in workshop



You should also be familiar with such concepts as a **mean** and a **median**.

These are statistics that show us what the main tendency is in a group of numbers. In common speech they are averages. The two most used measures of central tendency are the **mean** and the **median**.

Mean

This is the arithmetic **average**. In a list of numbers the mean is obtained by dividing the sum of the numbers by the number of cases. The symbol for the mean is 0.

Let me use as an example the salaries paid to seven employees of a small organisation. The mean salary paid each month would be the sum of the salaries divided by seven.

Rands	
7 750	
6 500	
6 500	
3 670	
3 600	
2 500	
2 030	
32 550	**Sum**
Divided by 7	**No of cases**
4 670 Mean	

The mean is often the most accurate measure of central tendency because it takes into account all the scores in a set of data. It is, however, affected by extreme scores (and hence not very useful if among a list of salaries there are a few extremely high earners).

Median

The median is the item occupying the middle **position** in a ranked list of numbers (a list of numbers from high to low). It is the score below which half the number of the scores fall. In a list of numbers from high to low, the median is the middle or midpoint number.

So, for example, in a list of the salaries seven people earn each month in a small organisation, the median would be the middle salary in the list (the fourth up or down).

Rands	
7 750	
6 500	
6 500	
3 670	**Median**
3 600	
2 500	
2 030	

Notice that it gives a different figure to the **mean**.

The concept of probability

Probability is the chance that something will happen. We commonly talk about probability. We will say that something is certain to happen, or it may happen, or that it is unlikely to happen, or that it will never happen (or actually be impossible to happen). So the probability of something happening ranges from the certain to the impossible.

We can also use decimals or percentages to express the probability of something happening.

Impossible can be represented by 0 – there is zero chance that it will happen. Your chances of winning the Lotto are also very small – probably less than 0.00001. If you have a choice of choosing from four different colours of clothing there is a 0.25 or 25% chance that you will choose one of them. If you flip a coin there is 0.5 or 50% chance that it will lands heads up. The likelihood that the sun will still rise tomorrow is near certain – a 1.0 or 100% certainty.

Number systems

A number or numeral system is a writing system for expressing numbers in a consistent, standardised way. Because we are so familiar with the tens number system it is important to remember that there are different number systems. The tens number system which we use today is very common (probably because we have ten fingers).

There is of course the way numbers are spoken of in languages. Languages usually have names for the first nine numbers and then names for numbers likes tens, hundreds, thousands, and so on. More useful still are systems which employ special abbreviations for repetitions of symbols; for example, using the first nine letters of our alphabet for these abbreviations, with A standing for "one occurrence", B "two occurrences", and so on, we could then write C+ D/ for the number 304. The numeral system of English is of this type ("three hundred [and] four"), as are those of virtually all other spoken languages, regardless of what written systems they have adopted.

In our system when you see 11 you know that it stands for the number eleven and is represented by a ten and a 1 – thus 11. The number ten is its **base**.

However, in other number systems, such as the binary code (with a base of two) used in computers, an "11" stands for the number three. In the hexadecimal system (with a base of sixteen) (it would stand for the number seventeen). We are familiar with remnants of a base twelve system (dozens), a system that was popular because it is easier to multiply and divide than a ten-base system. The 60-base system we can see in our system of time with minutes and hours.

The way we write and use numbers today is based on the Hindu-Arabic numeral system, borrowed from India, which is a positional base 10 system. It is the system used today throughout the world.

3 FURTHER READING

There are a wide variety of books and internet sites on numeracy and basic mathematics. The following sites all have interesting resources:

http://www.bbc.co.uk/skillswise/numbers/wholenumbers/

http://www.purplemath.com/

http://www.thejournal.com/highlights/roadmap.aspx?h=18&s=85

http://www.staff.vu.edu.au/mcaonline/gen/index.html

http://www.staff.vu.edu.au/mcaonline/tool/index.html

http://www.excellencegateway.org.uk/page.aspx?o=195644

http://www.mathsisfun.com/

An excellent South African book on teaching numeracy to adults is

USWE. 1996. ***Teaching Adult Numeracy 1.*** *Cape Town: USWE*

It is no longer in print but will be in many libraries.

Unit 2

Understanding the requirements of the unit standards

1 INTRODUCTION

Adult Basic Education and Training practitioners working in the current South African ABET system will invariably be teaching courses in numeracy that are designed according to so-called unit standards. What are the particular unit standards applying to numeracy and mathematics for ABET learners? And how useful are they for educators?

2 AIMS OF THE UNIT

This unit aims to briefly explain the purpose of the unit standards for numeracy and mathematics learning in ABET and of those for ABET practitioners teaching numeracy and mathematics.

LEARNING OUTCOMES

By the end of this unit you should be able to apply the requirements of the unit standards for numeracy learners at ABET levels 1 and 2.

You will demonstrate this by being able to

- describe the knowledge and skills required of learners in terms of the unit standards for numeracy at ABET levels 1 and 2
- use the unit standards for numeracy at ABET levels 1 and 2 to plan and facilitate learning
- describe learners' performance in relation to the ABET levels and outcomes
- apply core ETD (ABET) practitioner unit standards to the facilitation of numeracy at ABET Levels 1 and 2

THE UNIT STANDARDS FOR ABET LEVELS 1 TO 2 IN MATHEMATICS

ID number	Unit standard title	Credits	ABET level
Level 1			
119370	Work with numbers; operations with numbers and relationships between numbers	6	1
119366	Work with shape, space and measurement concepts	2	1
119374	Demonstrate an understanding of patterns, functions and algebra	2	1
Level 2			
119378	Work with numbers; operations with numbers and relationships between numbers	5	2
119369	Work with shape, space and measurement	3	2
119365	Demonstrate an understanding of patterns, functions and algebra	4	2
119372	Demonstrate an understanding of data-handling and probability	3	2

You will notice that these standards look very similar – and they are – they are simply at two different levels.

There are some technical terms in the titles of these standards but they are really quite simple in the numeracy knowledge that is expected of learners at this level.

All these unit standards can be found on the South African Qualifications Authority (SAQA) website at:

http://regqs.saqa.org.za/search.php?cat=unit

THE UNIT STANDARDS FOR ABET LEVELS 3 TO 4 IN MATHEMATICS

ID number	Unit standard title	Credits	ABET level
Level 3			
119367	Work with numbers; operations with numbers and relationships between numbers	**5**	**3**
119363	Apply concepts of shape, space and measurement to make decisions relative to the world around us	4	3
119375	Work with patterns, functions and algebra in different contexts	4	3
119376	Demonstrate an understanding of data-handling and probability concepts	4	3
Level 4			
119362	**Work with numbers; operations with numbers and relationships between numbers**	**4**	**4**
7447	Work with numbers in various contexts	6	4
119373	**Describe and represent objects in terms of shape, space and measurement**	**5**	**4**
7450	**Work with measurement in a variety of contexts**	**2**	**4**
7461	Use maps to access and communicate information concerning routes, location and direction	1	4
7463	Describe and represent objects and the environment in terms of shape, space, time and motion	2	4
7464	**Analyse cultural products and processes as representations of shape, space and time**	**2**	**4**
119368	Describe, interpret and represent mathematical patterns, functions and algebra in different contexts	6	4
7453	Use algebraic notations, conventions and terminology to solve problems	3	4

ID number	Unit standard title	Credits	ABET level
7452	**Describe, represent and interpret mathematical models in different contexts**	**6**	**4**
7448	**Work with patterns in various contexts**	**4**	**4**
119364	**Evaluate and solve data-handling and probability problems within given contexts**	**5**	**4**
7451	Collect, analyse, use and communicate numerical data	2	4
7449	**Critically analyse the way mathematics is used in social, political and economic relations**	**2**	**4**
Note: Level 4 unit standards in **bold** are required for the General Education and Training Certificate: Adult Basic Education and Training.			

All these unit standards can be found on the South African Qualifications Authority (SAQA) website at:

http://regqs.saqa.org.za/search.php?cat=unit

USING STANDARDS TO PLAN TEACHING

It is important to understand that the **ABET numeracy standards** are **exit** standards. They describe what learners need to be able to achieve at the end of a course of instruction in numeracy for which the learners will receive credit for their demonstrated knowledge and skills. So, in using these standards to plan teaching, you must recognise that they describe **end goals**. Although the standards help you by stating what the course of instruction in numeracy has to achieve, they do not necessarily help you decide on how to reach those goals in the actual course.

The standards do, however, make it clear that all the instruction and activities must be directed towards finally reaching these end goals.

The main essential then in planning numeracy instruction is to ensure that there are content and exercises that will build up the learners' knowledge and practical numeracy skills so that each specific outcome listed in the unit standards can be achieved. Most existing ABET numeracy course materials will have been designed to do this.

In planning your course of instruction or in using an existing set of course materials you need to know what the unit standards require at the particular ABET level, both for your own planning and budgeting of class time as well in selecting suitable course material.

The unit standards are also very important in your preparations for the assessment of the learners. What you assess should be what the unit standards require you to assess at the particular level and for the particular specific outcomes.

You can also gain guidance for your numeracy teaching by looking at the mathematics related unit standards for the training of ABET practitioners at NQF level 4 and 5.

These are:

ID number	Unit standard title	Credits	NQF level
NQF 4			
7394	Facilitate numeracy at ABET levels 1 and 2	16	4
7485	Demonstrate understanding of real and complex number systems	3	4
7470	Work with a wide range of patterns and inverses of functions and solve related problems	6	4
7465	Collect and use data to establish complex statistical and probability models and solve related problems	5	4
7468	Use mathematics to investigate and monitor the financial aspects of personal, business, national and international issues	2	4
NQF 5			
10298	Facilitate mathematics at ABET levels 3 and 4	20	5
10293	Mediate language, literacies and mathematics across the curriculum	20	5

These look a bit intimidating but when you read them you will see that they are simply using technical terms for quite straightforward numeracy and mathematical knowledge.

All these unit standards can be found on the South African Qualifications Authority (SAQA) website at:

http://regqs.saqa.org.za/search.php?cat=unit

OUTCOMES AND PERFORMANCE IN ABET MATHEMATICS

To get some idea of how one would use a unit standard to get a sense of the outcomes and performance required by learners, let us take some examples from the ABET level 1 Unit Standard No 119370: **Work with numbers; operations with numbers and relationships between numbers**.

Firstly the unit standard lists a number of **purposes**. These are stated in outcomes-based language as they describe what the learner will be able to **do** to achieve certain outcomes. These are to

Recognise, order, describe and compare numbers

Perform calculations to solve realistic and abstract problems

Use different techniques and strategies to calculate

Solve problems in contexts (social, economic, environmental, human rights)

Describe and illustrate a historical number system

Secondly a number of **specific outcomes** are listed. In this unit standard these are identical to the purposes.

For each specific outcome you are told about any special **outcome range** of conditions.

What is to be assessed to show that the outcome has been achieved is found in the **assessment criteria** for each specific outcome.

Let us take the example of the first specific outcome: "Recognise, order, describe and compare numbers".

The **outcome range** says that you have to learn about whole numbers up to three digits long (that is the numbers 1 to 999) and proper fractions where the denominators are only one digit (2 to 9).

There are nine **assessment criteria** which tell you how you are to recognise, order, describe and compare numbers. You have to use the correct names and symbols for the numbers; use place values correctly, order numbers correctly according to size, describe the relationship between numbers correctly (using words such as "greater than", "less than" or "equal to"; use the correct names and symbols for fractions and identify and draw diagrams representing fractions; and compare whole numbers and fractions using number lines and diagrammatic representations.

So you can see how useful the unit standard is in telling you what knowledge and skills the learner has to finally demonstrate and what criteria are to be used in assessing whether the performance has achieved the outcomes. But, as we have said before, it does not tell you how, for example, you are going to teach fractions and make them understandable to the learners. That is your task as the teacher aided by a course or textbook or workbook.

Choosing numeracy and mathematics courses

Many ABET practitioners will make use of prepared numeracy or mathematics courses in teaching. When you look at courses described as numeracy courses you will usually find that they deal with numbers only (eg writing numbers, counting, addition, subtraction, multiplication and division).

These will be mainly suitable for ABET level 1 and 2. Courses described as mathematics courses will usually also include basic numeracy but will include such things as measurement of lengths, weights and capacities, the calculation of areas and quantities, and some very simple statistics and statistical charts and graphs.

3 FURTHER READING

All the mathematics unit standards listed in this unit can be found on the South African Qualifications Authority (SAQA) website at:

http://regqs.saqa.org.za/search.php?cat=unit

You are advised to have copies of all the relevant unit standards for the level or levels of numeracy or mathematics you may teach.

Unit 3

What do the learners know already and how can it be used?

1 INTRODUCTION

We are often told that teaching adults is very different from teaching children because adults, even if illiterate and poorly educated, have a lot of practical knowledge and experience of things in the world. How then can we as educators make use of the knowledge that you and the learners already have about numeracy.

2 AIMS OF THE UNIT

To discuss the importance of making use of the learners' prior knowledge of numbers and their experience of using numbers in their everyday lives.

LEARNING OUTCOMES

By the end of this unit you should be able to help learners to identify and develop own knowledge of numeracy.

You will demonstrate this by being able to

- explain and apply the principle of active construction of mathematical knowledge by learners and reflective discussion
- encourage and facilitate reflective discussion during the construction of mathematical knowledge
- seat learners in such a way that they can all see and hear one another comfortably during reflective group discussions

USING LEARNERS' PRIOR KNOWLEDGE IN MATHEMATICS

All people have an innate sense of number and quantity and geometry and all languages have words for numbers. Illiterate and poorly educated adults will have this basic number competency, even though it has remained undeveloped and may be limited to small numbers. Even poorly educated adults can usually do simple addition and subtraction with one or two digits (eg when using money) and have a conception of "more than" or "less than" (Abadzi, 2006).

Instruction is necessary if people are to become fully numerate. This is because people do not have an innate, biologically designed command of large numbers, large sets of numbers, and things such as fractions, multicolumn addition and subtraction, carrying borrowing, multiplication tables, and so on. To do all this they need instruction and considerable practise. Some of this instruction must enable the adult learners to understand mathematical symbols and concepts. Practise is needed for the rote memorisation of routine calculations and procedures such as number bonds and multiplication tables (without knowing number bonds and multiplication tables by heart, learners have to spend time calculating the results and run out of working memory and time. This is equivalent to reading letters one by one rather than recognising words). The use of rote learning of some low-level concepts may help students grasp higher-level concepts, such as negative numbers (Abadzi, 2006).

The challenge for numeracy educators then is to make use of what innate knowledge learners have of numbers, shapes and measurement and of their experience in using numbers at a simple level (as with money) and at the same time provide instruction to help them to build a more sophisticated sense of numbers and how to use them. One of the main problems you will face as a numeracy educator is to enable the learners to make the connection between abstract symbols of numbers and calculation rules and the natural number sense that they already have and use in daily things like making a good buy at the supermarket or giving change.

The language used about numbers

People tend to use in calculations the language in which they first learnt maths skills. It would therefore be unwise to keep switching the language you use in instruction. To help the learners do fast and accurate calculations, the language used should not be switched mid-way through the programme. For example, if the learners are to learn calculations later on in English, perhaps they should start with calculations in English early on.

How do learners "see" numbers?

It is important to understand that to become fluent in calculations learners need to "see" numbers in three different ways.

- Firstly they must see the number as visual digit (eg they must recognise the signs, 1, 2, 3 etc) and hold them in their mind.
- Secondly, they must associate this sign, this number, with the words they hear spoken or read (eg the words one, two, three, etc).

- Thirdly, they must understand that these both represent quantities, so, for example, two is bigger than (and is in fact twice as big).

When a person is doing calculations all these three ways of seeing or thinking about numbers are in operation.

Mathematics in everyday activities

What is the relationship between the numerical skills which many people use in their everyday lives, and the mathematics taught in the ABET class?

Helping the learners to see that there is a relationship between numbers and everyday life is important for making numeracy "real" and useful.

When people go shopping they deal with concepts of quantity all the time, for example *money, weight,* and *capacity*. Working with quantity is the basis of mathematics, because numbers are in fact the way in which we express quantities. Comparing one thing with another, or thinking about how two things relate to each other, is a mathematical principle. Looking for common patterns in our everyday life experiences, working out common lessons from different kinds of experience, ordering our activities in a diary or a weekly, monthly or yearly planner are also examples of mathematical principles.

Think of all the occasions in your everyday life where you have to make use of numbers. If numeracy is applied to everyday life situations, learners can more easily see the reason to learn the numeracy skills required. These everyday activities could be the basis for practical exercises.

How then can we make use of this wealth of practical knowledge that learners already have?

ACTIVITY 1

■ ■ ■

Think about all the situations and activities in everyday life where numeracy skills are important.

Write these down in a list.

(This is also an exercise that you should do with numeracy learners.)

■ ■ ■

Our response

Here are some everyday activities that we thought of:

Telling the time	Using analogue and digital watches. Using 12 hour and 24 hour time. Adding and subtracting time. Using time in cooking. Converting minutes into hours and vice versa. Using a calendar. Using a diary. Reading bus or train or air travel timetables. Using a stopwatch in sport.
Money	Understanding money units. Relating money to decimals. Calculating costs. Calculating shares of money. Understanding interest. Understanding hire purchase.
Shopping	Estimating costs of a basket of goods. Counting totals. Counting change. Calculating best buys.
Banking	Filling in bank deposit and withdrawal slips. Filling out forms and cheques. Writing numbers in words on cheques. Reading bank statements.
Paying bills	Total amounts, budgeting, working out repayments, cost of loans, cost of credit cards or hire purchase.
Buying a house or car	Comparing interest rates. Working out repayment costs per month.
Measuring for dressmaking, decorating, building, gardening	Using measurement tools. Understanding metric measures. Understanding metric abbreviations. Making estimations. Using rules to work out perimeter, area, volume, mass, fractions, percentages, temperature, capacity, etc.
Cooking	Reading recipes. Shopping for ingredients. Measuring weight, volume, temperature, and time.
Driving	Understanding speeds, distances, fuel consumption. Calculating insurance and servicing costs. Budgeting for expenses.
Drinking	Measuring amounts in bottles and cartons. Working out the cost of alcohol. Working out if consumption is over the limit for driving.
Travel	Comparing fares for train, bus, taxi and car. Costing, organising and booking transport. Booking accommodation and renting cars. Budgeting for travel. Working out exchange rates. Reading timetables. Understanding time differences. Map reading.
Health	Following medical directions and quantities. Claiming hospital or medical expenses.
Income tax	Filling in income tax returns.
Telephoning	Cellphone costs. Understanding a phone bill.
Reading newspapers and brochures	Interpreting tables and graphs. Using an index. Using page numbers.

Sport	Shapes and sizes of fields and equipment. Scoring games. Recording times.

We can help learners make the connection between everyday activities and numeracy/mathematics by showing that having numeracy skills can help them solve everyday problems – as in many of the areas listed above.

We can also explain to them that it is the very usefulness of numeracy/mathematics that makes it such a powerful set of skills and why, in the modern world, possession of such skills helps people advance in their careers. This also explains why the school curriculum now insists that all learners study some form of mathematics right up to Senior Certificate ("matric") [Grade 12] level.

HELPING LEARNERS RECOGNISE WHAT THEY ALREADY KNOW ABOUT NUMBERS

If, then, we recognise that learners will already know something about numbers and will have had some experience of using numbers, the challenge is to help the learners connect their existing knowledge with new knowledge about numbers and to so build up their numeracy. This building up or construction process can be made easier when learners can see the practical, concrete usefulness and applications of number knowledge.

The active construction of mathematical knowledge requires that learners be given an opportunity to realise what they know. This is best done by discussing how they have experienced number use in practical situations. So even in a numeracy/mathematics class there is scope for discussion and describing of learner experiences. Learners should be seated and taught in such a way that they are able to see and hear one another comfortably during reflective group discussions.

This approach also requires flexibility from the educator because different learners will have had different experiences and will have different ways of coping with number challenges. Though some of these ways may be "wrong" (in the sense that the methods some learners use may not work systematically) one must be sensitive to the efforts that the learners have made to make sense of numbers and gently help them to build more appropriate understanding and calculation techniques.

In all this it is also vital that numeracy teaching has a strong practical component. This means that, while discussion and reading are important when we are learning mathematics, it is also important to write, calculate and, in some cases, to make drawings as well.

ACTIVITY 2

■ ■ ■

Think of ways in which you could build on learners' experience to develop their sense of numbers and their usefulness.

■ ■ ■

Our response

These are some of the things we would try:

- We would choose tasks that allowed for success.
- Create lots of opportunities for learning through discussion and interaction with the other learners.
- Provide practical hands-on activities that build on the learners' own experiences and activities.
- Respond to the learners' interests and experiences.
- Use adult contexts and examples that draw on the learners' own background, interests and experiences.
- Encourage the learners to reflect on their own learning.

3 FURTHER READING

Abadzi, H. 2006. The development and teaching of numeracy. In Abadzi, H.(ed.), *Efficient learning for the poor: insights from the frontier of cognitive neuroscience*. Washington, DC: The World Bank, pp 58–61.

> A state of the art summary of current ideas and research on numeracy teaching. Although the book mainly addresses school education it has valuable insights for all adult educators.

REFERENCES

Abadzi, H. 2006. The development and teaching of numeracy. In Abadzi, H. (ed.), *Efficient learning for the poor: insights from the frontier of cognitive neuroscience*. Washington, DC: The World Bank, pp 58–61.

Unit 4

Developing mathematical concepts and skills

1 INTRODUCTION

In practice, when we teach a class of learners, we follow a systematic sequence of learning activities. Developing such a common path or sequence of activities is often done in the form of a prepared course, programme or textbook.

2 AIMS OF THE UNIT

This unit aims to take you through some of the processes and exercises that would serve to provide a systematic development of mathematical concepts and skills.

LEARNING OUTCOMES

You will demonstrate this by being able to

- explain the common path of development of numeracy concepts and skills
- analyse numeracy materials and learning programmes in terms of the structured development of numeracy concepts and skills
- describe each learner's numeracy concepts and skills with reference to the methods he or she uses to solve numeracy problems

FINDING A COMMON DEVELOPMENTAL PATH FOR MATHEMATICAL CONCEPTS AND SKILLS

Although we know that there are many differences among learners and that many adults have developed their own ways of doing simple calculations (such as with money), we often cannot devise an individual learning programme for each learner – especially if we are teaching a large class. In addition, there is a lot of educational experience that has led to the development of appropriate programmes of numeracy development. There is a general accepted common developmental path that will work well for most learners. Many of these ideas are, of course, built into good courses and workbooks on numeracy.

There is general consensus that it is vitally important to build a sound understanding of mathematical concepts. Without that base of conceptual understanding, although the learner may learn some "tricks" of calculation, the future learning of more advanced concepts and procedures will be built on a weak base.

In interacting with learners it is important that you as the teacher understand the methods the learner is currently using to solve numeracy problems (often of a practical nature). This enables you to determine the right way to build on the existing concepts the learner has or to change them if they are leading the learner into a dead end.

THE STRUCTURED DEVELOPMENT OF NUMERACY CONCEPT

A good numeracy course or programme will have a clear structure and will build the more advanced concepts on knowledge of the basic mathematical operations.

If you have to choose materials or a course you need to check that such materials do in fact provide for the structured development of numeracy concepts and skills.

Learners, as they progress, should not only be able to calculate, but should also

- understand the purpose of the methods they use to calculate and what is achieved by them
- understand why mathematical methods work
- know when to use a particular method
- be able to interpret and understand the solutions and answers they come to

Thinking about the method that you like best, do the following subtraction:

42 – 29.

ACTIVITY 1

■ ■ ■

Prepare steps in a lesson plan on addition for adult learners.

These learners have been in an ABET Level 1 language group for six months, but you have only just started teaching them numeracy.

The lesson should last for one-and-a-half hours. To help you plan the lesson, try to think of what you want learners to be able to understand or do at the end of the lesson. In other words what outcome you hope to achieve. We have included a list of other points for you to think about before you write down your lesson plan.

- It must be clear to learners what the operation "addition" means.
- You need to establish what skills the learners already have.
- You need to use a context for practising addition which is meaningful for them.

- You need to ensure that your learners can add adequately by the end of the lesson.
- How will you establish that, firstly, your learners understand what addition means and, secondly, whether they have acquired the skill of addition?

■ ■ ■

Some ideas for teaching content areas of mathematics that are crucial and that many learners find difficult

Teaching numbers

In many ways, numbers are like teaching the alphabet: they are the tools for working with quantity just as the alphabet is our tool for working with written language. Whilst many learners may not be able to read or write numbers, they will have a *functional sense* of quantity from their everyday lives in familiar activities like cooking and shopping.

ACTIVITY 2

■ ■ ■

Make a list of the most important things you think ABET learners need to know to be competent in using numbers. Then think of some ideas for developing these skills.

■ ■ ■

Our response

This is what our list looks like:

- Learners need to be able to count in sequence.
- Learners need to be able to write numbers.
- Learners should understand that numbers are the symbolic representation of quantity.

An understanding of numbers can be developed by counting in different ways, for example by counting in ascending order (going up) and descending order (going down), by beginning at different starting points, and by counting in groups (of say twos, threes, fives, tens). This activity is called *generating number patterns.* Whilst numbers are fairly specific things, they can also be described in terms of each other (eg 7 is somewhere between 5 and 10), and in terms of their parts (eg 8 can be 7 + 1, 2 + 2 + 2 + 2, 4 x 2, 10 – 2, etc).

We will look at some ideas for building an understanding of numbers.

Helping learners to understand place value

Our number system has ten symbols which are called *digits:*

0 1 2 3 4 5 6 7 8 9

We combine these digits with each other to build up all the other numbers, and the way we combine them gives a different value to each digit. This is what is called "place value" – the **value** of the digit changes according to the position or **place** which it occupies in the number. For example, look at the following numbers:

132, 321, 231, and 213

The three digits, 1, 2 and 3 are combined differently so as to make four different numbers. In each number, the value of each numeral is different. In the first example the "2" has the value of two; in the second example it has a value of twenty; and in the third and fourth examples, the "2" has a value of two hundred.

The number 231 can be read in a number of ways:

- two hundred and thirty-one
- two groups of 100, three groups of 10 (which we call thirty), and a single one
- 231 = 200 + 30+1

ACTIVITY 3

■ ■ ■

(a) What is the value of "1" in each of these numbers: 123, 231, 312 and 213?
(b) Can you make up another number using the same three digits once each?
(c) What is the biggest three-digit number which you can make with these three digits?
(d) And the smallest?

■ ■ ■

Our response

Here are our answers:

(a) The value of "1" is *one hundred* in the first one, *one* in the second, *ten* in the third and fourth.
(b) The other numbers are 132 and 321.
(c) The biggest number you can make is 321 and the smallest is 123.

Some useful resources for teaching place value are illustrated below:

Number cards

These are useful for teaching the concept of place value.

You need one set of cards for your group:

(1) 0–9: zero, one, two … nine,
(2) 10–90 in tens: ten, twenty … ninety,
(3) 100–900 in hundreds: one hundred … nine hundred
(4) 1000–9000 in thousands: one thousand … nine thousand

Take turns to make up different numbers. What is the biggest number you can make?

Place a few sets of number cards face down. Decide on the number of digits your number is to have. Then take turns selecting from the number cards. With the digit you pick up, decide in which place it should be if you want to make the biggest number possible or the smallest number possible.

Dealing with learner errors

Analysing learners' errors is a useful way of gaining us insight into our learners' thinking as well as a way of finding out whether our teaching is achieving the goals we have set for ourselves.

ACTIVITY 4

Study the following set of calculations done by a learner. Some errors have been made. Think about why the learner has made these errors. Do you think the learner has just been careless, or do you think that she doesn't really understand the concepts? How could you help her to correct the errors?

46
+29
65
38
+54
82
128
+75
193

(**Source:** Frankenstein 1981)

Our response

We see that the learner has made the *same* mistake three times. We therefore feel that the learner hasn't just been careless – the learner doesn't know that he or she should add another 1 to the tens column. We think that we could help by doing more work on place value so that the learner can develop a better understanding of why it is necessary to add the figure to the tens or hundreds column.

Using a number quiz

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

THINKING ABOUT NUMBERS

Where does our number system come from?

There is evidence from all the continents of a long history in the development of our modern number system. Developed originally from a system of tallying (making a mark for each item, or setting aside a pebble, or making a knot in a piece of string), these marks were then grouped in different ways. The system which we use is called the Hindu-Arabic system and it is particularly well suited for doing calculations. It also makes it possible to work in fractions. The Roman numerals used in Europe 2000 years ago were very cumbersome for doing any calculations. Try, for example, to multiply the Roman numerals XIII and XXV!

Using "10" as the basis for our number system probably happened quite naturally, because it was natural for people to use their fingers to tally up numbers and to use our ten fingers as a natural group. In fact, the word "ten" seems to have come from *taihun* or *taihund* which meant "two hands" in Old English.

Perhaps one of the interesting debates about the development of our number system was the argument about whether "0" is a digit or not. It has been argued that because "0" is "nothing", it does not require a tally mark or a pebble. In Zulu, zero is called *iqanda,* an egg, probably taken from its shape. We often still talk about "0" as a "place holder".

Similarly, until the 1600s there was a heated debate in Europe about negative numbers – many mathematicians felt that you could not quantify something that did not exist. Clearly, one cannot talk about having minus three chairs to sit on, but it is very useful to know that the temperature is minus 5 degrees on a cold winter's night!

One of the ways in which historians of mathematics have traced the way different number systems may have developed is to examine how numbers are named. For example, it is clear from languages like isiXhosa and isiZulu that numbers are grouped in tens. There is a unique name for each of the ten digits up to ten, but from eleven onwards these same names are used in combination to name the numbers higher than ten: eleven is spoken of as "ten and one", twenty-three is "twice ten and three". This formulation also suggests that there is a relationship between the language we use to describe certain things and their concepts. Do you know if this is the same for other South African languages? This way of naming numbers also makes teaching place value to speakers of these languages much easier than in English.

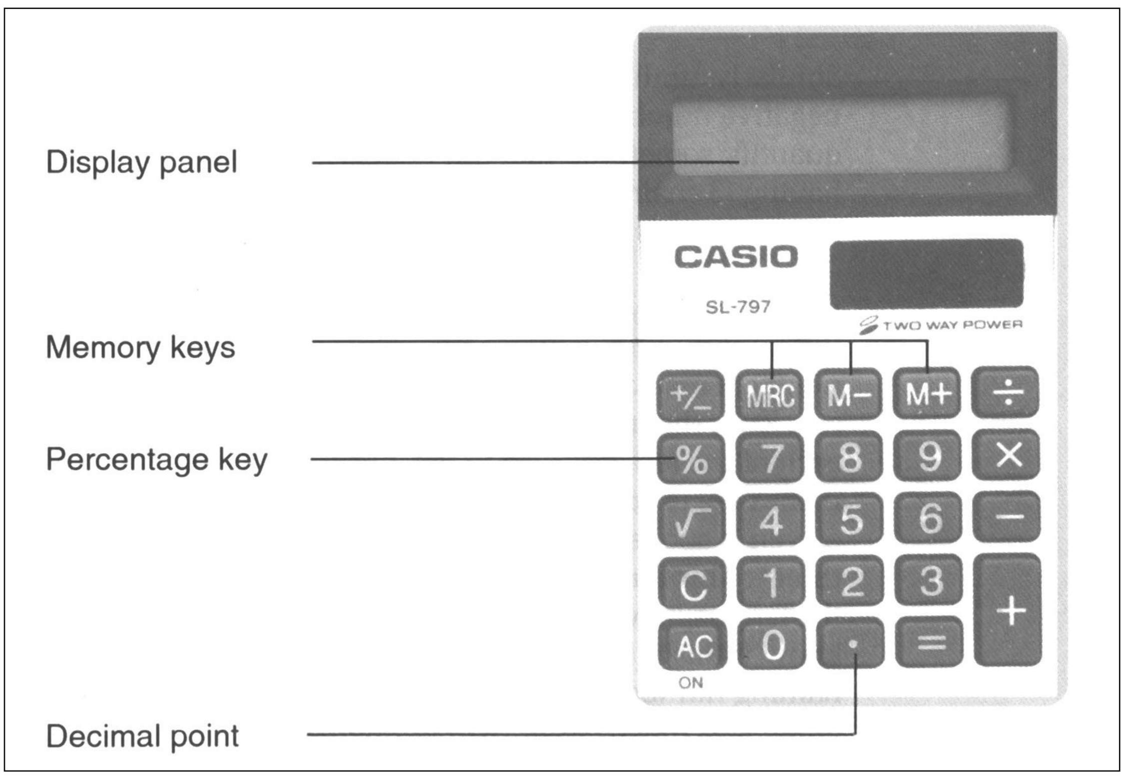
Modern	1	2	3	4	5	6	7	8	9	10
Bengali (India)	১	২	৩	৪	৫	৬	৭	৮	৯	১০
Urdu (Pakistan)	۱	۲	۳	۴	۵	۶	۷	۸	۹	۱۰
Burmese (Burma)	၁	၂	၃	၄	၅	၆	၇	၈	၉	၁၀

(**Source:** Irons & Burnett 1994)

HELPING LEARNERS LEARN HOW TO USE A CALCULATOR

Calculators are a useful teaching and learning aid. They also make the mechanics of calculations much easier and are useful for doing counting exercises in groups.

Here is a picture of a basic calculator.



(**Source:** TURP 1992:11)

Calculators cannot replace developing an understanding of numbers, nor of the four operations (adding, subtracting, multiplying, and dividing). Calculators can be used to check an estimated (guessed) outcome. For example, in a multiplication like: 347 x 42 a reasonable estimation could be found by multiplying 350 x 40, which gives 14 000 and this serves as a check on the exact outcome from the calculator. If the answer you get on the calculator is very different from this number, you know that something went wrong in the way you used the calculator.

Although calculators can save us time doing big calculations, a calculator cannot solve the thinking part of a problem for you. You need to know how the required calculations should be carried out (and you should be able to make a rough estimate of the likely answer). If you do not understand the operation you are performing it is a mistake to use a calculator.

ACTIVITY 5

■ ■ ■

A group of learners used their calculators to calculate 499 – 105. Three learners got the following answers.

(1) 499 – 105 = 499 105
(2) 499 – 105 = 4.752
(3) 499 – 105 = 484

What did each learner do wrong?

■ ■ ■

Use your calculator to see if you can get the same wrong answers so that you can see how the mistakes arose.

Do you think that the learners estimated their answers first?

Are their errors conceptual ones or simply a slip?

What can you do about these errors?

Our response

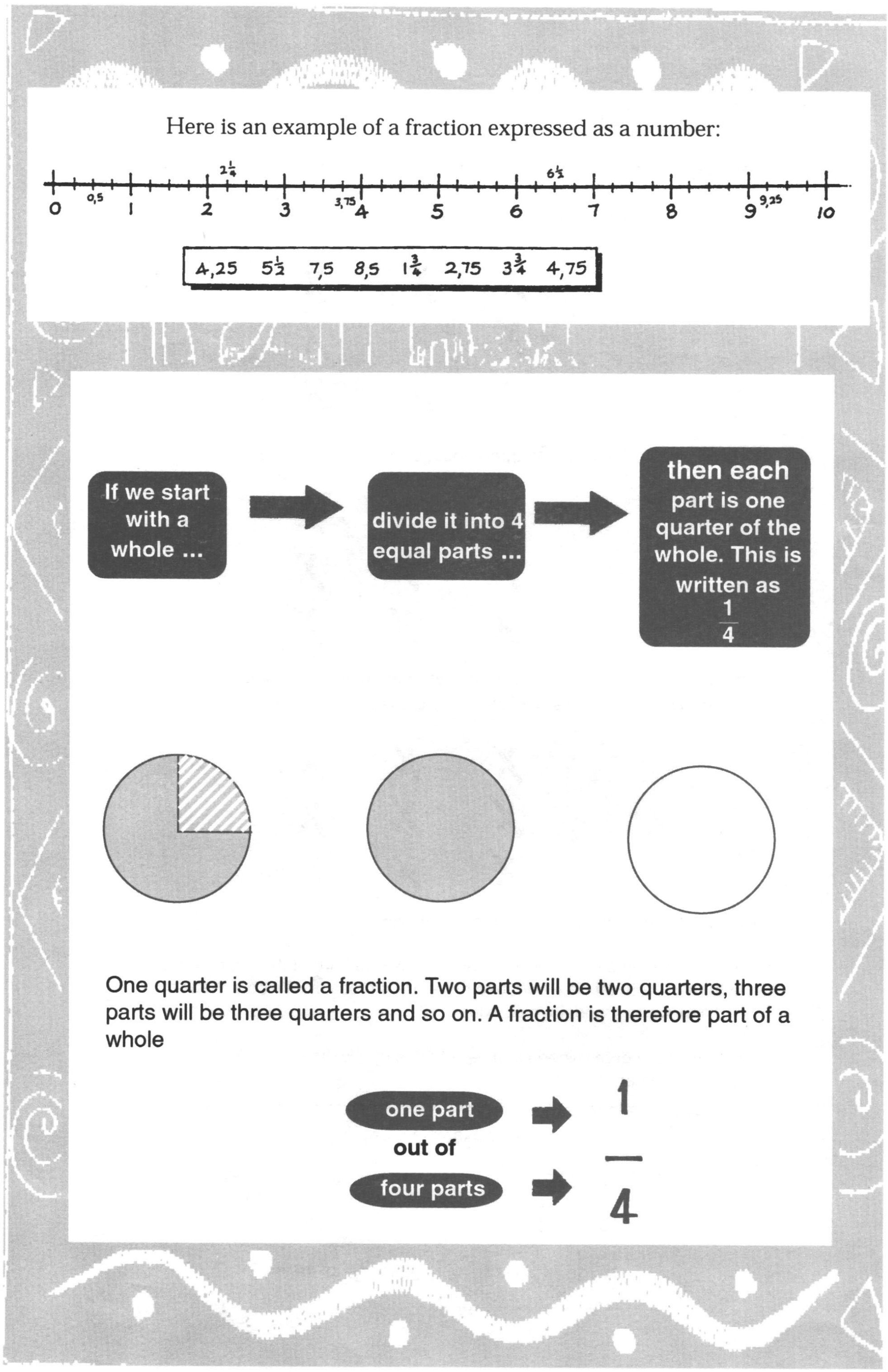
Here are our answers:

(1) The first learner pressed the two numbers and didn't press the minus sign in between.
(2) The second learner pressed the divide sign and not the subtraction sign.
(3) The third learner pressed 499 minus 15 rather than minus 105.

Before deciding how to correct the mistakes, it is important for us to try to understand *why* the mistakes were made. These are some of the questions we would consider: Do the learners understand the meaning of the operations? Do the learners know how to use the calculator? Do the learners understand the difference between 105 and 15? Then we would develop a teaching strategy to solve the problems we have identified.

HELPING LEARNERS UNDERSTAND HOW TO DO FRACTIONS

Decimals and proper fractions are very much part of our daily lives, yet they seem to cause a lot of headaches. There are different ways of showing fractions.



This is how we find out what a fraction of a number is

How would we find out what $\frac{3}{4}$ of 100 is?

Step 1

We divide 100 by the denominator

$100 \div 2 = 25$

This tells us what $\frac{1}{4}$ is

Step 2

Multiply the answer by the numerator

$25 \times 3 = 75$

So $\frac{3}{4}$ of $100 = 75$

The formula which we used was:
Divide by the denominator & multiply the answer by the numerator

Other examples are:

1. $\frac{2}{3}$ of 60

   $60 \div 3 = 20$

   $20 \times 2 = 40$

   so 40 is $\frac{2}{3}$ of 60

2. $\frac{5}{8}$ of 16

   $16 \div 8 = 2$

   $2 \times 5 = 10$

   so 10 is $\frac{5}{8}$ of 16

So, if each number to the left is 10 times bigger, then each number to the right is ten times smaller.

If we move beyond the units, then our numbers are made up of tenths, hundredths, thousandths, etc. These are really fractions that are divisible by 10

thousands hundreds tens units tenths hundredths thousandths

2 2 2 2 2 2 2

This number will be two thousand two hundred and twenty two point two two two

Note that a point is used to show when the rest of the numbers to the right are less than the units. This point is called a decimal point and when numbers are written in this way then they are written in a decimal form

We could have written this number is a fraction form

$$2\,222\ \frac{222}{1000}$$

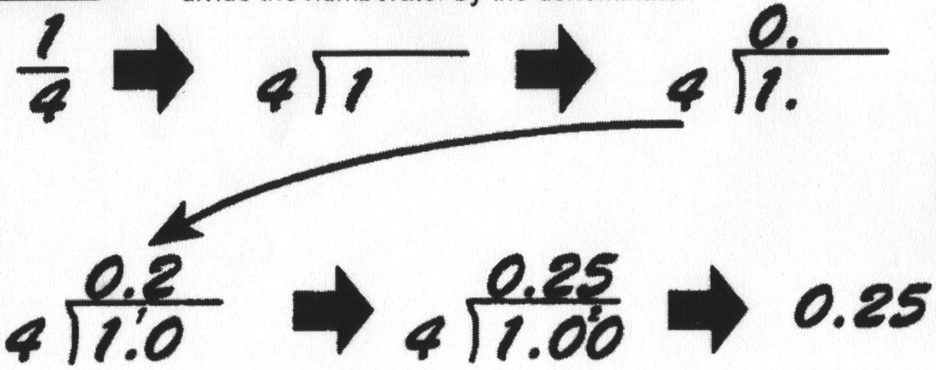
(two thousand two hundred and twenty two and two hundred and twenty two thousandths)

Other examples are

1. 353.32 = $353\ \frac{32}{100}$
2. 456.913 = $456\ \frac{913}{1000}$
3. 1091.056 = $1091\ \frac{056}{1000}$

This is how we change a fraction to a decimal

Fractions can be changed into decimals. To do this, we divide the numberator by the denominator.



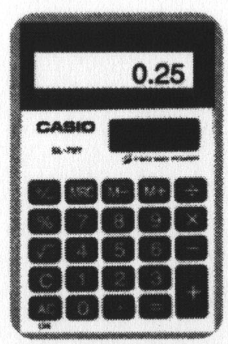
So $\frac{1}{4}$ written in a decimal form will be 0.25

The dot (.) between 0 and 25 is called a decimal point

You will notice that 0.25 is $\frac{25}{100}$

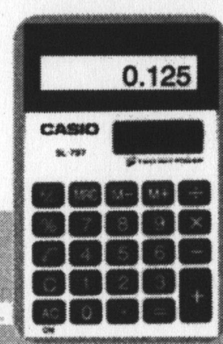
You can do this sum the easy way on the calculator

1 ÷ 4 = 0.25



Another example is $\frac{1}{8}$

1 ÷ 8 = 0.125



So $\frac{1}{8}$ is 0.125 or $\frac{125}{1000}$

This is how we determine the size of a decimal

Thembeka is wondering ....
Is 0.3 smaller or bigger than 0.05?

Step 1

We change both decimals to a common denominator

This means that they have the same denominator
This is the only way we can compare them

$0.3 = \frac{3}{10}$

which could be written as $\frac{30}{100}$

$0.05 = \frac{5}{100}$

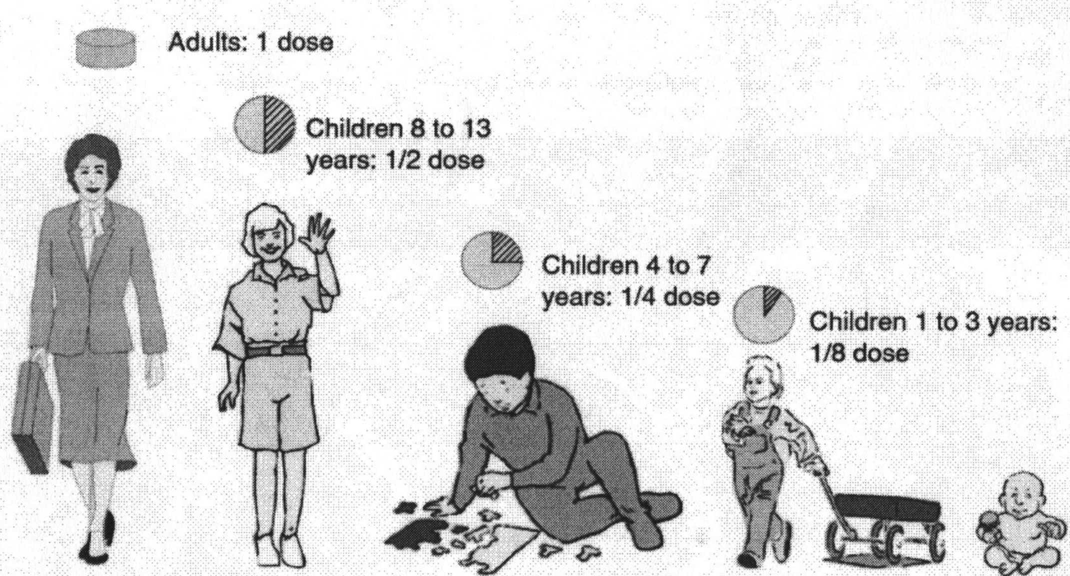
Step 2

Now that both decimals have changed to a number over a hundred, we can compare the two

$\frac{30}{100}$ is bigger than $\frac{5}{100}$

This means 0.3 is bigger than 0.05

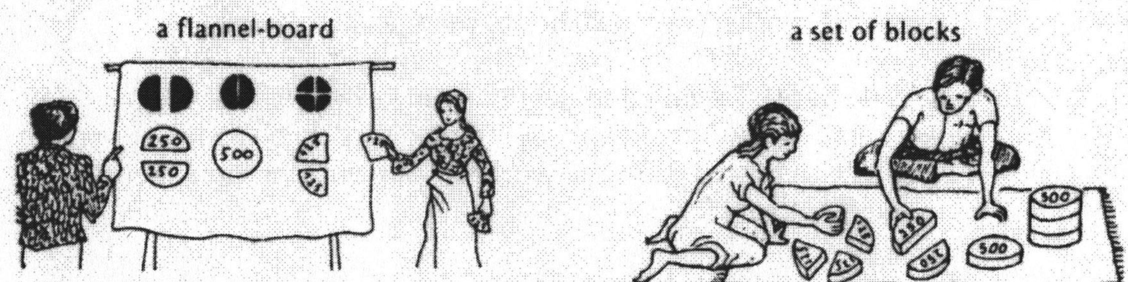
It is important to remember this especially when we work with rands and cents as we will show later on in this chapter.



Give a child under 1 year old the dose for a child of 1 year, but ask medical advice when possible

Teaching aids for learning about fractions and milligrams

Many health workers at first have difficulty in understanding the use of fractions and milligrams for medicing dosages. Teaching aids that can help them to 'see' what fractions mean are shown below. Students can help make these teaching aids themselves.



The flannel-board pieces or blocks that stand for tablets can be labeled 500 mg., 250 mg., or 400,000 units, to represent different medicines. Then students can practice figuring out dosages for adults and children.

(**Source:** Werner & Bower, 1986)

HELPING LEARNERS TO CARRY OUT THE FOUR OPERATIONS: ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION

For each of the operations, addition, subtraction, multiplication and division, it is important that learners should understand the concepts and the relationships between them. For example, addition and subtraction are opposites, and multiplication is the opposite of division. Multiplication is sometimes also described as repeated addition. What do you think the relationship is between division and subtraction?

Some basic assumptions about the operations

Most learners, even if they are not able to read and write, can usually add and subtract numbers using non-written forms.

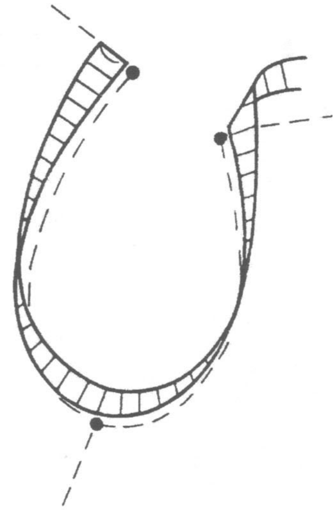
- Learners, and indeed you too, may use different methods of calculation. The method used may depend on the context in which the calculation is being done. For example, it is fairly usual to use the "add on" method of subtraction when checking your change when doing shopping. In most cases, you should not try to change the various methods which the learners already use. The only advantage of using columns (hundreds, tens, units), which is the most common method taught in schools, is that it is efficient; however, many students who use this method at school are able to get correct answers without any understanding of what they are doing. Also, for this method to be successful, learners must have a very thorough grasp of place value. In fact, most learners can add and subtract adequately if there is no "borrowing" and "carrying". Problems that emerge here are to do with a poor grasp of place value.
- Being able to round off and estimate are important skills for working with the operations.

Rounding off

R238.47 can be rounded off:

to the nearest R100	=	R200,00
to the nearest R10	=	R240,00
to the nearest R1	=	R238,00

The degree of accuracy required when rounding-off will depend on the situation. Here is an example.



(Source: ALBSU 1991)

You need to make a sleeve for a jacket you are sewing. You need to measure the sleeve and the armhole before you buy the material. Do you need an exact measurement? Would you round off to give yourself extra material or a bit less?

Whether we choose to round off to ten, a hundred, a thousand and so on depends on:

- how big the numbers are
- how close we want to be to the exact answer
- how quickly we need to work the answer out

ACTIVITY 6

■ ■ ■

Imagine that you have been asked to accompany a touring jazz band on its travels around South Africa. The band travels 1 413 km from Cape Town to Johannesburg. From there it travels 583 km to Durban and then 436 km to Umtata.

- Estimate the total distance which the band will have travelled by the end of its trip by rounding if off to the nearest ten.
- Estimate the total distance it travelled by rounding off to the nearest hundred.
- Use your calculator to find the exact distance travelled.
- Which method is quickest?
- Which method gives the exact answer?
- Describe one or two situations where you might use estimations for solving a problem. Would you also use the calculator method?

■ ■ ■

Adding

At the moment, there are two very commonly used methods for addition: the one is working in tens and the other method is the traditional method of adding in columns by "carrying". Here is an example of each:

Example of working in tens:

234 + 79 = 200 + 30 + 4 + 70 + 9

= 300 + 10 + 3

= 313

Example of "carrying":

234 + 79 = (4 + 9 = 10 + 3) (30 + 70 + (10 + 3) = 100 + 10 + 3) ((100 + 10 + 3) + 200)

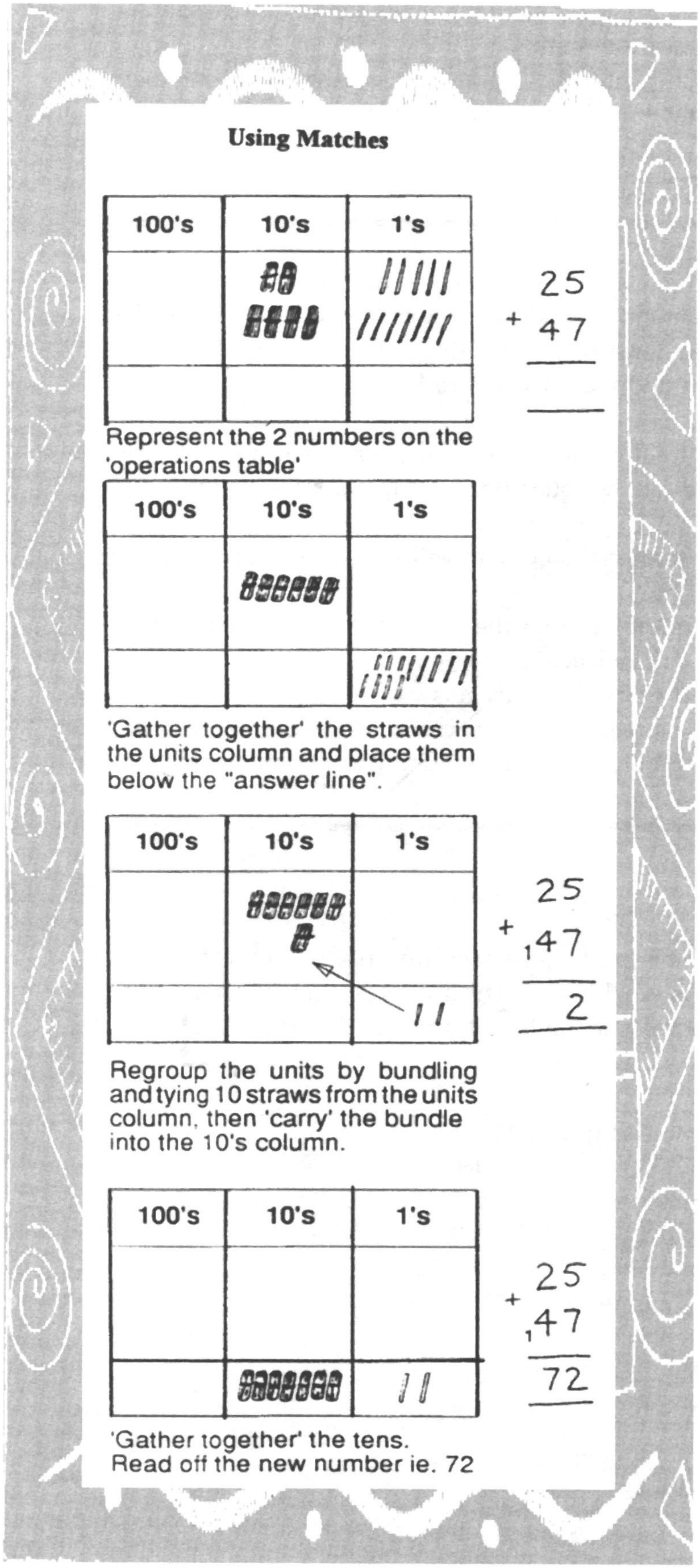
= 313

Subtracting

The "adding on" method is often used for subtraction, so that 234 – 79 becomes:

79 to 80 is 1, 80 to 100 is 20 + 1, 100 to 200 is 100 + 21, and 34 gives 155.

The traditional columns method involves "borrowing". As with addition, the most important concept to understand is place value. Here is an example of understanding place value which uses bundles of 10 matches



(**Source**: Goddard, Marr & Martin 1991)

Multiplication

Here are examples of learning materials taken from a course book on multiplication produced by the English Literacy Project. This book gives some ideas for teaching multiplication.

Write the answers in the blocks where the two lines meet.
This gives you the 2 times table.
Do the same thing for all the tables between 3 and 10.

x	2	3	4	5	6	7	8	9	10
2	4								
3	6	9							
4	8		16						
5				25					
6					36				
7						49			
8							64		
9								81	
10									100
11									
12									

Read
You have filled in the times tables going down.
Find another way to read the times tables in the chart.

Discuss and write

Circle all the 40s in the multiplication chart on page 42..
Use the chart to write the sum for each 40.

10 x 4 = 40

_______ x _______ = 40

_______ x _______ = 40

_______ x _______ = 40

x	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	23	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	961	08	120

Circle all the 21s in the multiplication chart

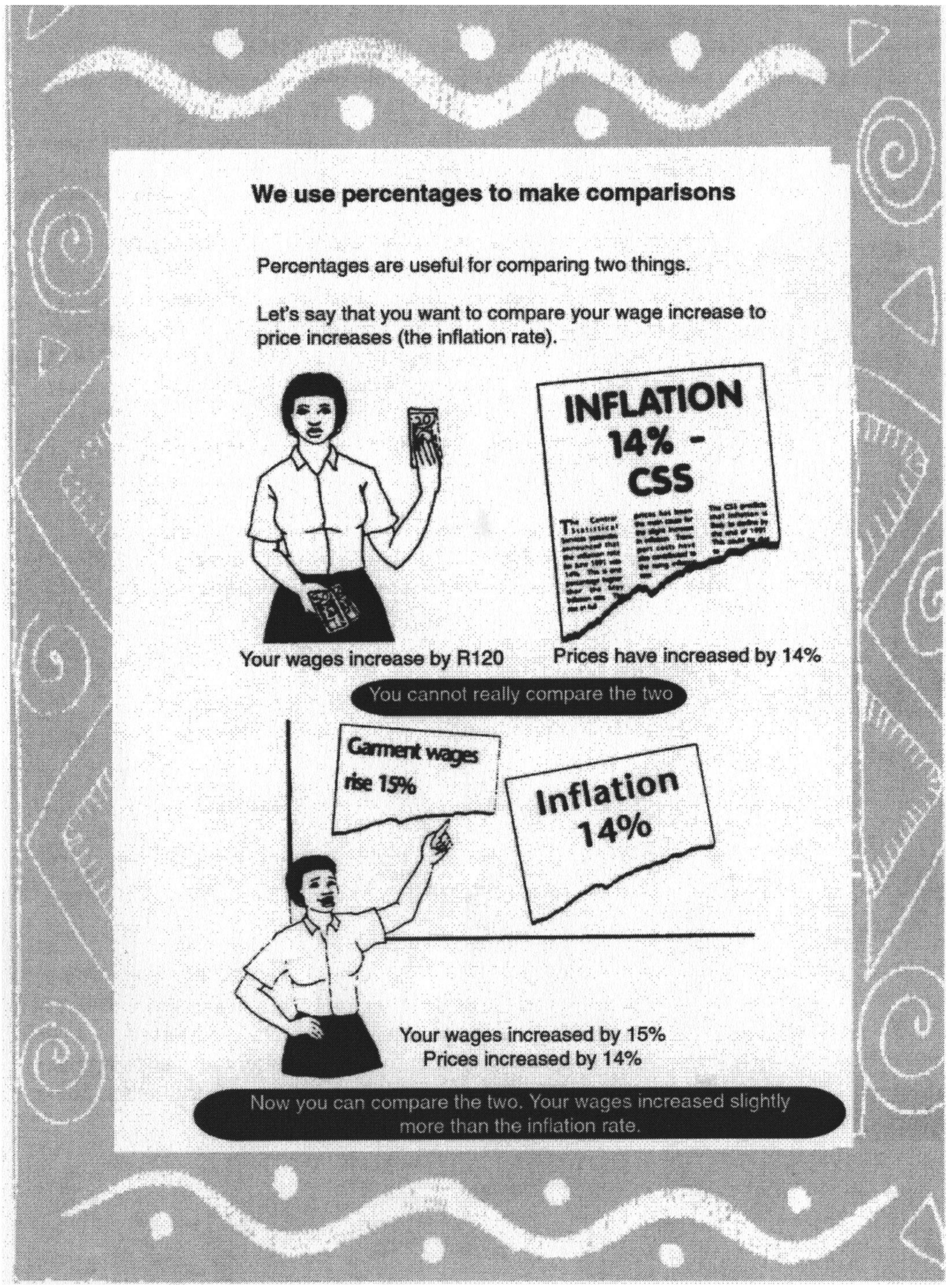
Use the chart to write the sum for each 21.

_______ x _______ = 21

_______ x _______ = 21

Doing percentages

We will now look at doing percentages. Percentages are a kind of fraction and are also very closely linked to decimals. Most learners will have seen the symbol "%". Percentages are used in many different contexts in our everyday lives. Changing amounts to percentages is a very useful way of comparing different amounts, as we can see from the following sketch.



(**Source:** TURP 1992:47)

Percentages

What does a percentage mean?

Percent means part of a hundred.

"Cent" is short for the Latin word "centum" for 100.

It is a kind of fraction where the whole thing is broken down into 100 parts.

The symbol % means percent.

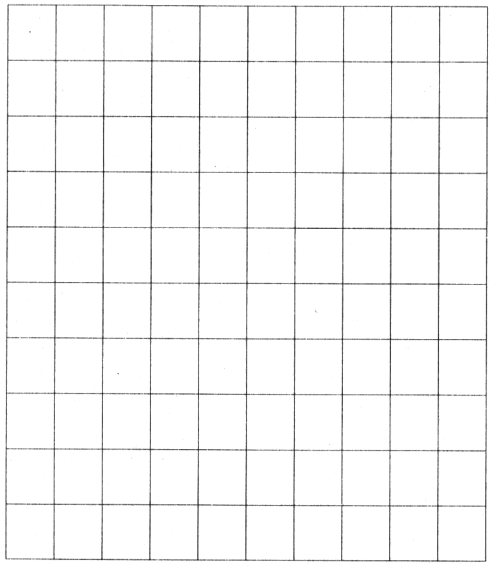
20% = 20 per cent = $\frac{20}{100}$ = 20 parts of 100 parts

75% = 75 per cent = $\frac{75}{100}$ = 75 parts of 100 parts

33.3% = 33.3 per cent = $\frac{33.3}{100}$ = 33 and 3 tenths of 100 parts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

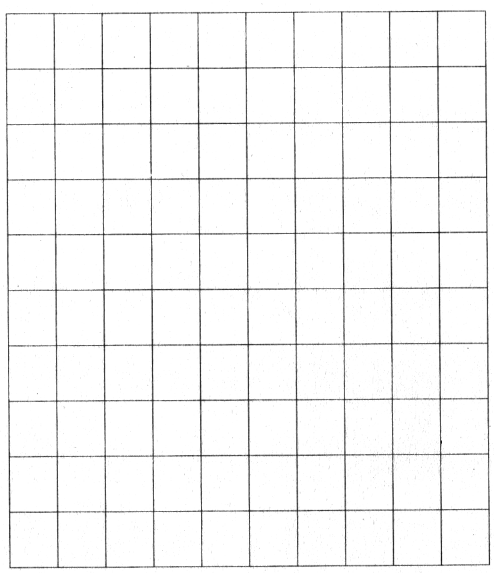
25 parts out of 100 parts are shaded = $\frac{25}{100}$ = 25% is shaded



$\_\_\_\_\_ = \frac{\_\_\_\_\_}{100}$

= _______ hundredths

= _______ %

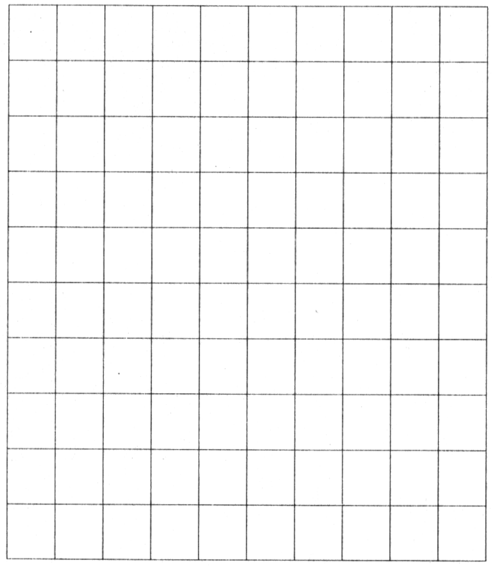


$\_\_\_\_\_ = \frac{\_\_\_\_\_}{100}$

= _______ hundredths

= _______ %

SHADE AS PER GRAPHIC ON PAGE 48 OF TEACHING NUMERACY AT ADULTS



______ = ______
100

= ______ hundredths

= ______ %

ACTIVITY 7

■ ■ ■

Think of all the many situations in which percentages are used.

Make notes of these in a mind map. We have drawn one here and have started by filling in some of our suggestions. Can you list other situations where percentages are used?

■ ■ ■

ACTIVITY 8

■ ■ ■

Let us look at a workplace example where members of a trade union need to have an understanding of percentages because in their wage negotiations with management they come across statements like:

- Management is willing to offer a 5% wage increase.
- The present inflation rate is 10%.
- Company profits rose by 25%.

Calculating percentages is very important for trade unionists. They have to deal with percentages and fractions in negotiations over wages, pensions, and medical aids.

■ ■ ■

Example:

In a factory the following wages have been paid:

Grade A workers R800 per month
Grade B workers R960
Grade C workers R1 050
Grade D workers R1 450
Grade E workers R2 400

In the first round of negotiations management offered:

a 4% increase across all grades
a 1.3% increase in sick pay
a 0.5% increase in maternity benefits
a 0.4% increase in educational benefits
a 6.0% increase in overtime pay

This is how we find a percentage from a number.

Management offers a 7% wage increase. You want to work out what this means in rands and cents for a Grade A worker who earns R800 per month.

$$7\% \text{ of } R800$$

$$= \frac{7}{100} \times \frac{R800}{1}$$

$$= \frac{7}{{}_{1}\cancel{100}} \times \frac{R\cancel{800}^{8}}{1}$$

$$= 7 \times R8$$

$$= R56$$

So 7% of R800 is R51.

CHANGING PERCENTAGES TO DECIMALS

Percentages (which are a special kind of fraction of 100 parts) can be changed to other fractions and to decimals (which are fractions based on ten parts).

How would we change 50% to a decimal form?

Step 1

Write the percentage in fraction form:

$$50\% = \frac{50}{100}$$

Step 2

Reduce this fraction to its simplest form:

$$\frac{50}{100} = \frac{1}{2} = 0.5$$

This means that if, for example, you spend half your wages on food, you can also say that you spend 50% of your wages on food.

Other examples are:

$$25\% = \frac{25}{100} = \frac{1}{4} = 0.5$$

$$33.3\% = \frac{33.3}{100} = \frac{1}{3} = 0.333$$

$$75\% = \frac{75}{100} = \frac{3}{4} = 0.75$$

MEASUREMENTS

When we work with measurement we think about things like working with time, weight, capacity, distance and temperature.

For each of these, there are special units of measurement and special instruments for taking these measurements. However, many learners may use other ways of measuring in familiar situations without using the standard units or the usual measuring instruments. These are important skills which should be recognised and built on when you are teaching measurements more formally.

Standard units and instruments for all measures, other than the units for time, have been developed largely to assist in communication and trade. Many older learners may remember when we changed our units of distance from miles to kilometres, and of weight from pounds to kilograms.

We start with time below.

Time

Adults are very aware of the "passing of time". In fact, time seems to go faster as we get older!

At ABET level 1, adults need to be able to show that they can tell the time using the 12- and 24-hour clocks. From ABET level 2, learners should be able to do calculations to answer questions like: "How long does it take to get from one place to another?"

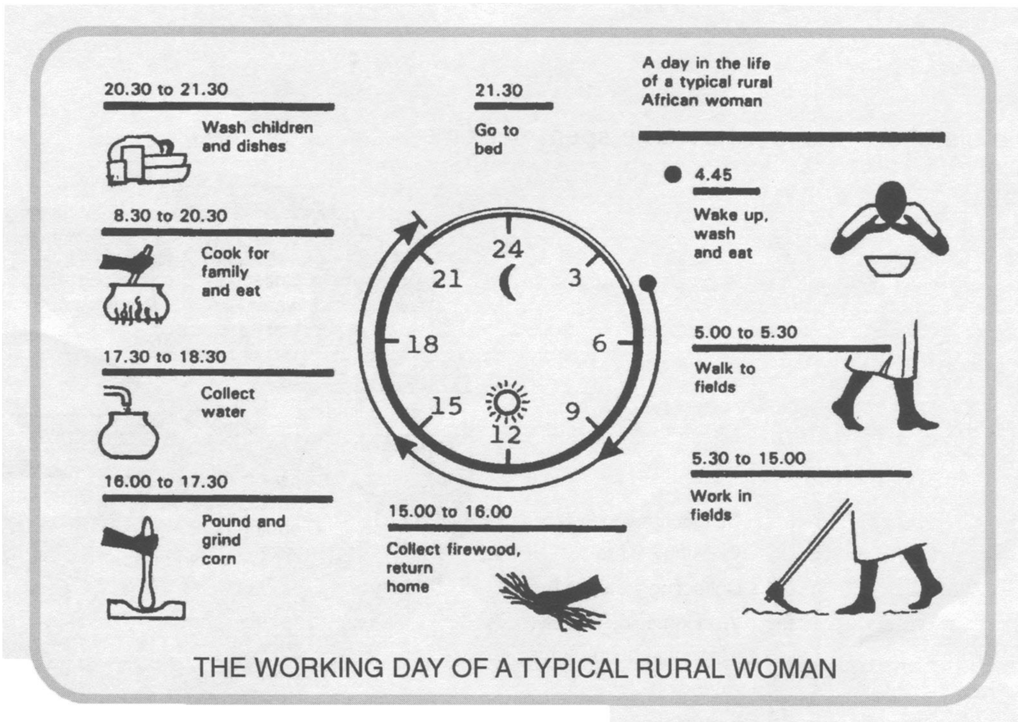
The most common conversions are between hours and minutes, and minutes and seconds and each of these requires working with 60.

Converting hours to days involves working with 24.

Converting months to years involves working with 12.

Let's think about the lives of rural women. The following diagram, which you will remember from module 1, shows that the lack of water and electricity in rural areas has meant that women have had to spend many more hours doing household tasks, like washing and cooking, than women in urban areas – who have running water and electricity.

The following diagram illustrates how Nomsa spends her day:



(**Source:** Morley & Lovel 1988:82)

Last month, as a result of the electrification and water projects in Gladys and Nomsa's villages, electricity is now available for the first time. They have each also been provided with a tap with running water outside their houses. This has resulted in major changes in their lives. For the first time ever each of these women now has some free time to get involved in other activities in the community.

Graphs

ACTIVITY 9

At ABET Level 2, learners should be able to represent quantitative data graphically in ways that adequately show the important features of data sets (limited to bar, line and pie graphs).

Why do you think that a picture graph was used in the sketch to describe Gladys's day rather than a bar graph?

Can you draw another pie graphs to show how Gladys's day looked after the installation of electricity and a water tap at her house?

Now compare the two pie graphs and suggest how much free time Gladys would now have for other activities.

■ ■ ■

Here is table showing the time breakdowns to help you:

	Before	**After**
Washing children and childcare	1	1
Personal care	.25	.25
Walking to fields and back	1	1
Work in fields	8	8
Collecting water	1	–
Collect firewood	1	–
Grinding corn	1.5	1.5
Cooking	1.5	1
Eating	.45	.45
Attending literacy class	1	1
Sleep	7.30	7.30
Totals	**24**	**21.5**

Note that making tables like these is usually the first stage in drawing a graph.

ACTIVITY 10

The days of Gladys and Nomsa have been described using graphs or pictures. Can you use this information to write a short descriptive paragraph about Nomsa's and Gladys's social conditions?

Drawing on some of the ideas that we have presented here regarding picture graphs, what approach would you use when teaching pie graphs to factory workers in an ABET level 3 group?

DISTANCE, WEIGHT AND VOLUME

Learners need to get practical experience in using various measuring instruments like rulers and tape measures to measure distance, scales to measure weight, and measuring cups and jugs to measure volume.

They also need to learn to use the right units for each kind of measurement. For example, litres and millilitres are a measure of volume, kilograms and grams are measures of weight, and kilometres, metres, centimetres and millimetres are measures of distance. As we know, money is divided into different coins representing different amounts of cents, like a ten-cent coin being equal to 10 cents, a one rand coin being equal to 100 cents and a five rand coin being equal to 500 cents.

It is useful to give the learners a great deal of practise in converting one unit into another, like the following examples:

(1) 250 ml = ___ litres
(2) 750 ml = ___ litres
(3) 2 kms = ___ metres
(4) 500 g = ___ kilograms

For example here are a set of measurements:

943 ml, 200 ml, 122 ml, 451 ml, 1 251 ml, 505 ml, 888 ml, 750 ml

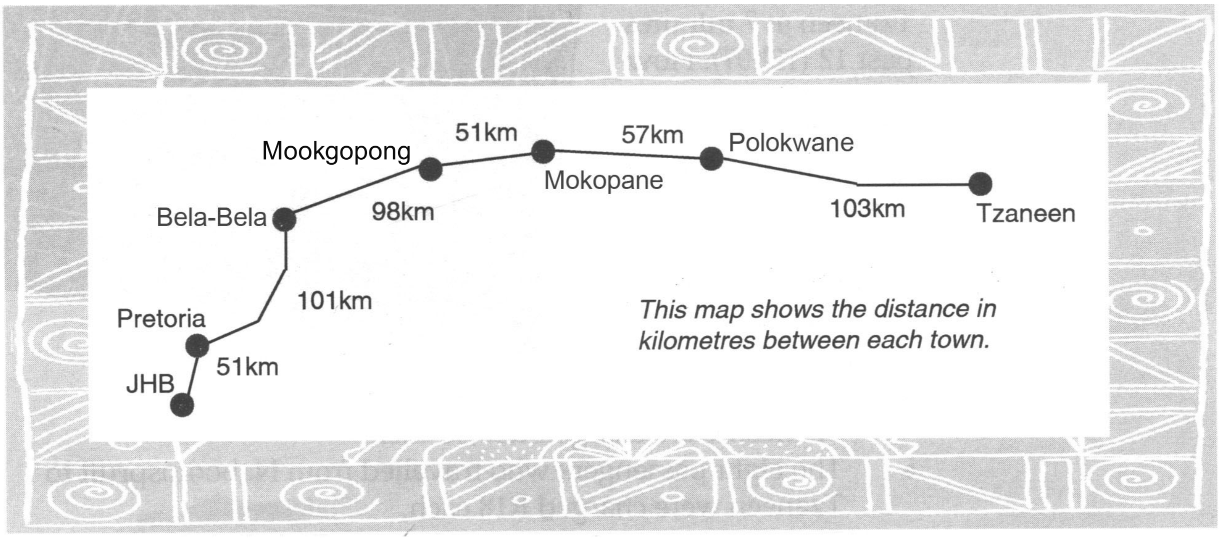
Place them under the appropriate headings:

Less than a quarter of a litre	**Between a quarter and a half a litre**	**More than half a litre**	**More than one litre**

MAPS

Let us look at the map below.

A taxi takes passengers from Johannesburg to Tzaneen. Here is a map of the route it travels:



The map shows the distance in kilometres between each town. The questions below show how we can use an example from people's everyday lives, like using maps, to ask questions which would require the learners to practise the skills of adding, subtracting, dividing and multiplying. Try and answer the questions yourself to see what steps you go through to get the answers.

(1) How far is it from Johannesburg to Mookgopong (Naboomspruit)? (Answer in kilometres.)

(2) The taxi left Johannesburg at 07h00 (in the morning) and arrived at Mookgopong at 23 minutes past 9 (09h23). How long did it take the taxi to travel from Johannesburg to Mookgopong?

(3) There were eleven passengers from Johannesburg. Six got out at Bela-bela (Warmbaths) and four new passengers got in. Five passengers got out at Mookgopong and eight new passengers got in. How many passengers were there in the taxi when it left Mookgopong?

(4) The driver charged the passengers as follonws: Each passenger that got out at Bela-bela was charged R20.00. Each of the five passengers from Johannesburg that got out at Mookgopong was charged R34.00. How much money did the driver collect from those passengers who got out at Bela-bela and Mookgopong?

(5) The taxi left Mookgopong at ten to ten (09h50), and arrived at Polokwane (Pietersburg) at five to eleven (10h55). How much time did it take to get from Mookgopong to Mokopane?

(6) How far is it from Mookgopong to Mokopane? (Use the map.)

(7) The taxi arrived at Tzaneen at 7 minutes past 12 (12h07). How much time did the whole journey take from Johannesburg to Tzaneen?

(8) At Tzaneen the driver charged the passengers as follows:

- The two passengers who travelled from Bela-bela to Tzaneen were charged R40.00 each.
- The eight passengers who travelled from Mookgopong to Tzaneen were charged R30 each.
- There were two passengers who travelled all the way from Johannesburg to Tzaneen. They were charged R60.00 each.
- How much money did the driver collect in total from all these people?

(9) How far is it from Johannesburg to Tzaneen? (Use the map.)

(10) How much money did the driver collect from all the people who used his taxi?

(11) Do your calculations suggest that the driver works out the cost of the fares in terms of the distance travelled? Explain what you mean

These questions are suitable for advanced ABET level 1 learners. However, you can use the same kind of approach for designing questions for ABET level 2 learners. How would you do this?

Usually it helps the learners to construct some kind of table to show how many travellers covered which section of the route:

	1	2	3	4	5	6	7	8	9	10	11	12
Route												
Johannesburg to Bela-bela												
Bela-bela to Mookgopong												
Mookgopong to Mokopane												
Mokopane to Polokwane												
Polokwane to Tzaneen												

We think that we would do this by using part of a local map of a community or town, and then by using more questions and bigger numbers. What ideas do you have?

The above exercises are about taking a journey to various towns. It is useful because it helps to consolidate the operations whilst working with distance, time, numbers of passengers and costs of travelling.

SHAPES

Study the case study below:

Case study

Nelson and Thembisa are both rural development workers in the Herschell District. They were chosen by their communities to attend a training course which will help them to upgrade education and other resources in their communities. The training course expects them to set up education and development projects as part of their "homework" or community tasks. They have both chosen to do a project on food gardens. This involves teaching members of their community to grow food for their families and even possibly to sell some of this food to others. They have decided to work together on this project.

The members of Zwalaza community have shown a lot of interest and have formed an association – the Zwalaza Food Gardens Association. Through this association, Nelson and Thembisa have persuaded the chief to allocate 800 square metres of community land for the project. They are now trying to decide how to divide up this land into pieces for each family who is interested.

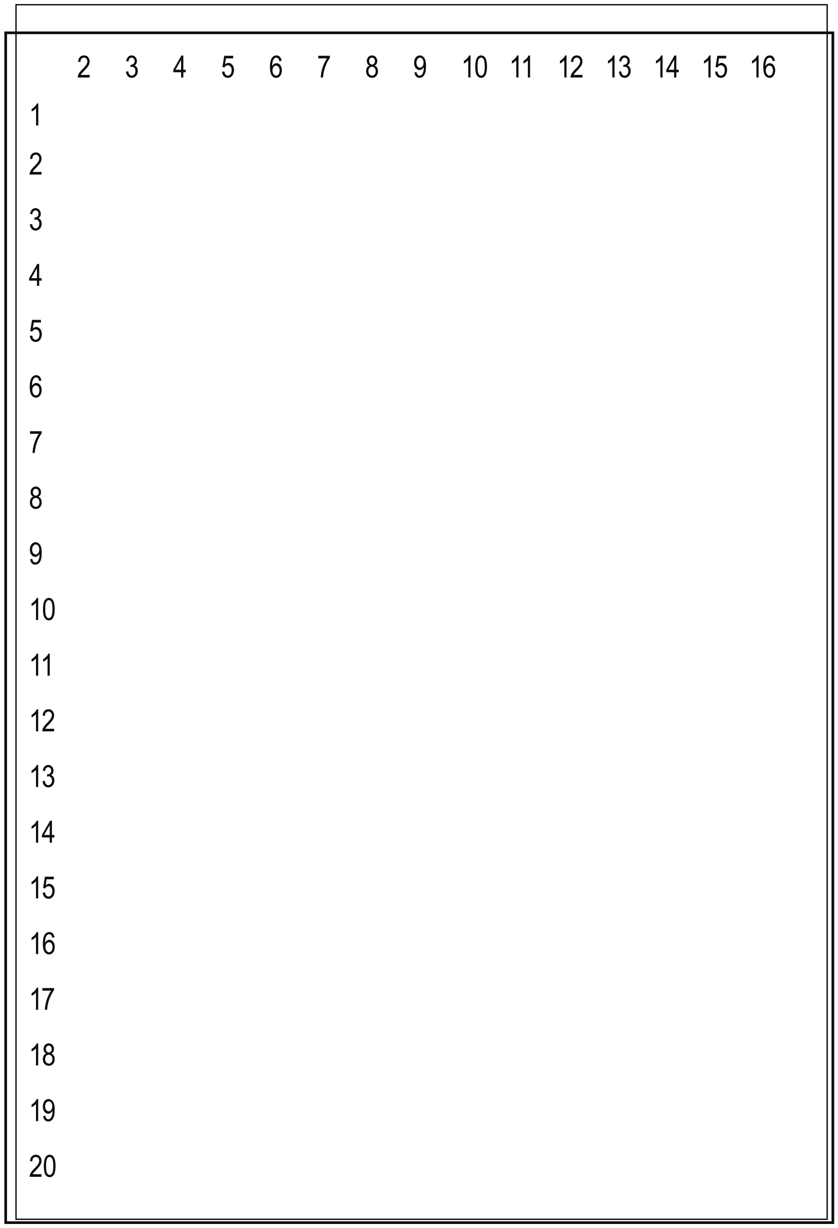
Nelson says that he has spoken to the Sterkspruit Agricultural Group, and they have promised poles and wire for fencing the food gardens.

They are now trying to decide what shape to make each garden. Nelson says that, to get the greatest area (ground space) for the same amount of fencing, you should use a circle shape. He says that this is the reason why the traditional kraals and huts are circular. Thembisa says that, while this might be true, a circle shape would be very impractical for their purposes as the shapes will not fit together and space will be wasted between the circles. She says rectangular or square plots are the most practical. What do you think?

In this exercise we will examine how we can go about testing different solutions. We have decided to check the various options by testing them practically and then evaluating them. We therefore hope that you will work through the practical exercises as you read them.

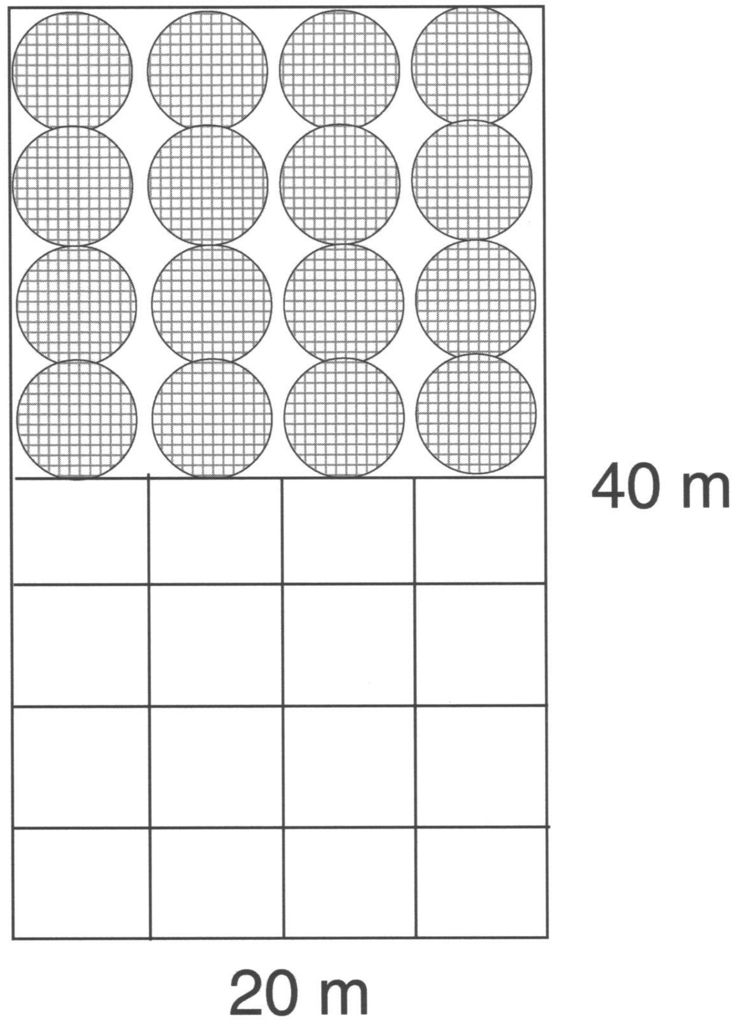
We have a set of square boxes below, representing an area. You will try out various shapes on this to see which shape covers the most area and at the same time fits in with other similar shapes.

Measure a piece of string along 15 of the boxes below. Now, with the string, create various shapes – triangles, circles, squares, rectangles, and so on to see which is the most practical shape.



You can see that Nelson is right in saying that the circle will give the biggest amount of ground. However, we think that Thembisa is also right when she says that a circle shape is impractical. “It won’t work for us,” she says. Why is she right?

We have drawn a plan of the 800 square metres allocated to the food gardening project for you. Divide half of the squares into circular plots (as was Nelson’s suggestion), and the other half of the squares into rectangular or square plots (as was Thembisa’s suggestion).



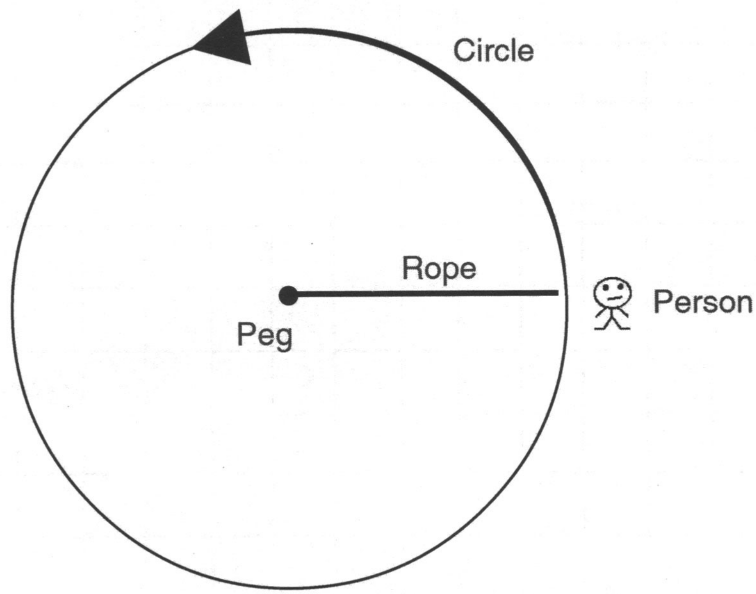
Do you think that using circles for the vegetable garden would work? What have we learnt about rectangles and squares?

We think that the practical exercises have shown us that circles shapes will not work because they waste space in between the circles. The practical exercises have also shown us that rectangles and squares can fit together. This quality of shapes fitting together is called tessellation, that is, we say that rectangles tessellate but circles do not tessellate. Can you think of other shapes that tessellate? Draw them on the squared paper on the following page.

We have seen that the circle gives the greatest area for the least amount of perimeter. This means that for the same amount of wax you could store more honey in a circular shape than a rectangle, or even a hexagon. BUT, since circles do not fit together, this would not help the bees. On the other hand, while rectangles, squares and triangles do tessellate, they do not keep enough honey inside them. So the bees chose the perfect compromise: a hexagon shape will both tessellate and keep more honey because of its more circular shape. *Check this with your piece of string on the square paper on the previous page.*

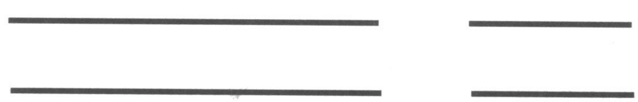
- What shape do you think would be best for dividing up the land for the vegetable gardens?
- Can you remember the first exercise you did with the piece of string? Which shape gave the greatest area?

- Let's see how to draw a perfect circle and a perfect rectangle or square for the food garden. Here are two methods that have been used in the rural areas for many, many years.



This way of drawing a circle uses the knowledge that a circle always has a constant radius (the rope in this case), that is, the distance from the centre of the circle (the peg) to the edge must always be the same.

To draw a perfect rectangle, you need:



two sticks of equal length, another two shorter ones also of equal length, and two pieces of rope.

Put these down to form your rectangle:

Place the rope so that it is touching the opposite corners. Now, to make this rectangle accurate, adjust or move the sticks until the diagonals (the ropes), are the same length. When these are the same length you have a perfect rectangle.

This method is often used for drawing the floor lines of rectangular houses.

Nelson and Thembisa have now registered over 250 people for the food garden project. Each of them is to be given a piece of land to grow their food on. They also want to keep at least another 50 plots of land for future use.

Show how you would divide up the land if you were them. Use a sheet of squared paper. Think about the reasons why you would do it this way.

ACTIVITY 11

■ ■ ■

What can we learn from the above study skills exercises about the kinds of exercise we can organise to help learners develop problem-solving skills using information from numeracy?

Here are some of the lessons we learnt:

Giving learners practical exercises to do themselves can really help facilitate learning.

- Using problems or the experiences of the learners as the basis of your lessons helps to make the lesson more meaningful. It also gives learners practise in using the numeracy skills they learn in class by trying to solve real problems in their lives.

(**Source:** Rural Education Facilitators Project, CAE, Wits)

- We developed a better understanding of different kinds of shapes.
- We have included some examples on teaching three-dimensional shapes which come from an Intermediate Mathematics Course produced by Project Literacy, to give you ideas on how to teach three-dimensional shapes.

■ ■ ■

ABET 3

3-DIMENSIONAL SHAPES

Case study

Sipho has been given an assignment to do. His facilitator has instructed him to look at the difference between two-dimensional and three-dimensional shapes. Because Sipho has a rural background he has found it very difficult to do this assignment and is very upset about it. He sits under a tree and looks at the other trees around him. He notices that each tree has its own shadow. But something bothers him about the shadows because they do not reflect the exact size and shape of the trees. He decides to discuss this with his facilitator.

(a) Do you think that his example has any bearing on two- and three-dimensional shapes? Why?

Based on your drawing, which of the shapes are three-dimensional and two-dimensional (shade or tree)?

Differences between 2-D and 3-D shapes

After Sipho's discussion with his facilitator she gives the class the following guidelines to differentiate between two- and three-dimensional shapes:

(a) 3-D means three dimensions or measurements, for example

This rectangular box has length, width or breadth, and height. A swimming pool has length, width and depth. It is also 3-D.

2-D means two dimensions or measurements:

breadth

length

This rectangle has length and width or breadth. It is flat.

Try to point out other two- and three-dimensional shapes in your classroom.

How many blocks are there in each shape?

Working with statistics

If we study the standards for ABET levels 1 to 4 in numeracy, we will see that learners need to be able to work with graphs at all levels. Even though we are surrounded by statistics, graphs and tables all the time – statistics on the radio, statistics, graphs and tables on the television and in the newspapers, many people do not read them, or they stop listening when someone on the radio starts talking about statistics. However, if people are able to read and understand tables and graphs it is easier for them to form their own opinions about problems. They will also be able to see if the graphs represent an accurate picture of a problem or situation, or whether they have been drawn to put across a particular point of view, which might not even be accurate. Being able to use statistics to support arguments can be very useful.

STUDY SKILLS **Statistics**

Before you read the rest of this section, we suggest that you read pages 88 - 92 in the *Good Study Guide*. There are some very useful exercises on how we can use statistics to make sense of information. Try to work through the exercise which has been set in the *Good Study Guide*, because it will give you an opportunity to practise the steps involved in reading and analysing tables.

We have included a story on how to teach graphs, which comes from an organisation called TELL (an NGO which provides literacy and numeracy training to ABET educators working with learners up to ABET level 3). Most of the educators come from communities where unemployment is high, and most of them have attained a level of Standard 8 (Grade 10) in the formal school system.

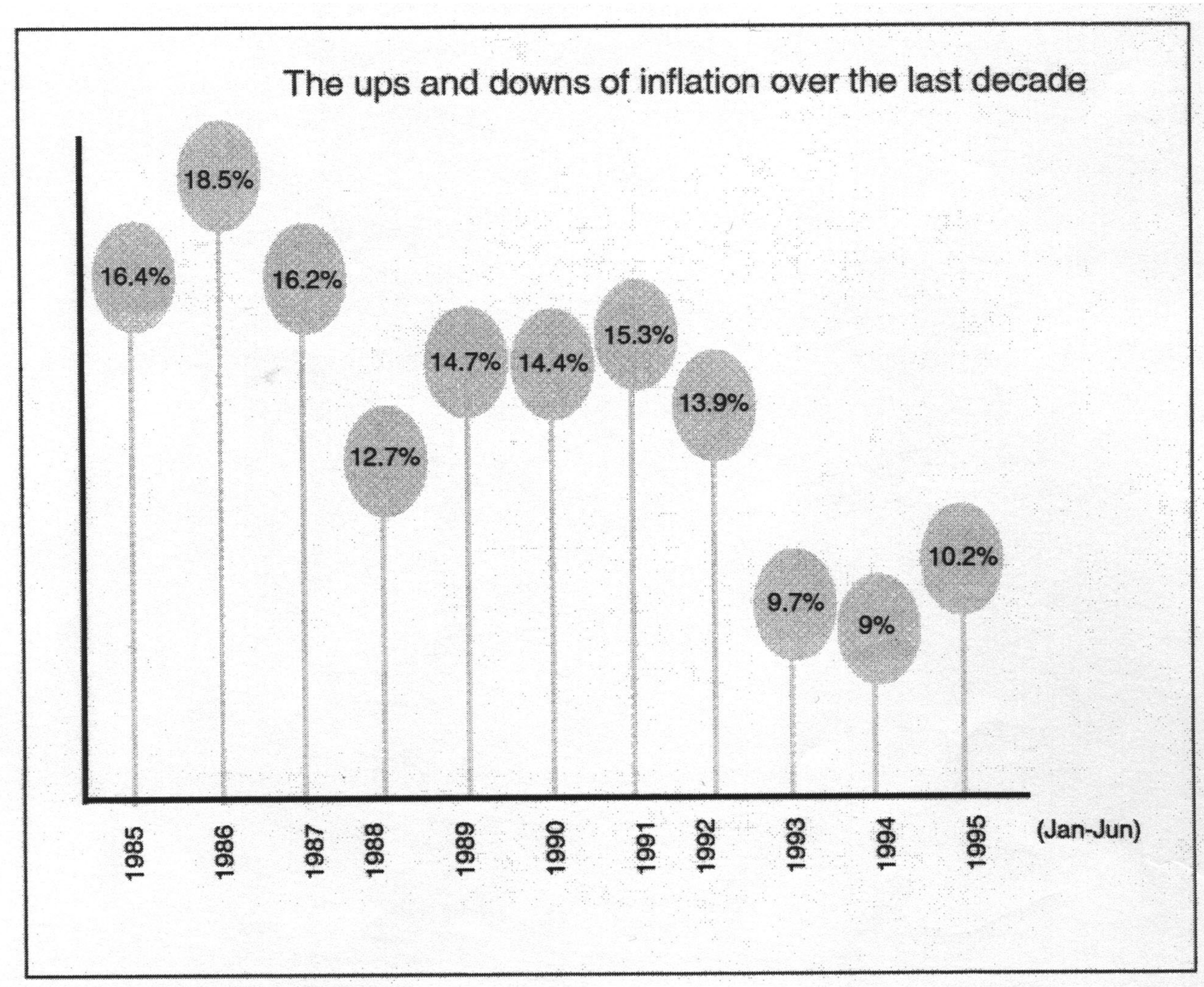
In the tutor training course they discuss the purposes of using graphs. Tutors are shown a range of graphs and these are then discussed. Certain key items are highlighted, such as the purpose of using graphs, different types of graph and how they are used, as well as the critical issue of who collects data and for what purpose.

When planning to teach their learners how to draw bar graphs, the TELL tutors felt that their learners, and they themselves, would find that bar graphs can be useful for organising one's life. One learner group, which looked at learner attendance at class, said that they were able to give reasons for low attendance in July, high attendance in February, and a lack of attendance by certain group members. This whole exercise resulted in a greater awareness of the importance of attending class regularly. It also resulted in the scolding of some learners by other group members!

The TELL educators spent time discussing what numeracy skills learners would need to have before they would be able to understand graphs. They came up with the following list:

- reading numbers
- counting
- writing numbers
- measuring to construct the graph
- knowing how to arrange the categories on the horizontal and verticle axes
- be comfortable about reading in different directions, that is, backwards, upwards and towards the left-hand side
- why graphs can be useful

We have also included another kind of graphic presentation of information. Study the list of skills suggested by the TELL tutors, look at the pie graph in the sketch, and then draw a bar graph of the information on the pie graph.

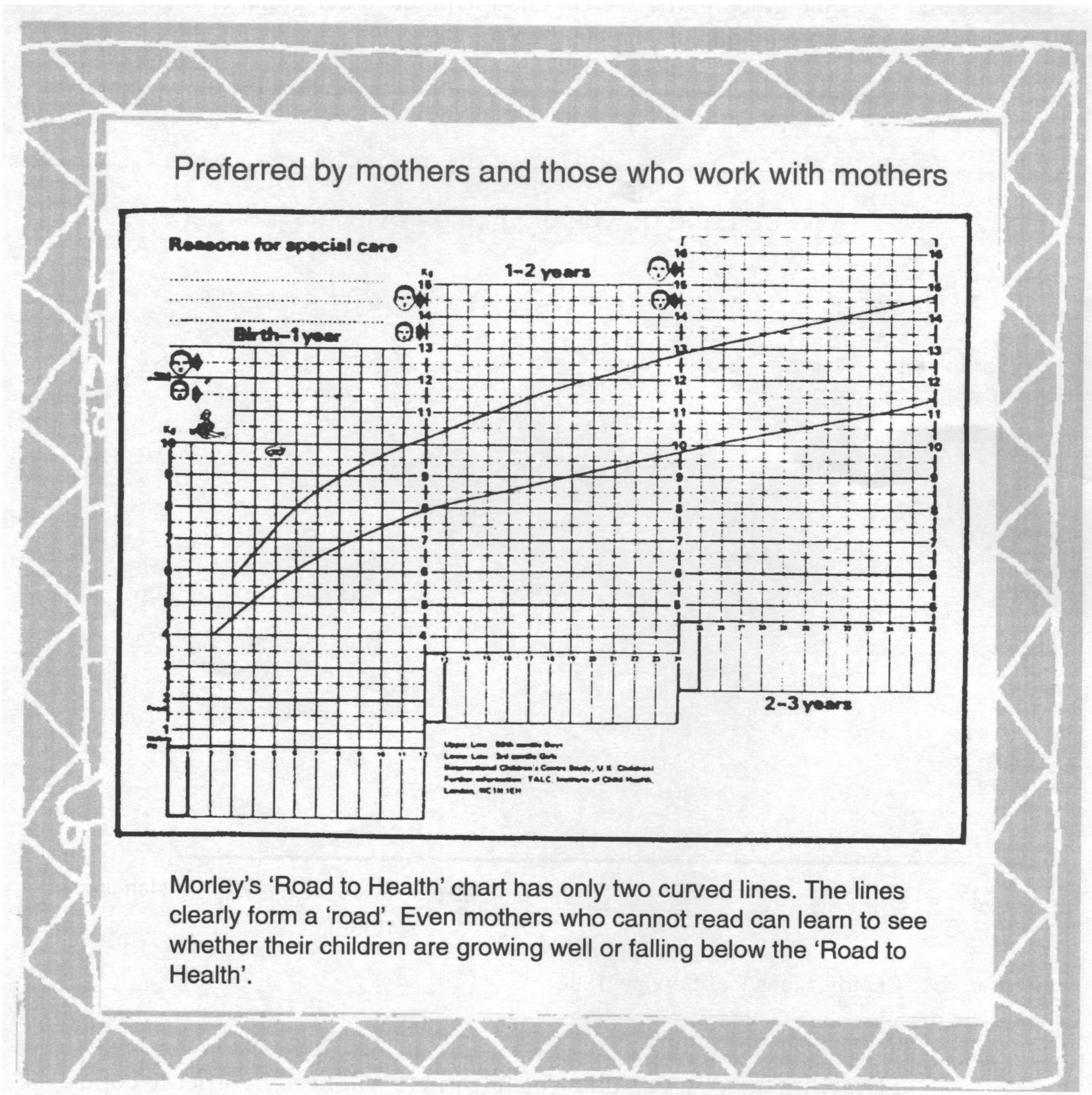


(**Source**: Central Statistical Service 1996)

Drawing up a list of skills that learners will need in order to cope with a particular task like reading graphs is useful for a number of reasons. You can use the list from TELL as a checklist before introducing graphs to your learners. Or you can use the list to work out what new skills you will need to teach the learners before giving them graphs to do. This kind of list can also help you try to work out possible reasons why your learners may be having problems with graphs. Perhaps they might not have adequately mastered one of the skills needed and you may need to go back and practise this skill more. Can you think of other reasons?

We have included another story about using graphs, because it gives us some ideas about how to help adult learners understand graphs.

On the following pages are two graphs used to collect and show the same information of how a baby is growing over a period of time. It is called a *baby's growth chart*. Many of you who are parents will be familiar with the second one from your visits to the clinic.



Morley's 'Road to Health' chart has only two curved lines. The lines clearly form a 'road'. Even mothers who cannot read can learn to see whether their children are growing well or falling below the 'Road to Health'.

Let's look at what is going on here.

On the horizontal axis is the baby's age, and on the vertical axis is the baby's weight. Each point can be read down or across. If you go down from the point you can read the age. If you go across from the point, you can read the baby's weight. These points can be joined with lines so that the curve gives a picture which represents the baby's growth.

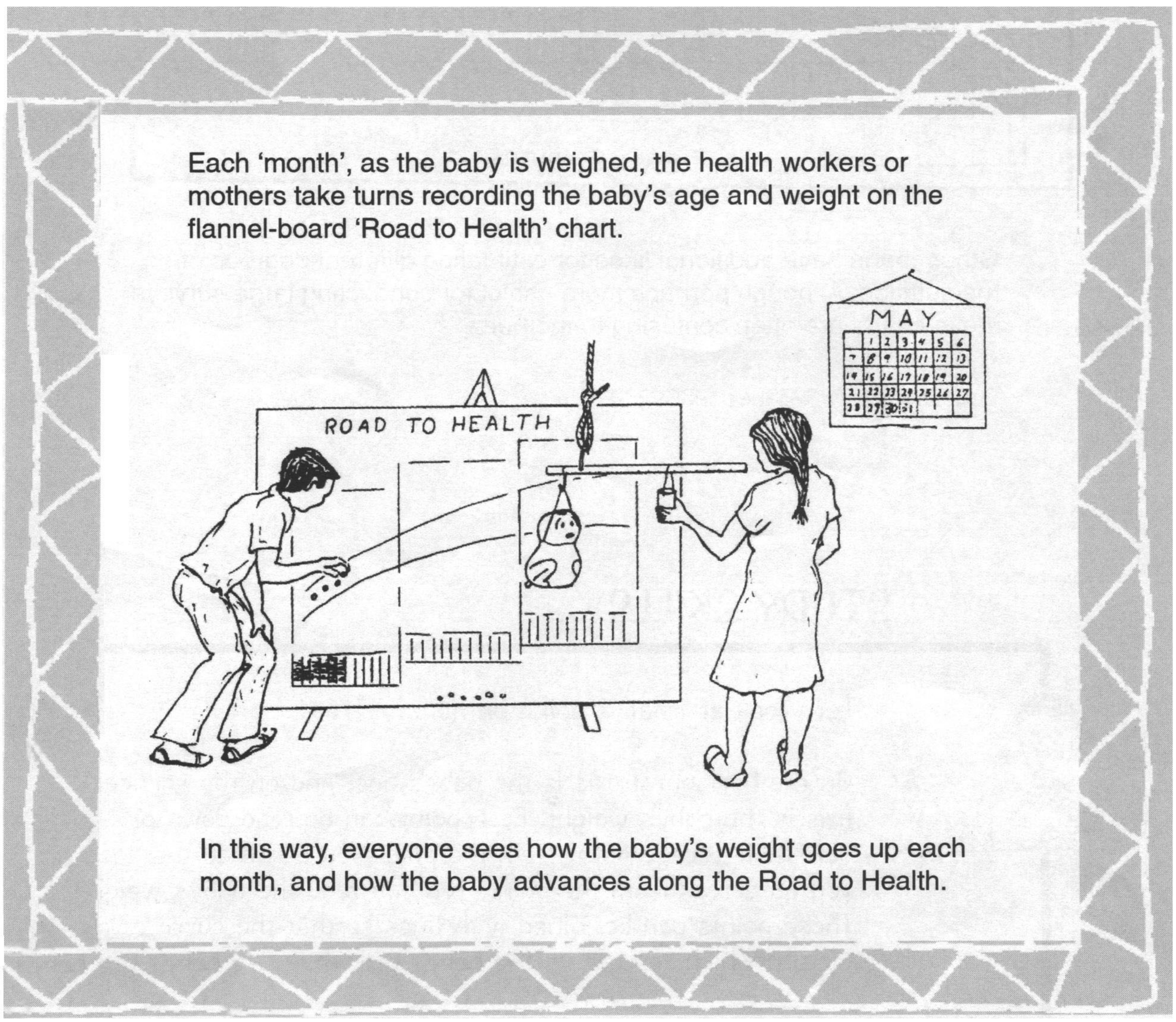
If the baby is growing well, which way do you think the curve should be going? And if the curve is "flat", do you think that the baby is healthy? Do you think that social and economic conditions will affect the baby's growth?

As you know, all babies of the same age do not weigh the same, but that does not mean that they are unhealthy.

From the graph, can you find the weight range for a 9-month old baby?

This book, called *Helping health workers learn,* also describes a method to make it easier for the mothers to understand what this complicated diagram is about. It shows how the mothers are actively involved in gathering the data (information) about their baby's growth, and how to put it on the chart.

Each "month", as the baby is weighed, the health workers or mothers take turns recording the baby's age and weight on the flannel-board "Road to Health" chart.



ACTIVITY 12

■ ■ ■ Study the case study on reading growth charts again. Then draw up a short lesson plan for health workers who need to assist mothers to understand the growth chart of their babies. Think carefully about how you would introduce the lesson.

■ ■ ■

3 CONCLUSION

Well, you have reached the end of this study guide. How did you find it? We hope that you have found this study guide useful and that you feel you are now in a better position to help learners develop an understanding of mathematics. We also hope that you have got some ideas of different approaches for teaching mathematics and dealing with common problems that learners may have in mathematics.

4 LIST OF SOURCES

Much material was drawn from the following programmes. In some cases, programmes have directly contributed by submitting examples of their work:

(1) The Independent Examinations Board (IEB).
(2) English Literacy Project (ELP).
(3) Project Literacy (PROLIT).
(4) Teaching English Language and Literacy (TELL).

5 REFERENCES

"Adults learning maths – a research forum", ALM-1, Proceedings of the Inaugural Conference, Fircroft College, Birmingham UK, July 1994.

ALBSU. 1991. Crediting numeracy, foundation stages 1 & 2.

Ambrosio, UD. 1985. Mathematics in a cultural setting. *Science & Technology* 16(4).

Baker, D, Semple, C & Stead, T. 1990. How big is the moon? Oxford University Press.

CAE, Wits. Rural education facilitators' project. Johannesburg: University of the Witwatersrand.

TURP (Trade Union Research Project). 1992. *Calculations for wage negotiations: a workbook*. Durban: University of Natal.

Frankenstein. M. 1987. *Relearning mathematics: a different third R – radical numeracy.* London: Free Association Press.

Gerdes, P (ed). 1994. *Explorations in ethnomathematics & ethnoscience in Mozambique*. Instituto Superior Pedagogico.

Goddard, R, Marr, B & Martin J. 1991. *Strength in numbers*. Eastern Metropolitan Council of Further Education, Division of Further Education, Min of Education and Training.

Harris, M (ed). 1991. School mathematics & work. London: Falmer.

Irons, C & Burnett, J. 1994. *Mathematics from many cultures*. Kingscourt.

Mathematics Education Project (MEP). 1995. Percentages and Area 2 Perimeter 2. University of Cape Town.

Morley, D & Lovel, H. 1988. *My name is today.* London: Macmillan.

Nunes, T, Schliemann, AD & Carrahi, DW. 1993. *Street mathematics and school mathematics.* Cambridge University Press.

Soares, D & Ismail, A. 1994. Explorations in ethnonumeracy and ethnoscience in Mozambique.

Tout, D & Johnston, B. (In press.) *Adult numeracy teaching: making meaning in maths*. Victoria.

TURP (Trade Union Research Project). 1992.

Werner, D & Bower, B. 1986. *Helping health workers learn*. Hesperian Foundation.

6 FURTHER READING

Abadzi, H. 2006. The development and teaching of numeracy. In Abadzi, H. 2006. *Efficient learning for the poor: insights from the frontier of cognitive neuroscience*. Washington, DC: The World Bank, pp 58–61.

A state of the art summary of current ideas and research on numeracy teaching. Although the book mainly addresses school education it contains valuable insights for all adult educators.

Unit 5

Teaching problem solving and error avoiding in beginner mathematics

1 INTRODUCTION

In practice, when we teach learners, we follow a systematic sequence of learning activities which focus on certain skills. In this unit the focus falls on problem solving skills.

2 AIMS OF THE UNIT

This units aims to take you through some of the processes and exercises that would serve to provide you with problem solving skills

LEARNING OUTCOMES

By the end of this unit you should be able to explain and use the rationale for problem-solving tasks in numeracy teaching and identify typical misconceptions of mathematical concepts and causes, and apply prevention strategies to avoid them.

You will demonstrate this by being able to

- explain the role of problem solving in numeracy learning
- explain the role played by problems in extending and developing number concept
- explain the role played by problems in extending and developing calculation methods
- describe the different problem types that involve the addition, subtraction, division and multiplication of whole numbers and fractions
- identify typical misconceptions of whole number and fraction arithmetic, spatial concepts and probability concepts
- evaluate numeracy materials and learning programmes in terms of the range and mix of problem types
- analyse problem-solving tasks in course materials in terms of purpose and likely challenges/difficulties for learners
- relate mathematical misconceptions to their causes
- describe, justify and apply strategies to remedy and prevent misconceptions

WHAT IS PROBLEM SOLVING IN MATHEMATICAL LEARNING?

ACTIVITY 1

■ ■ ■

Study the six statements below. Each statement tells us something about a member of the Nkosi family. Can you work out how old each member of the family is from the information which we have given you? The members of the family are Brian, Sibongile, Dyameko, Thabo and Joe.

(1) Anja is three years older than Brian.
(2) Sibongile was 27 when her son Brian was born.
(3) Dyameko, Sibongile's partner, is four times older than Anja.
(4) Yesterday they had a party for Brian's sixth birthday.
(5) Sibongile's third child, Thabo, is half Brian's age.
(6) Joe, Sibongile's brother, is twice as old as her daughter Anja.

This activity was taken with modifications from *Strength in numbers* (Goddard et al, 1999)

■ ■ ■

Our response

We have given our answers below and explained how we got them.

You will see that we first went through the list of statements looking for a definite piece of information.

We found one statement (no. 4) which gives us the age of one person – Brian is six. This gave us a definite number to start with.

All the ages of the other people are given in relation to someone else's age.

So we then went about answering the questions by starting with what was known (Brian's age) and relating one age to another.

By following a chain of reasoning, we could eventually work out the age of each member of the family.

(a) Brian is six years old (we know this from item 4).
(b) Thabo is three years old, because he is half Brian's age (item 5).
(c) Anja is nine years old, because she is three years older than Brian (item 1).
(d) Dyameko is 36 years old because he is four times older than Anja (item 3).
(e) Sibongile is 33 years old, because Brian is six years old and she was 27 when he was born (item 2).
(f) Joe is 18 years old, because he is twice as old as Anya (item 6).

Note that to solve this problem you need to be able to do three things:

- simple **calculations** involving addition (e.), subtraction (b.), and multiplication (c., d., f.)
- making **comparisons** and thus being able to working out the age of each member of the family *in relation to* the age of another member of the family
- **thinking logically** step by step

This example shows us how people can use mathematical principles or techniques to work out everyday problems, and it is by solving everyday problems that we organise our world.

Often learners who have some familiarity with mathematical procedures may not recognise that a problem phrased in words is in fact solvable using number operations. I certainly remember when I was in primary school the horror we often had at what were called "word sums". That is why becoming truly numerate requires a degree of logical thinking that can look at a problem and recognise if number work can help solve the problem.

It is helpful to take learners through exercises where they examine problems and see whether there are ways they can or cannot be solved with number help.

It is also helpful for the examples of problems to be real ones that can be solved (are done so daily) by using numbers and calculations.

USING PROBLEM SOLVING TO EXTEND CONCEPTS AND CALCULATIONS IN MATHEMATICS

In looking at problem solving as a way of helping learners to build up and strengthen their knowledge of mathematics, it is important to remember that the reason that mathematics is so important is that it enables us to solve practical everyday problems – problems that can only be solved by using numbers.

PROBLEMS INVOLVING ADDITION, SUBTRACTION, DIVISION AND MULTIPLICATION OF WHOLE NUMBERS AND FRACTIONS

A numeracy educator will need to build up a set of resources of exercises and examples of numerical problems that can be used with learners. Even where you have a pre-existing course or programme with its own materials you will generally find that you need extra sets for learners who want or need to practise more.

Even your own experiences in daily life will provide many examples that you can adapt for learner use.

However, given that most problems of this type are practical applications of mathematical knowledge, it is pointless to give learners real problems to work on if the fundamental conceptual understanding of numerical operations has not been built. One should note, however, that it is precisely through such problems challenging the learners that you can identify whether the learners have learnt the fundamental concepts. So the practical problems must test all the basic addition, subtraction, division and multiplication operations, including knowledge of fractions, percentages and decimals.

EVALUATING NUMERACY MATERIALS AND COURSES IN RELATION TO THE RANGE AND MIX OF PROBLEM TYPES IN TERMS OF PURPOSE AND LIKELY CHALLENGES AND DIFFICULTIES FOR LEARNERS

As an educator you may have to choose what courses and materials to use or advise the organisation you work for on the choice of course materials. The same principles apply – does the course or material cover the foundations of numeracy, good examples that will help learners develop conceptual understanding and practical numerical problem-solving skills?

Because many learners **have a negative attitude to maths, either** through the bad influence of others who have had difficulty with mathematics or from their own bad experiences in primary education, the more attractive and interesting the materials are the better.

TYPICAL MISCONCEPTIONS IN ABET LEVEL MATHEMATICS, THEIR CAUSES AND REMEDIES

There is a range of typical misconceptions and errors that learners have and make in relation to whole number and fraction mathematics, and about spatial concepts and probability concepts. There are number of texts that deal with such predictable problems (although such works are usually about mathematics teaching in primary school with young children).

In your own practice it is helpful to keep records of errors the learners make so that you can then develop exercises and further teaching content to remedy such errors.

In looking at errors it is important to distinguish between errors based on conceptual misunderstanding and errors based on calculations. The former are really a problem because one has to go back to the start and rebuild a sounder conceptual understanding. The latter calculation errors may simply be a matter of carelessness.

It is in this corrective work that your true skills as a teacher of numeracy to adults will be shown.

3 FURTHER READING

Goddard, R, Marr, B & Martin, J. 1996. *Strength in numbers: a resource book for teaching adult numeracy*. Melbourne: ARIS Language Australia.

Remember that there is a wide range of useful material freely downloadable from the internet.