COS 1501 May / June 2016 Question 1 (1)

$U = \{a, b, c, d, \{b, c\}, \{a, b, c\}\}$

$A = \{a, b\}$, $B = \{b, c, \{b, c\}\}$, $C = \{b, \{a, b,$

1.1) $A \cup B \cup C$

$= \{a, b\} \cup \{b, c, \{b, c\}\} \cup \{b, \{a, b, c\}\}$

$= \{a, b, c, \{b, c\}, \{a, b, c\}\}$

$= \underline{3}$

1.2) $A \cap B \cap C$

$= \{a, b\} \cap \{b, c, \{b, c\}\} \cap \{b, \{a, b, c\}\}$

$= \{b\} \cap \{b, \{a, b, c\}\}$

$= \{b\} = \underline{2}$

1.3) $A - C$

$= \{a, b\} - \{b, \{a, b, c\}\}$

$= \{a\}$

$= \underline{3}$

1.4) $B + C = (B \cup C) - (B \cap C)$

$= \{b, c, \{b, c\}, \{a, b, c\}\} - \{b\}$

$= \{c, \{b, c\}, \{a, b, c\}\}$

$= \underline{2}$

1.5) $C' = U - C$

$= \{a, b, c, d, \{b, c\}, \{a, b, c\}\} - \{b, \{a, b, c\}\}$

$= \{a, c, d, \{b, c\}\}$

$= \underline{1}$

1.6) $P(c) = \{\emptyset,\ \{b\},\ \{\{a,b,c\}\},\ \{b, \{a,b,c\}\}\}$.

$\therefore \{\{\{a,b,c\}\}\} \subseteq P(c)$

$\{\{b\}\} \subseteq P(c)$

$\therefore \{\{b, \{a,b,c\}\}\} \subseteq P(c)$

$\therefore$ <u>3</u>

1.7) $P = \{(b,c), (b,a), (c,a), (a,a)\}$

on $U = \{a, b, c, d, \{b,c\}, \{a,b,c\}\}$.

$\therefore P =$ antisymmetric & transitive

$\forall a \forall b\ (a \neq b \wedge (a,b) \in P \rightarrow (b,a) \in P)$

$\forall a \forall b \forall c\ (((a,b) \in P \wedge (b,c) \in P \rightarrow (a,c) \in P)$.

$\therefore$ <u>4</u>

1.8) $A \cup B = \{a, b, c, \{b,c\}\}$.

$T = \{(a,a), (b,b), (c,c), (a,c), (c,b)\}$

$\therefore$ equivalence rel = reflexive,

symmetric,

transitive.

$\therefore$ reflexive? ✓

symmetric? × → needs (c,a) and (b,c)

$\therefore T = \{(a,a), (b,b), (c,c), (a,c), (c,a), (b,c), (c,b)\}$

transitive? × → $(b,c)(c,a)$ needs (b,a)

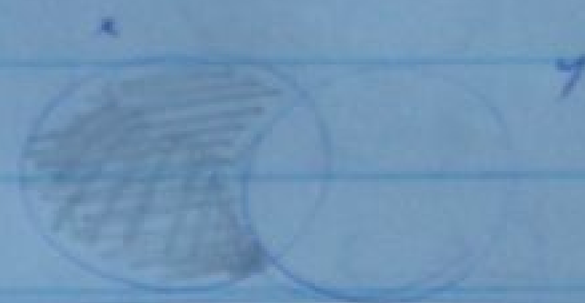
to be transitive

$\therefore$ <u>4</u>

COS 1501 May / June 2016 Question 2

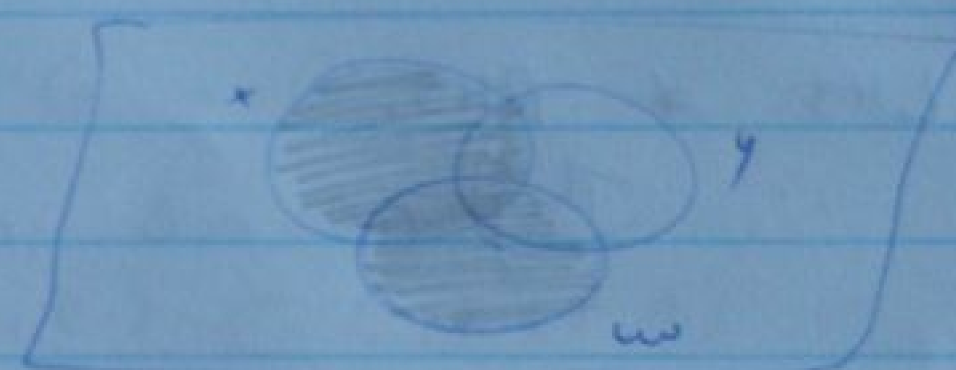
2.) a) $(X - Y)' \cap (X \cup W) = X' \cup (Y \cap W)$

$\therefore X - Y =$



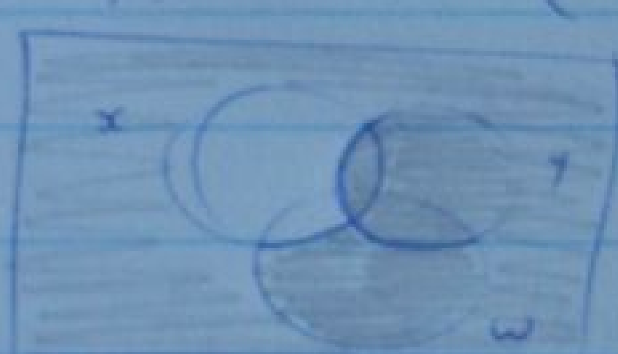
$\therefore (x - y)' =$

$X \cup W =$

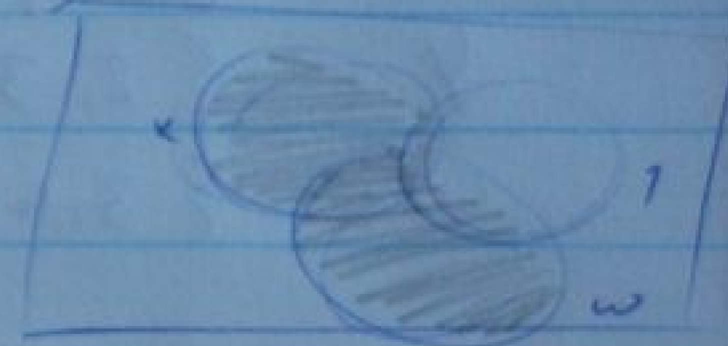


$\therefore (X - Y)' \cap (X \cup W)$

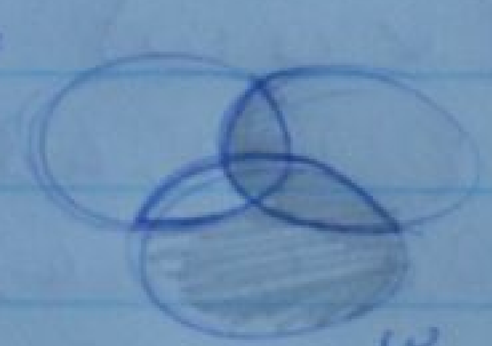
$=$



$\cap$



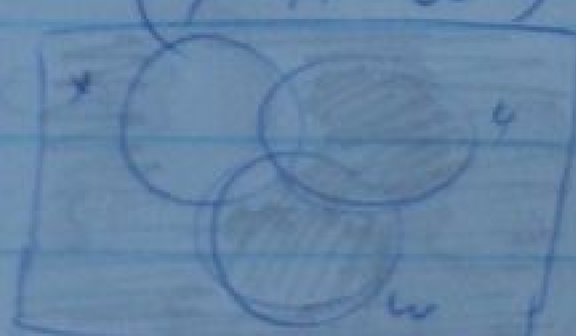
$=$



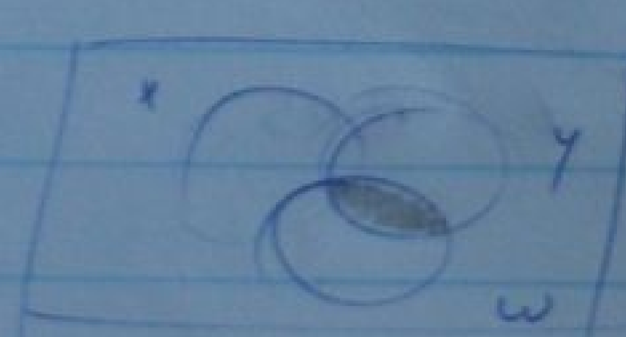
$=$ LHS.

RHS $= X' \cup (Y \cap W)$

$\therefore X' =$



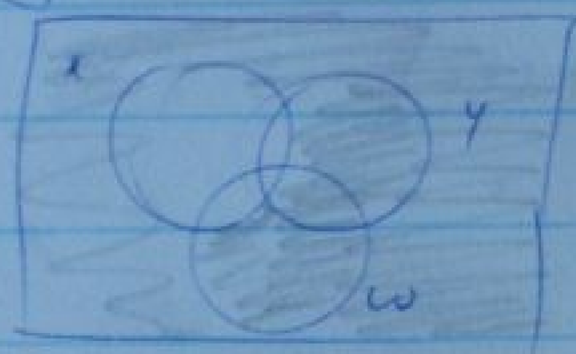
$Y \cap W =$



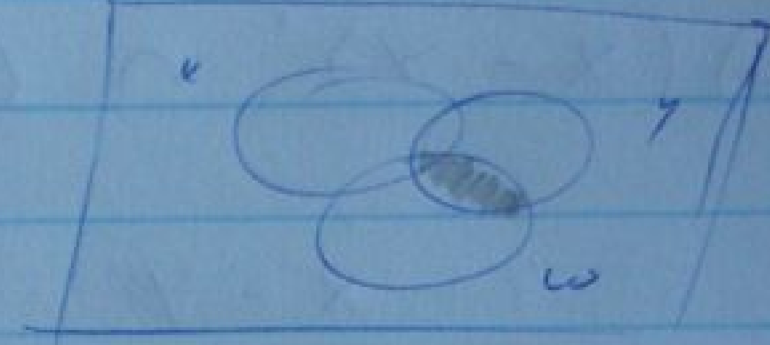
$\therefore$

$\therefore \quad x' + (y \cap w)$

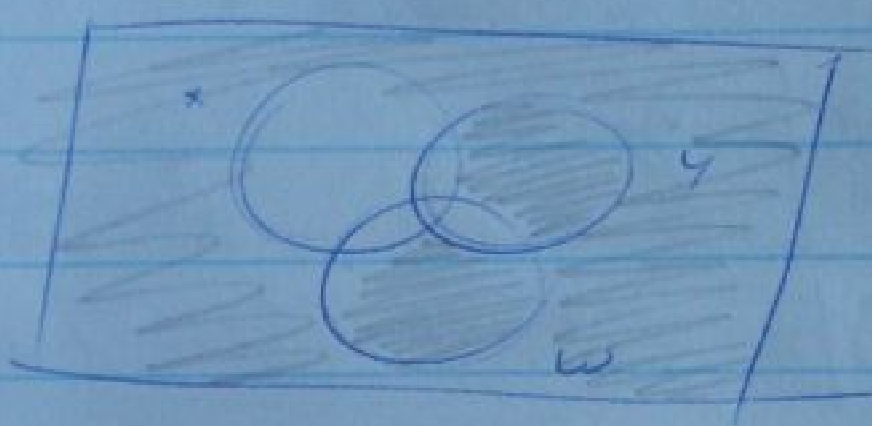
$=$



$+$



$=$



$= RHS$

$\therefore \quad LHS \neq RHS.$

$\therefore$ They are not identities.

2.1) b) $U = \{1, 2, 3, 4\}$
$X = \{1\}$
$Y = \{2, 3\}$
$W = \{3, 4\}$

$$\therefore \quad LHS = (x - y)' \cap (x \cup w)$$
$$= \{1\}' \cap \{1, 3, 4\}$$
$$= \{2, 3, 4\} \cap \{1, 3, 4\}$$
$$= \{3, 4\}$$

$$RHS = x' + (y \cap w)$$
$$= \{2, 3, 4\} + (\{2, 3\} \cap \{3, 4\})$$
$$= \{2, 3, 4\} + \{3\}$$
$$= \{2, 4\}$$

$\therefore \quad LHS \neq RHS.$

May / June 2016 — Question 2.2.

2.2) $z \in (A' \cap B) \cup (A \cap C)$

iff $(z \in A'$ or $z \in A)$ and $(z \in A'$ or $z \in C)$
... and $(z \in B$ or $z \in A)$ and $(z \in B$ or $z \in C)$
iff $(z \in A' \cup A)$ and $(z \in A' \cup C)$ and $(z \in B \cup A)$
... and $(z \in B \cup C)$
iff $z \in U \cap (A' \cup C) \cap (B \cup A) \cap (B \cup C)$

Since $U \cap G = G$
we have
iff $z \in (A' \cup C) \cap (B \cup A) \cap (B \cup C)$ $\square$

Question 3

3.1) a) A bijective function is (1) injective
(2) surjective.

b) injective functions are: $\forall a \forall b\ (a \neq b \rightarrow f(a) \neq f(b))$.
or $\forall a \forall b\ (f(a) = f(b) \rightarrow a = b)$.

$\therefore$ $R = \{(1, a), (2, b), (3, c)\}$

c) strict partial order = irreflexive, antisymmetric & transitive
$\therefore T = \{(3,4), (4,5), (3,5)\}$

d) $B = \{0, 1\}$
$\therefore$ Partition $= \{\{0\}, \{1\}\}$
$\hookrightarrow$ no empty sets.
$\{0\} \cap \{1\} = \emptyset$
$\{0\} \cup \{1\} = B$.

Question 3.2.

3.2) $(x, y) \in R$ iff $y - x = 7m$, $m \in \mathbb{Z}$

a) $y - x = 7m$

$z - y = 7k$

$\therefore z - x = 7m + 7k$

$\therefore z - x = 7(m+k)$

$\therefore (x, z) \in R.$

b) $[0] = \{ \dots -21, -14, -7, 0, 7, 14, 21, \dots \}$

Question 3.3

3.3) $(x, y) \in f$ iff $y = x^2 + 2x$ $\quad f: \mathbb{Z}$

$(x, y) \in g$ iff $y = -x + 3.$ $\quad f: \mathbb{Z}$

a) i) $ran(g) = \{ y \mid \exists y \in \mathbb{Z}, (x, y) \in g \}$

$= \{ y \mid \exists y \in \mathbb{Z}, y = -x + 3 \}$

$= \{ y \mid x = 3 - y$, is an int$\}$.

$= \mathbb{Z}$

ii) Since the range and ~~domain~~ co-domain is $\mathbb{Z}$.

$\therefore$ it is surjective.

b) $f \circ g(x)$

$= f(g(x))$

$= (-x+3)^2 + 2(-x+3)$

$= x^2 - 6x + 9 - 2x + 6$

$= x^2 - 8x + 15.$

May / June 2016 Question 4

4.1) $A = \begin{bmatrix} 5 & 2 & -4 \\ 3 & 6 & 1 \\ 2 & 0 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}$

$\therefore A \cdot B = \begin{bmatrix} 0 - 6 - 8 \\ 0 - 18 + 2 \\ 0 + 0 - 10 \end{bmatrix} = \begin{bmatrix} -14 \\ -16 \\ -10 \end{bmatrix}$

4.2) a)

*	a	b	c
a	a	b	c
b	b	b	b
c	c	a	b

b) $(a * b) * c$
$= b * c$
$= b$

$a * (b * c)$
$= a * b$
$= b$

c) No, you would need to check all pairs to see if it is associative

Question 5

5.1) a) $\neg r \vee p = r \rightarrow p$.

b) p = "it is November"
q = "I am sick"
r = "I am lazy"
t = "I write the exam"

$\therefore (p \wedge (\neg q \vee \neg r) \rightarrow t)$.

c) i)

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \to (\neg r)$	x	$q \vee r$	$p \to (q \vee r)$
T	T	T	T	F	F	F	T	T
T	F	F	F	T	T	F	F	F

$x = (p \wedge q) \to (\neg r) \leftrightarrow p \to (q \vee r)$

ii)

p	q	r	$q \to r$	$\neg(q \to r) = x_1$	$x \to p$	x_2	$\neg q$	$\neg q \wedge r$	$p \vee (\neg q \wedge r)$
T	F	F	T	F	T	T	T	F	T
F	T	F	F	T	F	T	F	F	F

$x_2 = \cancel{\neg p}\ \neg(q \to r) \to p \leftrightarrow p \vee (\neg q \wedge r)$

Question 5.2

5.2) a) $\neg(\exists x \in D,\ 2x^2 + 4 > 15)$

$\equiv \forall x \in D,\ 2x^2 + 4 \leq 15$

b) Since the highest value is $f(3)$

$= 2(3)^2 + 4$

$= 2 \cdot 9 + 4$

$= 22.$

The original statement is true (there is one element (3) such that $2x^2 + 4 > 15$).

Question 5.3

5.3) a) $\neg(\forall x \in \mathbb{Z}, [((x-5) \leq -1) \vee (x^2 + 2x \neq 15)]$.

b) $\equiv \exists x \in \mathbb{Z}, \neg[((x-5 \leq -1)\cancel{)} \vee (x^2 + 2x \neq 15)]$.

$\equiv \exists x \in \mathbb{Z}, [\neg(x-5 \leq -1) \wedge \neg(x^2 + 2x \neq 15)]$

$\equiv \exists x \in \mathbb{Z}, [(x-5 > -1) \wedge (x^2 + 2x = 15)]$.

May / June 2016 Question 6

6.1) $x^2 - 3x - 10$

$\therefore (x-5)(x+2)$

$\therefore x^2 - 3x - 10 \leq 0$

~~below~~ for $-2 \leq x \leq 5$.

or: $x = 4$

$\therefore 4^2 - 3(4) - 10$

$= 16 - 12 - 10$

$= -6$

4^2
16
6

6.2) if $n^2 - 5n + 4 < 0$, then $n > 0$.

$\rightarrow$ 6.2 & 6.3 $\rightarrow$ just check practice paper
to exact same questions.