Tutorial Letter 101/3/2015

NUMERICAL METHODS 1

COS2633

Semesters 1 & 2

Department of Mathematical Sciences

This tutorial letter contains important information about your module

BAR CODE

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1 INTRODUCTION AND WELCOME

Dear Student,
Welcome to *Numerical Methods 1* (COS2633) which is available as part of a major in Computer Science *and* Applied Mathematics. We hope that you will find it interesting and exciting. We certainly do!
This letter (COS2633/101/3/2015) contains important information that you are going to need during the year. Please read it carefully. Another publication that can be of real help is the brochure entitled *My Studies @ Unisa*, which you received with your tutorial matter. It contains information about computer laboratories, the library, *my*Unisa, assistance with study skills, etc.

1.1 Tutorial Matter

You will receive a number of tutorial letters during the semester. A tutorial letter is our way of communicating with you about teaching, learning and assessment. Some of this tutorial matter may not be available when you register. Tutorial matter that is not available when you register will be posted to you as soon as possible, but is also available on *my*Unisa.

Tutorial Letter 101 contains important information about the scheme of work, resources and assignments for this module. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignments, preparing for the examination and addressing questions to your lecturers.

In Tutorial Letter 101, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed textbook and how to obtain this textbook. Please study this information carefully and make sure that you obtain the prescribed textbook as soon as possible.

Please make sure that you read *all* the tutorial matter and follow the correct procedures for submitting assignments. Also note that all tutorial matter will be downloadable from the Internet. Keep an eye on *my*Unisa and if, for whatever reason, you do not receive a printed copy of a tutorial letter in time, download it from either of the two sites, *my*Unisa and *osprey*.

If you have internet access, please visit our departmental website for information on the Department of Mathematical Sciences. To reach our website, follow the links on the main Unisa website, **http://www.unisa.ac.za**

2 PURPOSE AND OUTCOMES FOR THE MODULE

2.1 Purpose

This module is available as part of a major in Computer Science *and* Applied Mathematics.The *abbreviated syllabus* comprises the numerical solution of nonlinear equations and systems of equations, the construction and use of interpolating polynomials, least square approximation, numerical integration and differentiation.

In this module you will learn how to develop and use numerical methods to solve mathematical problems by means of a computer. While the emphasis is on the more practical aspects, a good mathematical background is essential. We therefore advise you to include second year mathematics, in particular MAT2611 and MAT2613, in your curriculum.

The module that follows Numerical Methods 1 is, of course, Numerical Methods 2 (APM3711) which is also available as a subject in Computer Science *and* Applied Mathematics. Although numerical methods are not dependent on any specific programming language, many software packages are available as an aid to the study of numerical methods. *You are therefore expected to learn one or two programming languages (like Matlab, python, C++ or Maple) on your own and to be able to write relatively simple programs in the language.*

2.2 Outcomes

1. Be able to draw a rough graph of any given function.
2. Solve different nonlinear equations using different numerical methods and interpret the results.
3. Solve sets of equations using different numerical methods.
4. Be able to construct interpolating polynomials and fit curves to a given data
5. Be able to perform numerical Differentiation and Integration

3 LECTURER AND CONTACT DETAILS

3.1 Lecturer

The lecturer responsible for this module is as follows:

Dr. M.M. Kamga Pene

Office:	C-Block, Room 655
	Florida Campus
Telephone:	+27(0)11-670-9153
Fax:	+27(0)11-670-9171
E-mail:	kamgamm@unisa.ac.za

All queries that are not of a purely administrative nature but are about the content of this module should be directed to your lecturer(s). Please have your study material with you when you contact your lecturer.
E-mail and telephone numbers are included above but you might also want to write to us. Letters should be sent to:

The Module leader for COS2633
Department of Mathematical Sciences
PO Box 392
UNISA
0003

PLEASE NOTE: Letters to lecturers may not be enclosed with or inserted into assignments.

3.2 Department

Fax number:	011-670-9171 (RSA)	+27-11-670-9171 (International)
Departmental Secretary:	011-670-9147 (RSA)	+27-11-670-9147 (International)

3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication *my Studies @ Unisa* that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).

Always have your student number at hand when you contact the University.

Please note that all administrative enquiries should be directed to the Contact details from *my Studies @ Unisa*. Enquiries will then be channeled to the correct department.

4 MODULE RELATED RESOURCES

The Department of Dispatch should supply you with the Study Guide and Tutorial Letter 101 at registration and others later. Full particulars regarding the suggested use of the study guide in conjunction with the textbook can be found in the study guide.

If you have access to the Internet, you can view the study guide and tutorial letters for the modules for which you are registered on the University's online campus, *my*Unisa, at **http://my.unisa.ac.za**

4.1 Prescribed Book

Your prescribed textbook for this module for this semester is:

R.L Burden & J.D. Faires
Numerical Analysis.
Brooks/Cole Cengage Learning, 9th edition, 2010.

Please buy this book without delay. Prescribed books can be obtained from the University's official booksellers. Consult the list of official booksellers and their addresses listed in the brochure *my Studies @ Unisa*. If you have difficulty in locating your book at these booksellers, please contact the Prescribed Book Section at Tel: 012 429-4152 or email: vospresc@unisa.ac.za

4.2 Recommended Books

There are no recommended books for this module.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication *my Studies @ Unisa* that you received with your study material.

myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The *myUnisa* learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa all through the computer and the internet.
To go to the *myUnisa* website, start at the main Unisa website, **www.unisa.ac.za**, and then click on the **myUnisa** link below the orange tab labelled **Current students** . This should take you to the myUnisa website. You can also go there directly by typing **my.unisa.ac.za** in the address bar of your browser.
Please consult the publication *my Studies @ Unisa* which you received with your study material for more information on *myUnisa*.

6 MODULE SPECIFIC STUDY PLAN

The sections of the ninth edition that are prescribed for examination purposes are

- **Chapter 2:** sections 2.1 - 2.6;
- **Chapter 3:** sections 3.1 - 3.7;
- **Chapter 4:** sections 4.1 - 4.9;
- **Chapter 6:** sections 6.1 - 6.5;
- **Chapter 7:** sections 7.1, 7.3 - 7.5;
- **Chapter 8:** section 8.1;
- **Chapter 10:** section 10.2.

Semester 1		
Period	Assignment (due date)	Textbook (9th ed.)
20/01 - 21/02	1 (26/02)	**Study** chapters 1 and 2
21/02 - 31/03	2 (17/04)	**Study** chapters 3, 4, 6, 7, 8.1 and 10.2
20/01 - 23/04	3 (24/04)	**Study** all chapters
24/04 - examination date : **Revision**		

Table 1: Suggested study programme for Semester 1

In addition to the textbook you should also study the following:

Semester 2		
Period	Assignment (due date)	Textbook (9th ed.)
15/07 - 05/08	1 (06/08)	**Study** chapters 1 and 2
07/08 - 21/09	2 (22/09)	**Study** chapters 3, 4, 6, 7, 8.1 and 10.2
15/07 - 28/09	3 (29/09)	**Study** all chapters
29/09 - examination date : **Revision**		

Table 2: Suggested study programme for Semester 2

- *Tutorial letter COS2633/102/3/2015*, the use of which we discuss in its preface.
- *Tutorial letters*, which include detailed discussions and model solutions of the assignments. The assignments and the corresponding tutorial letters are important since they give you an idea of what we expect of you with respect to the *types of problems* to be solved, and their *solutions*. Please note, however, that you should not rely solely on the tutorial letters for your exam preparation. The examination covers the whole syllabus, theory as well as practice, and you should prepare accordingly. We also give *additional explanations* in these letters.

  The tutorial letters are dispatched to you in the course of the year as they become available and will also be downloadable from the Internet.
- *Inventory for the current academic year* that you received on registration and which lists the items available from the Department of Dispatch in Pretoria or the regional offices at the time of registration. Please check the tutorial matter you have received against this inventory and, if necessary, take appropriate action contacting the department of dispatch.

You should read the entire tutorial letter COS2633/102/3/2015, before working through the textbook. You should work through the sections of the prescribed textbook in the order indicated in table 1 or 2 and submit assignments 1, 2 and 3 before the respective due dates.
See the brochure *my Studies @ Unisa* for general time management and planning skills.

7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8 ASSESSMENT

8.1 Assessment Plan

Assignments are seen as part of the learning material for this module. As you do the assignments, discuss the work with fellow students or tutors, you are actively engaged in learning. It is therefore important that you complete all the assignments.

- Assignment 1 is compulsory and will determine your exam admission. It contributes 20% towards your year mark. You should note that this assignment is a **multiple choice** assignment. The reason for this is that in the semester based tuition model that Unisa has embraced the turnaround time on submission and marking of assignments by hand is too long to allow for more than one hand marked assignment (in this module assignment 2) in the tuition period of one semester.

  We urge you to use the following approach: Solve the problems at hand on paper on your own and obtain answers to all the problems. Keep your written paper solutions and your answers in a safe place for later use. Then check which of the multiple choice answers correspond with yours and select the allocated numbers as your answers. Submit the marking sheet in adherence to the official due date. Once you have received our discussion of the solutions you should compare your (carefully kept) paper solutions with ours to see where you made mistakes, if any, or to see what our view on the solutions are. This is an important part of your learning process. In the exam there will be no multiple choice questions.

- Assignment 2 contributes 60% towards your year mark. It should be submitted and will be marked by us.

- Assignments 3 comprises of your participation in the online discussions on the module topics throughout each semester. It will also be submitted on or before the due date and *marked* by us. This assignment contributes 20% to the year mark, provided that it is clear that you made contributions in *all* the topics that are discussed. The idea is that you share with your fellow students your understanding of the module topics and how you arrive at your solutions to the assignments. This will also give you the opportunity to evaluate your own work against the work of your fellow students and against the model solutions, which will be provided by us during the discussions. You will be informed of what exactly should be submitted **two weeks** prior to the due date.

Assignments will be assessed not only on the mathematical correctness of your work, but also on whether you use mathematical notation and language to communicate your ideas clearly. Markers will comment on the work that you submit in your assignments. The assignments and the comments constitute an important part of your learning and should help you to be better prepared for the examination.

- **Meeting due dates**: Only those assignments that reach us *before* or *on* the appropriate due date will be marked. If this date falls on a Sunday or a public holiday, then the next working day will be considered as the due date.

- **Extension date**: Because of the tight two semester schedule, we will not be able to allow an extension of the due dates
  PLEASE DO **NOT** PHONE TO REQUEST ANY EXTENSION OF THE ASSIGNMENT DUE DATES.

- **Marking of questions**:

  – It is the prerogative of the lecturer to decide which question(s) of any particular assignment will be marked.

  – Students will not be notified of this in advance.

  - The same question or questions will be marked for all students.
  - Only the question(s) that is(are) marked will contribute towards the year mark (%) obtained for an assignment.

  It is therefore in your own interest to attempt all questions with care!

- **Examination dates**: Make a note of the dates of your examination and make arrangements for leave in advance.

Examination admission

Due to regulations imposed by the Department of Education the following applies for COS2633: In order to be considered for examination admission in COS2633, a student **must** submit assignment 1 BEFORE the due date (26 February for Semester 1 or 06 August for Semester 2). This means that assignment 1 is **compulsory**. Students who do not comply with this condition will not be considered as "active" students, will not qualify for government subsidy to the University and will therefore not be allowed to write the examination.

You will be admitted to the examination if and only if Assignment 1 reaches the Assignment Section by 26 February 2015 if you are registered for Semester 1, or by 06 August 2015 if you are registered for Semester 2.

Please note that lecturers are not responsible for examination admission, and ALL enquiries about examination admission should be directed to the Unisa Contact Centre.

Year mark

It is a University decision that the final mark of a module should consist of an examination mark and a year mark. For COS2633 the final mark will be calculated as follows:

- The June/November examination mark will constitute 80% of the final mark and the assignments will contribute 20% to the final mark.
- The year mark will not be taken into account in any special examination results.

This will have implications for you as the student, so please make sure that you *read this section very carefully*.

Composition

In this module, assignments 1, 2 and 3 will contribute towards the year mark according to the weights 20, 60 and 20. So, if you obtain P%, Q% and R% for assignments 1, 2 and 3 respectively, then the contribution of your year mark towards your final mark will be

$$M_y = \frac{1}{5}\left[\frac{20P + 60Q + 20R}{100}\right]\%.$$

Assignment	Evaluated by	% of year mark
1	Lecturer	20
2	Lecturer	60
3	Lecturer	20

Table 3: Summary of assignment contributions to year mark

Example: If you obtain 55%, 100% and 65% for assignments 1 to 3 respectively, then your year mark will contribute

$$M_y = \frac{1}{5}\left[\frac{20(55) + 60(100) + 20(65)}{100}\right] = 16.8\%$$

towards your final mark.

The June/November exam mark will contribute 80% towards the final mark. If you obtain $Y\%$ in the June/November exam, then its contribution to your final mark will be

$$M_e = \frac{80Y}{100}\%.$$

Example: If you obtain 58% in the examination, the contribution towards your final mark will be

$$M_e = \frac{80(58)}{100} = 46.4\%.$$

The final mark: $M_{final} = (M_y + M_e)\%$.

In terms of the above examples, your final mark will be $16.8 + 46.4 = 63.2\%$.

Table 3 contains a summary of the contributions of the various assignments to the year mark.

8.2 General Assignment Numbers

The assignments are numbered as 1, 2 and 3 for each semester.

8.2.1 Unique Assignment Numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

8.2.2 Due Dates of Assignments

The dates for the submission of the assignments are:

	ASSIGNMENT 1	ASSIGNMENT 2	ASSIGNMENT 3
SEMESTER 1	26 February 2015	17 April 2015	24 April 2015
SEMESTER 2	06 August 2015	22 September 2015	29 September 2015

8.3 Submission of Assignments

You may submit assignments either by post or electronically via myUnisa. **Assignments may not be submitted by fax or e-mail**. For detailed information and requirements as far as assignments are concerned, see the brochure *my Studies @ Unisa* that you received with your study material.

To submit an assignment via myUnisa:

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left-hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.

Also, you should NEVER submit your assignment answers directly to a lecturer, no matter what the circumstances are! (For example, even if the myUnisa website is down, do NOT email your assignment answers to a lecturer!)

8.4 Assignments

- The assignments consist mainly of *numerical problems*, which may be solved with the aid of a pocket calculator or a computer. You must not only supply the answers to problems, but also explain the methods used.
- When you solve these problems, make sure that you *understand* the underlying *theory* well, since questions on the theory will be included in the examination paper.
- The *submission* of the *answers* to the assignments must be done on the prescribed paper, stapled to an assignment cover with the front page completed. The student number, subject, module code and assignment number must also be given on each page. Assignments must be sent to the address given in *my Studies @ Unisa*. They may also be submitted electronically through *my*Unisa.

  Remember that your assignments must have exactly the *same* number as the one specified in this tutorial letter.
- If you submit your assignments *electronically*, you MUST use *standard notation*, that is, the notation of the textbook. We live in an era in which word processing tools allow for professional mathematical texts to be produced. Be warned: If you devise your own primitive, crude ”all on one line” notation, your assignment will not be marked. Remember to be RATHER SAFE THAN SORRY!
- A *discussion* of the *solutions* to the assignments is sent out to all students after the due date. They will also be available electronically.

SEMESTER 1
COMPULSORY ASSIGNMENT FOR THE EXAM

ASSIGNMENT 01
Due Date: 26 February 2015
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: 574320

ONLY FOR SEMESTER 1

Bisection, Secant and Regula Falsi Methods, Fixed-Point Iteration, Newton's Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomials and Müller's Method

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Question 1: **40 Marks**

Consider the function $f(x) = xe^x - 2$; we want to study the properties of $f(x)$ so that we can apply numerical methods to solve the equation $f(x) = 0$.

(1.1) Which option is false? (10)

(1) the function, $f(x)$ is well defined and continuous for all x in the interval $(0, 2)$

(2) the function, $f(x)$ has no discontinuity and no singularities

(3) the function, $f'(x)$ is well defined and continuous for all x in the interval $(0, 2)$

(4) the function, $f'(x)$ has no discontinuity and no singularities

(5) all of the above

(1.2) To have an idea on whether we should apply the regula falsi method to determine the root of $f(x) = 0$ in a given interval, we may: (10)

(1) study carefully the continuity of $f(x)$ and $f'(x)$ then conclude

(2) check if $f(x)$ and $f'(x)$ are differentiable then conclude

(3) draw the graph of $f(x)$ and observe the graphs then conclude

(4) apply the functions, $f(x)$ to a finite set of sample values then conclude

(5) none of the above is true

(1.3) Applying the regula falsi method, with starting points $x_0 = 0.5$ and $x_1 = 1$ yields the following result: (10)

(1) 0.847977577064739 after at most two iterations

(2) 0.852551206774865 after exactly four iterations

(3) 0.852604865962733 after exactly four iterations

(4) 0.852599625362406 after at least five iterations

(5) all of the above

(1.4) Applying the secant method, with starting points $x_0 = 0.5$ and $x_1 = 1$ yields the following result: (10)

(1) 0.810371774952277 after at least three iterations

(2) 0.852604956977597 after exactly four iterations

(3) 0.852758254751202 after exactly two iterations

(4) 0.847977577064739 after at most one iterations

(5) all of the above

Question 2: **60 Marks**

The function $f(x) = x^4 - 2x^3 - 5x^2 + 12x - 5$ has four distinct roots, we want to determine the roots by means of Müller's method

(2.1) Which option is false? (10)

(1) $f(x) = x^4 - 2x^3 - 5x^2 + 12x - 5$ has no singularities and no obvious symmetries and $f(0) = -5$

(2) $f'(x) = 4x^2 - 6x^2 - 10x + 12$, $f''(x) = 12x^2 - 12x - 10$ and $f'''(x) = 24x - 12$

(3) $f''(x)$ has a local *extremum* at $x = 0.5$

(4) the two zeros for $f''(x)$ are -0.5417 and 1.5417

(5) $f''(x)$ has a local *maximum* at $x = 0.5$

(2.2) Applying Müllers method to compute the zeros of $f'(x)$ yields the following result: (10)

(1) -1.5 with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$

(2) -1.5 with the starting points $x_2 = -0.5417$, $x_0 = 1.0417$ and $x_1 = 1.5417$

(3) 1 with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$

(4) 2 with the starting points $x_2 = -2.5417$, $x_0 = -1.5417$ and $x_1 = -0.5417$

(5) none of the above

(2.3) Which option is false? (10)

(1) $f'(x)$ has a local minimum at 1.5417

(2) $f'(x)$ has a local maximum at -0.5417

(3) $f(x)$ has a local minimum at -1.5 and 2

(4) $f(x)$ has a local maximum at 1

(5) all of the above

(2.4) Applying Müller's method to compute the zeros of $f(x)$ yields the following result: (10)

(1) -1.5 with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = -1.5$

(2) -2.4321 with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = -1.5$

(3) 1 with the starting points $x_2 = -1.5$, $x_0 = -0.25$ and $x_1 = 1$

(4) 0.5798 with the starting points $x_2 = 1$, $x_0 = 1.5$ and $x_1 = 2$

(5) none of the above

(2.5) Applying Müller's method to compute the zeros of $f(x)$ yields the following result: (10)

(1) -2.4321 with the starting points $x_2 = -3.5$, $x_0 = -2.5$ and $x_1 = -1.5$

(2) 0.5798 with the starting points $x_2 = -1.5$, $x_0 = -0.25$ and $x_1 = 1$

(3) 1.5206 with the starting points $x_2 = 1$, $x_0 = 1.5$ and $x_1 = 2$

(4) 2.3316 with the starting points $x_2 = 2$, $x_0 = 3$ and $x_1 = 4$

(5) all of the above

(2.6) Select the appropriate option: (10)

(1) secant method and Müller's method are similar in the sense that they both start with two initial points

(2) secant method yields a complex root even when initial approximation is a real number

(3) Müller's method determines the next approximation by considering the intersection of a parabola and the $x-$axis through three given points

(4) all of the above

(5) none of the above

ASSIGNMENT 02
Due Date: 17 April 2015
Total Marks: 120
UNIQUE ASSIGNMENT NUMBER: 574361

ONLY FOR SEMESTER 1

Linear Systems of Equations, Linear Algebra, Pivoting Strategies and Matrix Factorization, Jacobi and Gauss-Seidel Iterative Techniques and Error Bounds, Interpolation and Lagrange Polynomials, Data Approximation, Hermite Interpolation, Divided Difference, Cubic Splines, Parametric Curves and Discrete Least Squares Approximation, Numerical Differentiation and Integration

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Question 1: **30 Marks**

Consider the linear system

$$\begin{array}{rcrcrcrcr} 2.141x_1 & - & 2.718x_2 & + & 1.414x_3 & - & 1.732x_4 & = & 3.316 \\ 9.869x_1 & + & 2.718x_2 & - & 7.389x_3 & + & 0.428x_4 & = & 0 \\ 2.236x_1 & - & 2.449x_2 & + & x_3 & - & 1.414x_4 & = & 3.141 \\ 31.006x_1 & + & 7.389x_2 & - & 2.645x_3 & + & 0.111x_4 & = & 1.414 \end{array}$$

(1.1) Write the system in matrix notation. (2)

(1.2) Solve the system using:

(a) Gaussian elimination without pivoting. (2)

(b) Gaussian elimination with scaled partial pivoting. (2)

(c) LU decomposition. (2)

(1.3) Show that the total number of arithmetic (multiplication, divisions and additions) operations in the (1.2(a)) is approximately 58.66 (show all details). (2)

(1.4) Suppose we are to solve the equation

$$Ax = b.$$

We solve this by

$$x = \frac{1}{A}b = \frac{1}{\omega A}\omega b = \frac{1}{1-r}\omega b$$

where $\omega \neq 0$ is some real number chosen to weight the problem appropriately, and $r = 1 - \omega A$. Now suppose that ω was chosen such that $|r| < 1$ and $A \neq 0$. Then the following geometric expansion holds:

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + ...$$

This gives approximate solution to our problem as,

$$\begin{aligned} x &\approx \left[1 + r + r^2 + r^3 + \cdots + r^k\right] \omega b \\ &= \omega b + \left[r + r^2 + r^3 + \cdots + r^k\right] \omega b \\ &= \omega b + r\left[1 + r + r^2 + r^3 + \cdots + r^{k-1}\right] \omega b \end{aligned}$$

This suggests an iterative approach to solving the system $Ax = b$. Let $x^{(0)} = \omega b$ be the initial estimate of the solution, then for the k^{th} iterate we have

$$\begin{aligned} x(k) &= \omega b + r x^{(k-1)} \\ &= \omega b + (I - \omega A)\, x^{(k-1)} \end{aligned}$$

If $|r| < 1$ then the iterate $x^{(k)}$ is guaranteed to converge to the true solution.
For some matrix, M and some scaling factor ω, we obtain the following algorithm

$$Mx^{(k)} = \omega b + (M - \omega A)\, x^{(k-1)}$$

(a) Let A be the matrix from (1.1) and $\omega = 1$ (8)

- (i) Solve the system by choosing M to be the matrix consisting of the diagonal of A. [4]
- (ii) Solve the system using the Jacobi method. [2]
- (iii) Compare the two results. [2]

(b) Let A be the matrix from (1.1) and $\omega = 1$ (8)

- (i) Solve the system by choosing M to be the lower triangular part of A including the diagonal entries. [4]
- (ii) Solve the system using the Gauss-Seidel method. [2]
- (iii) Compare the two results. [2]

(c) Solve the system using Successive Over-Relaxation technique with $\omega = 0.5$ and $x_0 = (3, 0, 3, 1)^T$ (2)

(d) Which method do you prefer and why? (2)
Note: Use four-decimal arithmetic with truncation (*not* rounding)and do three iterations, starting at $x_0 = (3, 0, 3, 1)^T$ where applicable.

Question 2: **6 Marks**

Find the inverse of

$$\begin{bmatrix} 3 & \frac{3}{2} & 1 \\ \frac{3}{2} & 1 & \frac{3}{4} \\ 1 & \frac{3}{4} & \frac{3}{5} \end{bmatrix}$$

Use Gaussian elimination and work *exactly*.

Question 3: **8 Marks**

Write a computer program which uses Newton's method to obtain a solution, accurate to five decimal digits, of the pair of simultaneous equations

$$\begin{aligned} x^2 + y^2 &= 5 \\ x^3 + y^3 &= 2 \end{aligned}$$

taking $x = 1$ and $y = -1$ as the initial values in the iterative process.

Question 4: **18 Marks**

Consider the following data:

x	$f(x)$
-0.5	5
0	15
0.5	9
1	3
1.5	1

(4.1) Set up a difference table through third differences. (3)

(4.2) What is the minimum degree that an interpolating polynomial, that fits all five data points exactly, can have? Explain. (3)

(4.3) Give the (forward) Newton-Gregory polynomial that fits the data points with x values 0.5, 1, and 1.5. Then compute $f(1.25)$. (3)

(4.4) Compute an approximate bound for the error in the approximation to $f(1.25)$ in (4.3) using Newton's forward interpolating polynomial. (3)

(4.5) Compute $f(1.25)$ using the Lagrange interpolating polynomial through the data points with x values 0.5, 1, and 1.5. (3)

(4.6) Construct the *natural* cubic spline for the last four data points. (3)

Question 5: **12 Marks**

Let $(x_0, y_0) = (0, 0)$ and $(x_1, y_1) = (5, 2)$ be the end points of a curve. Use the following guide points to construct parametric cubic approximations $(x(t), y(t))$ and graph the approximations. Construct and graph the cubic Bezier polynomials:

(5.1) $(1, 1)$ and $(6, 1)$ (4)

(5.2) $(0.5, 0.5)$ and $(5.5, 1.5)$ (4)

(5.3) $(2, 2)$ and $(6, 3)$ (4)

Question 6: **19 Marks**

Given the data

x	y
0.2	0.050446
0.3	0.098426
0.6	0.33277
0.9	0.72660
1.1	1.0972
1.3	1.5697
1.4	1.8487
1.6	2.5015

(6.1) Construct the least squares approximation polynomial of degree three and compute the error. (4)

(6.2) Construct the least squares approximation of the form be^{ax} and compute the error. (4)

(6.3) Construct the least squares approximation of the form bx^a and compute the error. (4)

(6.4) Draw the graph of the data points and the approximations in (6.1), (6.2) and (6.3). (7)

Question 7: **17 Marks**

(7.1) Approximate

$$\int_{0.75}^{1.3} \left(\sin^2 x - 2x \sin x + 1\right) dx$$

by means of:

(a) Composite trapezoidal rule with $n = 8$ (3)

(b) Three-term Gaussian quadrature formula (3)

(c) Simpson $\frac{3}{8}$ rule (3)

(7.2) Estimate the respective truncation errors in (7.1(a)) and (7.1(c)). (4)

(7.3) Determine the integral analytically and then compute the actual errors in (7.1(a)), (7.1(b)) and (7.1(c)) respectively. [**Hint:** Use Taylor series expansion of $\sin x$.] (4)

Question 8: **10 Marks**

The area of the surface described by $z = f(x, y)$ for $(x, y) \in \mathcal{R}$ is given by

$$\int\int_{\mathcal{R}} \sqrt{1 + [f_x(x, y)]^2 + \left[f_y(x, y)\right]^2}\, dA.$$

Find an approximation to the area of the surface on the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$ that lies above the region in the plane described by $\mathcal{R} = \{(x, y)\,|0 \leq x \leq 1,\ 0 \leq y \leq 1\}$ using:

(8.1) Trapezoidal rule in both directions (3)

(8.2) Simpson $\frac{1}{3}$ rule in both directions (3)

(8.3) Three-term Gaussian quadrature formulas in both directions (4)

ASSIGNMENT 03
Due Date: 24 April 2015
UNIQUE ASSIGNMENT NUMBER: 574402

ONLY FOR SEMESTER 1

ALL TOPICS

Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.

SEMESTER 2
COMPULSORY ASSIGNMENT FOR THE EXAM

ASSIGNMENT 01
Due Date: 6 August 2015
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: 574529

ONLY FOR SEMESTER 2

Bisection, Secant and Regula Falsi Methods, Fixed-Point Iteration, Newton's Method and its Extensions, Error Analysis for Iterative Methods, Accelerating Convergence, Zeros of Polynomials and Müller's Method

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Question 1: **50 Marks**

Suppose we want to develop an iterative method to compute the k^{th} root of a given positive number y, i.e., to solve the nonlinear equation $x^k = N$ given the value of N. Consider the following functions

- $g_1(x) = N + x - x^k$
- $g_2(x) = 1 + x - \dfrac{x^k}{N}$

(1.1) Which option is false? (10)

(1) the function, $g_1(x)$ gives a fixed-point problem that is equivalent to the equation $f(x) = 0$

(2) the function, $g_2(x)$ gives a fixed-point problem that is equivalent to the equation $f(x) = 0$

(3) Newton's algorithm for $f(x) = 0$ is given by: $x_{n+1} = \dfrac{1}{k}\left((k-1)\,x_n + \dfrac{N}{x_n^{k-1}}\right)$

(4) The algorithm in option (3) would still apply if N was a negative number, with a complex number as the starting point

(5) not all of the above is true

(1.2) Consider $g_1(x) = N + x - x^3$ (10)

(1) the fixed-point iteration scheme $x_{n+1} = g_1(x_n)$ is (locally) convergent to $\sqrt[3]{N}$ if $N = 7$

(2) the (local) convergence of $g_1(x)$ cannot be guaranteed because $g_1(x)$ is not differentiable in the interval that contains $\sqrt[3]{7}$

(3) the interval of convergence where $|g_1'(x)| < 1$ does not contain $\sqrt[3]{7}$

(4) $g_1'(x)$ is not continuous

(5) None of the above is true

(1.3) Consider $g_2(x) = 1 + x - \dfrac{x^3}{N}$ (10)

(1) the convergence of $g_2(x)$ is not guaranteed because $g_2'(x)$ is not continuous

(2) the convergence of $g_2(x)$ is not guaranteed because the interval of convergence where $|g_2'(x)| < 1$ does not contain $\sqrt[3]{7}$

(3) the convergence of $g_2(x)$ is guaranteed because $g_2(x)$ and $g_2'(x)$ are continuous and the set of points of intersection of the graphs of $y = g_2(x)$ and $y = x$ is not empty

(4) $g_2'(x)$ is not continuous

(5) none of the above is true

(1.4) Consider $g_2(x) = 1 + x - \dfrac{x^3}{N}$ (10)

(1) there is a guarantee on the convergence of $g_2(x)$

(2) the convergence of $g_2(x)$ would not have been guaranteed if $g_2(x)$ were continuous and differentiable only in an interval that excludes $\sqrt[3]{7}$

(3) the convergence of $g_2(x)$ is guaranteed because the interval of convergence where $|g_2'(x)| < 1$ contains $\sqrt[3]{7}$

(4) $g_2'(x)$ is continuous

(5) all of the above is true

(1.5) the error of the fixed-point function, $g_3(x)$, given by Newton's method for $f(x) = x^3 - 7$ is: (10)

(1) $e_{n+1} = g_1(x_n)$

(2) $e_{n+1} = \dfrac{1}{3x_n}\left[\dfrac{g_1(x_n)}{x_n} - 1\right]$

(3) $e_{n+1} = g_2(x_n)$

(4) $e_{n+1} = g_2(x_n) - \dfrac{g_1(x_n)}{3x_n^2}$

(5) none of the above

Question 2: **50 Marks**

The difference between two numbers is 3. If the larger number is added to its square root and the smaller number is added to its square, the product of the two differences equals 66.33. We want to determine the two numbers to within 10^{-3}.

(2.1) the two numbers say x and y can be determined by: (10)

(1) simply using a non-programmable calculator

(2) solving the equation $(2x+3)\left(x+\sqrt{x+3}\right)\left(x^2+x\right)=0$ then deduce the value of y from the equation $y=x-3$

(3) solving the equation $(2x+3)\left(x+\sqrt{x+3}\right)\left(x^2+x\right)=0$ then deduce the value of y from the equation $y=x+3$

(4) solving the equation $x^2-\sqrt{x+3}-25.11=0$ then deduce the value of y from the equation $y=x+3$

(5) none of the above

(2.2) With the starting point 1.5 and applying the Newton's method to the appropriate equation yields the following result: (10)

(1) 6.6093246475 after at exactly two iterations

(2) 5.4243003271 after exactly four iterations

(3) 5.4243003271 after at least seven iterations

(4) 5.4243003271 after at least five iterations

(5) none of the above

(2.3) Applying the secant method to the same equation used for the Newton's methods in question (2.2), yields the following result: (10)

(1) 5.1715901498 with starting points 1.5, -1 and after at most two iterations

(2) 5.1715901498 with starting points 1.5, 7 and after exactly four iterations

(3) 5.2904130851 with starting points 1, 3 and after exactly six iterations

(4) 5.2904935273 with starting points 3, 5 and after at most four iterations

(5) none of the above

(2.4) Applying the regula falsi method to the same equation used for the Newton's methods in question (2.2), with the starting $x_0 = 1$ and $x_1 = 2$, yields the following result: (10)

(1) 3.9015664048 after at most two iterations

(2) 5.1163828931 after exactly five iterations

(3) 5.2840053817 after exactly seven iterations

(4) 5.2840053817 after at most five iterations

(5) none of the above

(2.5) Select the appropriate option: (10)

(1) the Newton's method yields a complex root with the following starting points -1, 0 and 0.5

(2) the Secant method yields a complex root with the starting points $x_0 = -3$, and $x_1 = 1$

(3) the Falsi method yields a complex root with the starting points $x_0 = -1$, and $x_1 = -2$

(4) all of the above

(5) none of the above

ASSIGNMENT 02
Due Date: 22 September 2015
Total Marks: 120
UNIQUE ASSIGNMENT NUMBER: 574583

ONLY FOR SEMESTER 2

Linear Systems of Equations, Linear Algebra, Pivoting Strategies and Matrix Factorization, Jacobi and Gauss-Seidel Iterative Techniques and Error Bounds, Interpolation and Lagrange Polynomials, Data Approximation, Hermite Interpolation, Divided Difference, Cubic Splines, Parametric Curves and Discrete Least Squares Approximation, Numerical Differentiation and Integration

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Question 1: **30 Marks**

Consider the linear system

$$\begin{array}{ccccccccc} 0.06x_1 & + & 0.08x_2 & + & 0.07x_3 & + & 0.08x_4 & = & 0.29 \\ 0.08x_1 & + & 0.20x_2 & + & 0.09x_3 & + & 0.07x_4 & = & 0.44 \\ 0.07x_1 & + & 0.09x_2 & + & 0.20x_3 & + & 0.10x_4 & = & 0.46 \\ 0.06x_1 & + & 0.08x_2 & + & 0.10x_3 & + & 0.20x_4 & = & 0.44 \end{array}$$

(1.1) Write the system in matrix notation. (2)

(1.2) Solve the system using:

- **(a)** Gaussian elimination without pivoting. (2)
- **(b)** Gaussian elimination with scaled partial pivoting. (2)
- **(c)** LU decomposition. (2)

(1.3) Show that the total number of arithmetic (multiplication, divisions and additions) operations in the (1.2(a)) is approximately 58.66 (show all details). (2)

(1.4) Suppose we are to solve the equation

$$Ax = b.$$

We solve this by

$$x = \frac{1}{A}b = \frac{1}{\omega A}\omega b = \frac{1}{1-r}\omega b$$

where $\omega \neq 0$ is some real number chosen to weight the problem appropriately, and $r = 1 - \omega A$. Now suppose that ω was chosen such that $|r| < 1$ and $A \neq 0$. Then the following geometric expansion holds:

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + ...$$

This gives approximate solution to our problem as,

$$
\begin{aligned}
x &\approx \left[1 + r + r^2 + r^3 + \cdots + r^k\right] \omega b \\
&= \omega b + \left[r + r^2 + r^3 + \cdots + r^k\right] \omega b \\
&= \omega b + r \left[1 + r + r^2 + r^3 + \cdots + r^{k-1}\right] \omega b
\end{aligned}
$$

This suggests an iterative approach to solving the system $Ax = b$. Let $x^{(0)} = \omega b$ be the initial estimate of the solution, then for the k^{th} iterate we have

$$
\begin{aligned}
x(k) &= \omega b + r x^{(k-1)} \\
&= \omega b + (I - \omega A)\, x^{(k-1)}
\end{aligned}
$$

If $|r| < 1$ then the iterate $x^{(k)}$ is guaranteed to converge to the true solution.
For some matrix, M and some scaling factor ω, we obtain the following algorithm

$$
M x^{(k)} = \omega b + (M - \omega A)\, x^{(k-1)}
$$

(a) Let A be the matrix from (1.1) and $\omega = 1$ (8)

(i) Solve the system by choosing M to be the matrix consisting of the diagonal of A. [4]

(ii) Solve the system using the Jacobi method. [2]

(iii) Compare the two results. [2]

(b) Let A be the matrix from (1.1) and $\omega = 1$ (8)

(i) Solve the system by choosing M to be the lower triangular part of A including the diagonal entries. [4]

(ii) Solve the system using the Gauss-Seidel method. [2]

(iii) Compare the two results. [2]

(c) Solve the system using Successive Over-Relaxation technique with $\omega = 0.5$ and $x_0 = (0, 0, 0, 0)^T$ (2)

(d) Which method do you prefer and why? (2)

Note: Use four-decimal arithmetic with truncation (*not* rounding)and do three iterations, starting at $x_0 = (0, 0, 0, 0)^T$ where applicable.

Question 2: **6 Marks**

Find the inverse of

$$\begin{bmatrix} 3 & 5 & 1 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{bmatrix}$$

Use Gaussian elimination and work *exactly*.

Question 3: **8 Marks**

Consider the nonlinear system

$$\begin{aligned} x^2 &+ y^2 &= 5 \\ e^x &+ xy &= 2 \end{aligned}$$

(3.1) Approximate the solutions graphically. (3)

(3.2) Use the approximations from part (3.1) as initial approximations for Newton's method to calculate solutions with an accuracy of 10^{-3} in the absolute error definition. (5)

Question 4: **15 Marks**

(4.1) Construct a divided-difference table from: (5)

x	$f(x)$
0.3	−1.1518
−0.4	0.7028
0.5	−1.4845
0.1	−0.14943
0.0	0.13534

(4.2) Use this divided-difference table to estimate $f(0.15)$, using:

(a) A polynomial of degree 3 through the first four points, (5)

(b) A polynomial of degree 4. (5)

Question 5: **13 Marks**

The following table lists the quarterly growth of Gross Domestic Product for the RSA from from 3rd quarter of 2010 to the end of 2011:

GDP (in thousands)	Quarter
1843318	10|3
1863705	10|4
1884627	11|1
1889101	11|2
1897054	11|3
1911890	11|4

(5.1) Find the Lagrange polynomial of degree five fitting this data, and use this polynomial to estimate the GDP growth in the quarters 10|2 and 12|1. (8)

(5.2) The GDP growth in 10|2 was approximately 1829347. How accurate do you think your 12|1 figures are? (5)

Question 6: **18 Marks**

Consider the following table:

x	$\sin x$	$\frac{d}{dx}\sin x = \cos x$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

(6.1) Use the above values and five-digit rounding to construct a cubic spline Q with boundary conditions (8)

$$Q'(x_0) = f'(x_0) \quad \text{and} \quad Q'(x_n) = f'(x_n)$$

which force the slopes of the spline to assume certain values (in our case the values $f'(x_0)$ and $f'(x_n)$, respectively) at the two boundaries. Use this spline to approximate $\sin 0.33$.

(6.2) Determine the error for the approximation in (6.1). (3)

(6.3) Use the spline constructed in (6.1) to approximate $\cos 0.34$. (3)

(6.4) Use the spline constructed in (6.1) to approximate (4)

$$\int_{0.30}^{0.35} \sin x dx$$

Question 7: **18 Marks**

Consider the following set of data points in the table below:

i	x	y
0	10	10
1	50	15
2	75	60
3	90	100
4	105	140
5	150	200
6	180	140
7	190	120
8	160	100
9	130	80

(7.1) Using guide-points of your choice from the data set, construct the connected Bezier curve from the set of points. (*Hint:* divide the set of points into three parts). (10)

(7.2) Draw the connected Bezier polynomial (6)

(7.3) Why is the graph smoothly connected at points 3 and 6? (2)

Question 8: **12 Marks**

Consider the following table:

x	y
0.2	0.050446
0.3	0.098426
0.6	0.33277
0.9	0.72660
1.1	1.0972
1.3	1.5697
1.4	1.8487
1.6	2.5015

Find the least squares polynomials of degree one, two and three for the data in the above table. Compute the total error in each case. Draw a graph of the data and the polynomials.

ASSIGNMENT 03
Due Date: 29 September 2015
UNIQUE ASSIGNMENT NUMBER: 574635

ONLY FOR SEMESTER 2

ALL TOPICS

Assignment 3 comprises of your participation in the online discussion forum throughout the semester. A detailed schedule and assessment rubrics for the discussion will be made available online at the beginning of the semester.

9 OTHER ASSESSMENT METHODS

There are no other assessment methods for this module.

10 EXAMINATIONS

This module is offered in a semester period of fifteen weeks. This means that if you are registered for the first semester, you will write the examination in May/June 2015and the supplementary examination will be written in October/November 2015. If you are registered for the second semester you will write the examination in October/November 2015and the supplementary examination will be written in May/June 2016.
During the relevant semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.
For general information and requirements as far as examinations are concerned, see the brochure *my Studies @ Unisa* which you received with your study material.

Examination paper

The exam is a two hour exam. You are allowed to use a **non-programmable calculator** in the exam. The examination questions will be similar to the questions asked in the study guide and in the assignments. You need not know any proofs of theorems in this module, however, you have to be able to apply the formulas stated in the theorems and definitions.

Tutorial letter with information on the examination.

To help you in your preparation for the examination, you will receive a tutorial letter that will explain the format of the examination paper, and set out clearly what material you have to study for examination purposes. This tutorial letter will also contain a sample exam paper.

11 FREQUENTLY ASKED QUESTIONS

The *my Studies @ Unisa* brochure contains an A–Z guide of the most relevant study information.

12 SOURCES CONSULTED

All the sources consulted in the compilation of this tutorial letter are acknowledged in the body of this letter.

13 CONCLUSION

In conclusion we hope that you found this module interesting and enjoyable. We wish you the best of luck with the examination and look forward to welcoming you to APM3711, which covers most of the remaining topics in numerical methods for your undergraduate studies next year!