COS 1501 Oct/Nov 2016 Question 1

1) $U = \{\{1\}, 2, a, c, \{c,d\}\}$

$A = \{\{1\}, 2, c\}$

$B = \{c, \{c,d\}\}$

$C = \{2, a, c\}$

1.1) $B \cup C = \{c, \{c,d\}\} \cup \{2, a, c\}$

$= \{2, a, c, \{c,d\}\}$

$= \underline{2}$

1.2) $A \cap B \cap C = \{\{1\}, 2, c\} \cap \{c, \{c,d\}\}$

$= \{c\} \cap \{2, a, c\}$

$= \{c\}$

$= \underline{3}$

1.3) $A - C = \{\{1\}, 2, c\} - \{2, a, c\}$

$= \{\{1\}\}$

$= \underline{1}$

1.4) $B + C = \{c, \{c,d\}\} + \{2, a, c\}$

$= \{2, a, \{c,d\}\}$

$= \underline{4}$

$[B + C = (B \cup C) - (B \cap C)]$

1.5) $C' = U - C$

$= \{\{1\}, 2, a, c, \{c,d\}\} - \{2, a, c\}$

$= \{\{1\}, \{c,d\}\}$

$= \underline{4}$

$A = \{\{1\}, 2, c\}$

1.6) $\therefore P(A) = \{\emptyset, \{\{1\}\}, \{2\}, \{c\}, \{\{1\}, 2, c\}, \ldots\}$

$\underline{\underline{2}}$ (note: it asks for an element in $P(A)$, $\emptyset$ is an element, but $\{\emptyset\}$ is a subset of $P(A)$, that is why 4 is out)

1.7) $P = \{(\{1\}, 2), (2, c), (\{1\}, c)\}$

antisymmetric? ✓

transitive? ✓ $(\{1\}, 2)(2, c) \rightarrow (\{1\}, c)$ ✓

trichotomy? ✓ (all unique pairs in)

$\therefore$ $\underline{\underline{1}}$ (or could just see that the relation is not reflexive and choose 1 automatically)

1.8) $T = \{(2, 2), (a, a), (a, c), (c, 2), (c, c)\}$

on $C = \{2, a, c\}$

reflexive? ✓

antisymmetric? ✓

transitive? × $(a, c)(c, 2) \rightarrow (a, 2)$

$\therefore$ $\underline{\underline{3}}$

Question 2 Oct/Nov 2016

2.1) $(A - B) \cap C$

$= \{1\} - \{1\} \cap \{1, 3\}$

$= \emptyset \cap \{1, 3)$

$= \emptyset = LHS.$

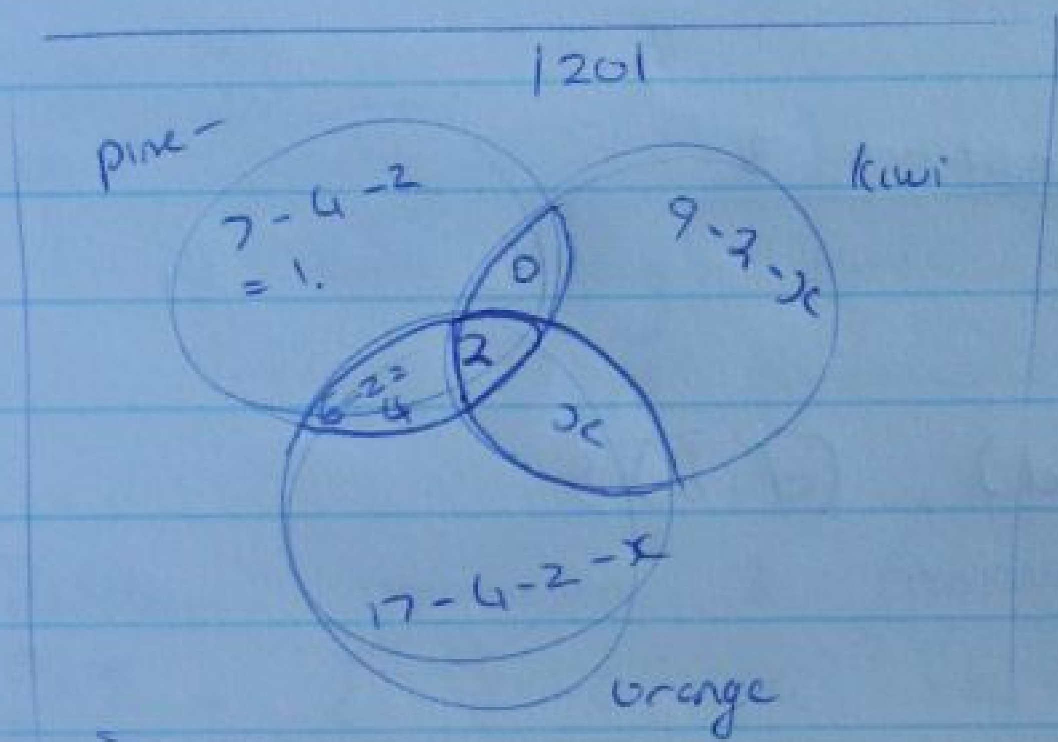
$A \cap (B - C)$

$= \{1) \cap (\{1\} - \{1, 3\})$

$= \{1\} \cap \emptyset$

$= \emptyset = RHS.$

$\therefore$ it is not possible to prove $(A - B) \cap C \neq A \cap (B - C)$, would need to check other sets.

2.2) 

$\therefore$ Kiwis $= 7 - x$ $(= 2)$

Oranges $= 11 - x.$ $(= 6)$

$\therefore 1 + 4 + 2 + x + (11 - x) + (7 - x) = 20$

$\therefore 25 - x = 20$

$\therefore -x = -5$

$x = 5.$

2.2)a) 5 kids like kiwis and oranges

b) $7 - 5 = 2.$ 2 kids only like kiwis.

c) ~~11-5 = 6 : 20-6 = 14.~~ ~~20 - 6 - 5 - 2 - 4~~

~~$= 3 \rightarrow$ 3 kids don't li~~

2.2) c) $20 - (11-5) - 5 - 2 - 4$

$= 3$. $\therefore$ 3 kids don't like oranges.

[note: for 'not like', it is asking for A'.
→ as such, you calculate U - A
(which includes A's intersections].

2.3) $(A \cup B) - C = (A - C) \cup (B - C)$

iff $x \in (A$ or $B)$ ~~and~~ $- C$.

iff $x \in (A$ or $B)$ and $x \notin C$.

iff $x \in (A$ and $x \notin C)$ or $($~~B~~ $x \in B$ and $x \notin C)$.

iff $x \in (A - C)$ or $x \in (B - C)$

iff $x \in (A-C) \cup (B-C)$.

Question 3

3.1) $A = \{a, b, d\}$

$B = \{(a, b), (b, d), (d, b)\}$

$C = \{(b, a)\}$.

3.1) a) i) (a, d) and/or (d, a), can be added to satisfy trichotomy.

ii) strict partial order = irreflexive? ✓

antisymmetric? × $(b,d)(d,b)$

transitive? × $(a,b)(b,d)$ but no (a,d)

$\therefore$ if you remove (b,d) it will be a strict partial order.

if you remove (d,b), you must add (a,d) as well.

iii) $\{(a, a), (d, a)\}$.

Question 3 (cont). Oct/Nov 2016

3.1) b) $S = \{(1,3), (3,3)\}$
$T = \{(1,3), (3,4), (5,3)\}$
on $A = \{1,3,5\}$ to $\{3,4\}$.

i) $(1,4)$ or $(3,4)$ or $(5,4)$
ii) No, T maps two unique domain elements to the same range value.
i.e.: $1 \neq 5$, but $f(1) = 3 = f(5)$
$\therefore$ it is not injective.

3.2) $(x,y) \in R$ iff $y - x = 2k$, $k \in \mathbb{Z}$

i) $y - x = 2k$
and $x - y = 2(-k)$
$\therefore$ both (x,y) and $(y,x) \in R$
$\therefore$ R is symmetric.

ii) Reflexive, symmetric & transitive.
iii) $[0] = \{\ldots -6, -4, -2, 0, 2, 4, 6, \ldots\}$.

3.2) b) $f = y = 2x - 3$ $f: \mathbb{Z} \to \mathbb{Z}$
$g: \mathbb{Z} \to \mathbb{Z} = y = x^2 + 1$

i) f
ii) c
iii) e
iv) b
v) a
vi) d.

$f \circ g = f(g(x))$
$= 2(x^2+1) - 3$
$= 2x^2 - 1.$

Question 4 Oct/Nov 2016

4.1) $A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

i) $A \cdot B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2-3 & 0-1 & 4+1 \\ 3 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 & 5 \\ 3 & 1 & -1 \end{bmatrix}$

ii) $2A + D = \begin{bmatrix} 7 & 1 \\ 3 & 3 \end{bmatrix}$

$\therefore \begin{bmatrix} 7 & 1 \\ 3 & 3 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = D.$

$\therefore \begin{bmatrix} 7 & 1 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 0 & 2 \end{bmatrix} = D$

$\therefore D = \begin{bmatrix} 3 & 3 \\ 3 & 1 \end{bmatrix}$

4.2) a) i) $a \lozenge b = b \lozenge a.$

$= a \neq b$

$\therefore$ it is not commutative

ii) $(a \lozenge b) \lozenge a = a \lozenge (b \lozenge a)$

$a \lozenge a = a \lozenge b$

$c \neq a$

$\therefore$ it is not associative

Question 4.2 (cont) Oct/Nov 2016

4.2) iii) c is the identity element

iv)

$\lozenge$	a	b	c
a	c	b	a
b	b	c	b
c	a	b	c

(change $a \lozenge b$ to ~~a~~ b)
(or change $b \lozenge a$ to a).

Question 5

5) a) i) $p \vee q \equiv \neg(p \wedge \neg q)$

$p \vee q \equiv \neg p \vee \neg q$

$\therefore$ False

ii) $(p \wedge p) \vee q \equiv p \wedge (p \vee q)$

$\therefore (p \vee q) \wedge (p \vee q) \equiv (p \wedge p) \vee (p \wedge q)$

$\therefore$ False

iii) $\neg p \vee \neg(\neg q) \equiv (p \rightarrow q)$

$\therefore \neg p \vee q \equiv \neg p \vee q$

$\therefore$ True

b) i)

p	q	r	$\neg q$	$p \vee q$	$\neg q \vee r$	$(p \vee q) \rightarrow r$	$\Leftrightarrow$	$p \rightarrow (\neg q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	F	F	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	F	F	F	T
F	F	T	T	F	T	T	T	T
F	F	F	T	F	T	T	T	T

ii) it is neither

5.2) $\forall x \in \mathbb{Z}^+, [((x-4) \geq 0) \wedge ((x+2) \geq 0)]$.
$\therefore x \geq 4$ and $x \geq -2$.

a) No, x-values from 1 till 3 (in $\mathbb{Z}^+$) falsify the ~~st~~ statement.

b) i) $\neg(\forall x \in \mathbb{Z}^+, [((x-4) \geq 0) \wedge ((x+2 \geq 0))])$
$\equiv \exists x \in \mathbb{Z}^+, \neg[((x-4 \geq 0) \wedge ((x+2 \geq 0))]$.
$\equiv \exists x \in \mathbb{Z}^+, [\neg(x-4 \geq 0) \vee \neg(x+2 \geq 0)]$.
$\equiv \exists x \in \mathbb{Z}^+, [((x-4 < 0) \vee (x+2 < 0))]$.

ii) Yes, the x-values ~~of~~ 1 satisfies this statement (x values from 1 -> 3 would) (but only need 1 ($\exists x$)).

Question 6

6.1) if n is a multiple of 3, then $2n^2 + 2n + 7$ is odd ($n \in \mathbb{Z}$).

$\therefore n = 3n$.
$\therefore 2(3n)^2 + 2(3n) + 7$
$= 2(9n^2) + 6n + 7$.
$= 18n^2 + 6n + 7$
$\therefore$ Since there is no common factor of 2, $2n^2 + 2n + 7$ is odd. $\square$

6.2) if $3x^2 - 5$ is odd, then x is even
$\equiv$ if x is odd, then $3x^2 - 5$ is even.
$\therefore x = 2n+1$.
$\therefore 3(2n+1)^2 - 5$
$= 3(4n^2 + 4n + 1) - 5$
$= 12n^2 + 12n - 4$
$= 2(6n^2 + 6n - 2)$
$\therefore 3x^2 - 5$ is even. $\square$