Question 1 May / June 2017

$U = \{\{a\}, b, \{b\}, c, d, e\}$

$A = \{\{a\}, b, c\}$, $B = \{b, \{b\}, d, e\}$ $C = \{b, c, d, e\}$

1.1) $A \cup B = \{\{a\}, b, c\} \cup \{b, \{b\}, d, e\}$

$= \{\{a\}, b, \{b\}, c, d, e\}$

$= \underline{4}$

1.2) $(C \cap A) \cap B = \{b, c, d, e\} \cap \{\{a\}, b, c\}$

$= \{b, c\} \cap \{b, \{b\}, d, e\}$

$= \{b\}$

$= \underline{1}$

1.3) $\cancel{A - B = (A \cup B) - (A \cap B)}$

$\cancel{= \{\{a\}, b, \{b\}, c, d, e\} - \{b\}}$

$\cancel{= \{\{a\}, \{b\}, c, d, e\}}$

$\cancel{= \underline{3}}$

1.3) $A - B = \{\{a\}, b, c\} - \{b, \{b\}, d, e\}$

$= \{\{a\}, c\}$

$= \underline{4}$

1.4) $B + C = (B \cup C) - (B \cap C)$

$= \{b, \{b\}, c, d, e\} - \{b, d, e\}$

$= \{\{b\}, c\}$

$= \underline{2}$

1.5 $A' = U - A$

$= \{\{a\}, b, \{b\}, c, d, e\} - \{\{a\}, b, c\}$

$= \{\{b\}, d, e\}$

$= \underline{3}$

$A = \{\{a\}, b, c\}$

1.6) $P(A) = \{\emptyset, \{\{a\}\}, \{b\}, \{c\}, \{\{a\}, b\}, \{\{a\}, c\}, \{b, c\}, \{\{a\}, b, c\}\}$

$= \underline{3}$

$B = \{b, \{b\}, d, e\}$

$R = \{(b,b), (\{b\}, d), (d,e), (e, \{b\})\}$

1.7) antisymmetric? ✓

reflexive? ×

transitive? × $((\{b\}, d)$ & (d,e) but no $(\{b\}, e)$

trichotomy? × $((b, \{b\})$ not present, + more pairs).

$= \underline{1}$

1.8) $U = \{\{a\}, b, \{b\}, c, d, e\}$

for partition : (1) no empty set

(2) Part $\cap$ Part $= \emptyset$

(3) Part $\cup$ Part $= U$.

$\therefore \underline{\underline{2}}$ (no element e, so ~~d~~ union of all partitions $\neq U$)

Question 2 May / June 2017

2.1) You are on your own. Pretty simple, just make sure you know your operators.

2.2) $X - (Y \cap W) = (X - Y) \cup (X - W)$

$x \in X - (Y \cap W)$

iff $x \in X$ and $x \notin (Y \cap W)$

iff $x \in X$ and $x \notin (Y$ and $W)$

iff $x \in X$ and $x \notin Y$ or $x \notin W$

iff $x \in X$ and $x \notin Y$ or $x \in X$ and $x \notin W$

iff $x \in (X - Y)$ or $x \in (X - W)$

iff $x \in (X - Y) \cup (X - W)$

2.3) $X = \{1, 2\}$ $Y = \{2, 3\}$

$(X \cup Y) \times (X \cap Y) = (X \times (X \cap Y)) \cup (Y \times (X \cap Y))$

$\therefore X \cup Y = \{1, 2\} \cup \{2, 3\}$

$= \{1, 2, 3\}$

$X \cap Y = \{1, 2\} \cap \{2, 3\}$

$= \{2\}$

$\therefore \{1, 2, 3\} \times \{2\} = (\{1, 2\} \times \{2\}) \cup (\{2, 3\} \times \{2\})$

$\therefore \{(1,2), (2,2), (3,2) = (\{(1,2), (2,2)\}) \cup \{(2,2), (3,2)\}$

$\therefore \{(1,2), (2,2), (3,2)\} = \{(1,2), (2,2), (3,2)\}$

$\therefore$ LHS = RHS.

2.4) a) $(B \cap C) - (A \cap B \cap C)$

b) $(A \cup B \cup C) - (A \cap B \cap C)$

Question 3

3.1) a) $T = \{(2,2), (2,4), (4,4), (4,6), (6,2), (6,6)\}$

[Trichotomy just needs each unique pair to be present in T, so (2,4) or (4,2), (2,6) or (6,2) and (4,6) or (6,4)]

b) Strict total order = irreflexive, antisymmetric, transitive & trichotomy

$A = \{a, b, c\}$

$\therefore S = \{(a,b), (a,c), (b,c)\}$

irreflexive? ✓

antisymmetric? ✓

transitive? ✓ ((a,b) & (b,c) → (a,c))

trichotomy? ✓ (all unique pairs present (order does not matter for this))

c) For every element $b \in B$ (co-domain), there is an element $a \in A$ (domain) such that

$f(a) = b$.

$\therefore \forall b \, \exists a \, (f(a) = b)$

Or

The range and codomain are identical.

(every element in the codomain is in the range).

d)i) $F = \{(1,2), (2,3), (3,4), (4,4)\}$

d)ii) $P \circ P = \{(1,1), (1,2), (3,2), (3,3)\}$

iii) ~~[illegible]~~ $(1,2)$

Question 3 (cont)

3.2) a) $(x, y) \in R$ iff $y = x^2 - 1$

No it is not transitive.

$y = 2^2 - 1$
$= 3$ $(2, 3)$ is in R

~~yes~~ $y = 3^2 - 1$
$= 8$ $(3, 8)$ is in R

yet $8 \neq 2^2 - 1$, so $(2, 8) \notin R$.
$\therefore$ it is not transitive.

b) $(x, y) \in f$ iff $y = 2x^3 - 3$
$(x, y) \in g$ iff $y = x + 7$

i) $g \circ f(x) = g(f(x))$
$= (2x^3 - 3) + 7$
$= 2x^3 + 4$

ii) ~~f~~ $y = 2 \cdot (2)^3 - 3$
$= 2 \cdot 8 - 3$
$= 16 - 3$
$= 13$
$\therefore (2, 13)$ is in f.

iii) $y = -1 + 7$
$= 6$
$\therefore (-1, 6)$ is in g.

Question 4

4.1) $B = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 7 & 3 \\ 3 & 6 & 2 \end{bmatrix} \quad c = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}$

$B + D = [\ ]$

$\therefore D = \begin{bmatrix} 5 & 1 & 6 \\ 8 & 0 & -1 \\ -7 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 4 \\ -1 & 7 & 3 \\ 3 & 6 & 2 \end{bmatrix}$

$D = \begin{bmatrix} 5 & 0 & 2 \\ 9 & -7 & -4 \\ -10 & -4 & 1 \end{bmatrix}$

$BC = \begin{bmatrix} 0 & 1 & 4 \\ -1 & 7 & 3 \\ 3 & 6 & 2 \end{bmatrix} \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}$

$= \begin{bmatrix} 0 + 4 + 28 \\ -1 + 28 + 21 \\ 3 + 24 + 14 \end{bmatrix}$

$= \begin{bmatrix} 32 \\ 48 \\ 41 \end{bmatrix}$

4.2) a) $(a * b) * c \overset{?}{=} a * (b * c)$

$a * c \overset{?}{=} a * b$

$a = a$

$\therefore$ it cannot be used

b) c is the identity column.

Question 4 (cont)

(4.2) c)

$*$	a	b	c
a	a	a	a
b	b	b	b
c	a	b	c

d) $a*(b*(b*c)) = (c*a)*(a*b)$

$a*(b*b) = a*a$

$a*b = a$

$a = a.$

Question 5

5.1) a) i) $(\neg p \vee q) \wedge p \equiv \neg p \vee (q \wedge p)$

$\equiv (p \wedge \neg p) \vee (p \wedge q) \not\equiv (\neg p \vee p) \wedge (\neg p \vee q)$

$= F$

ii) $\neg(\neg(\neg p)) \rightarrow q \equiv p \vee q$

$\equiv \neg p \rightarrow q$

$\equiv \neg(\neg p) \vee q$

$\equiv p \vee q$

$= T$

iii) $(\neg q \vee \neg q) \vee p \equiv q \rightarrow p$

$\cancel{\equiv (p \vee \neg q)}$

$\equiv \neg q \vee (\neg q \vee p)$

$\equiv \neg q \vee (q \rightarrow p)$

$= F.$

b) p = "I am a uni student"
q = "reg. for COS"
r = "writing on 15 may 2023"

$p \wedge \neg q \rightarrow \neg r$

c) i)
ii) } on your own here, too tedious to write out, & pretty simple

5.2) $\forall x \in \mathbb{Z}^+, [((x+2) \geq 2) \wedge ((x-2) \geq 0)]$

~~[illegible] $\mathbb{Z}^+$~~

a) No, since ~~[illegible]~~ $(1+2) \geq 2 \wedge (1-2) \geq 0$ is false ($-1 \geq 0$ is untrue)

b) $\neg[\forall x \in \mathbb{Z}^+, [((x+2) \geq 2) \wedge ((x-2) \geq 0)]]$
$\equiv \exists x \in \mathbb{Z}^+, \neg((x+2) \geq 2 \wedge (x-2) \geq 0))$
$\equiv \exists x, \in \mathbb{Z}^+, (\neg(x+2) \geq 2 \vee \neg(x-2) \geq 0))$
$\equiv \exists x, \in \mathbb{Z}^+, ((x+2) < 2 \vee (x-2) \leq 0)$

Question 6

6.1) If $n+1$ is a multiple of 3
then $n+1 = 3k$
$\therefore n = 3k-1$
$\therefore (3k-1)^2 + 3(3k-1) - 1$
$= 9k^2 - 6k + 1 + 9k - 3 - 1$
$= 9k^2 + 3k - 3$
$= 3(3k^2 + 1 - 1)$
$\therefore n^2 + 3n - 1$ is a multiple of 3.

Question 2

6.2) Let $x = 2$

then: $2^3 + 5(2) - 9$

$= 8 + 10 - 9$

$= 9 \geq 0$

6.3) a) if x is a multiple of 2 then $4x^2 + x + 2$ is a multiple of 2

b) if x is not a multiple of 2 $\rightarrow$ $4x^2 + x + 2$ is not a multiple of 2

6.4) Suppose $x^2 + 8x + 14$ is odd

then x is even

assume $x = 2n$

$\therefore (2n)^2 + 8(2n) + 14$

$= 4n^2 + 16n + 14$

$= 2(2n^2 + 8n + 7)$

However this shows it is even,

$\therefore$ x must be odd.