Question 1 Oct/Nov 2017

$U = \{\{1\}, 2, \{1,2\}, a, b, c\}$

$A = \{\{1\}, a, c\}$

$B = \{\{1\}, \{1,2\}, b, c\}$

$C = \{2, c\}$

1.1) $A \cup C = \{\{1\}, a, c\} \cup \{2, c\}$

$= \{\{1\}, 2, a, c\}$

$= \underline{3}$

1.2) $B \cap A = \{\{1\}, \{1,2\}, b, c\} \cap \{\{1\}, a, c\}$

$= \{\{1\}, c\}$

$= \underline{1}$

1.3) $C - B = \{2, c\} - \{\{1\}, \{1,2\}, b, c\}$

$= \{2\}$

$= \underline{2}$

1.4) $A + C = \{\{1\}, a, c\} + \{2, c\}$

$= \{\{1\}, 2, a\}$

$= \underline{4}$

~~1.5) $(A \cup C)'$ $\{\{1\}, a\}$ $\{\{1\}, 2, \{1,2\}, a, b, c\}$ $- \{\{1\}, c\}$ $= \{2, \{1,2\}, a, b\}$~~

1.5) $(A \cup C)' = \{\{1\}, 2, \{1,2\}, a, b, c\} - \{\{1\}, 2, a, c\}$

$= \{\{1,2\}, b\}$

$= \underline{1}$

1.6) $U = \{\{1\}, 2, \{1,2\}, a, b, c\}$

$\therefore$ $\underline{3}$

↳ should be $\{\{1,2\}, \{\{1\}\}, \ldots\}$

not $\{1\}$

1.7) $T = \{(a,c), (\{1\}, a), (c,c), (c, \{1\})\}$

on $A = \{\{1\}, a, c\}$

$\therefore$ reflexive? × → $(\{1\}, \{1\})$ not there

symmetric? × → (a,c) but no (c,a)

irreflexive? × → (c,c) present

$\therefore$ $\underline{4}$

1.8) $(A + C) \cap B' = (\{\{1\}, a, c\} + \{2, c\})$

$= \{\{1\}, 2, a\} \cap \{2, a\}$

$= \{2, a\}$

$\therefore |\{2, a\}| = 2$ #

$\therefore$ $\underline{1}$

Question 2

2.1) $(A - B) \cup (C - B) \neq (A \cap C) - B$

$= (\{1,2\} - \{2\}) \cup (\{2,3\} - \{2\}) \neq (\{1,2\} \cap \{2,3\}) - \{2\}$

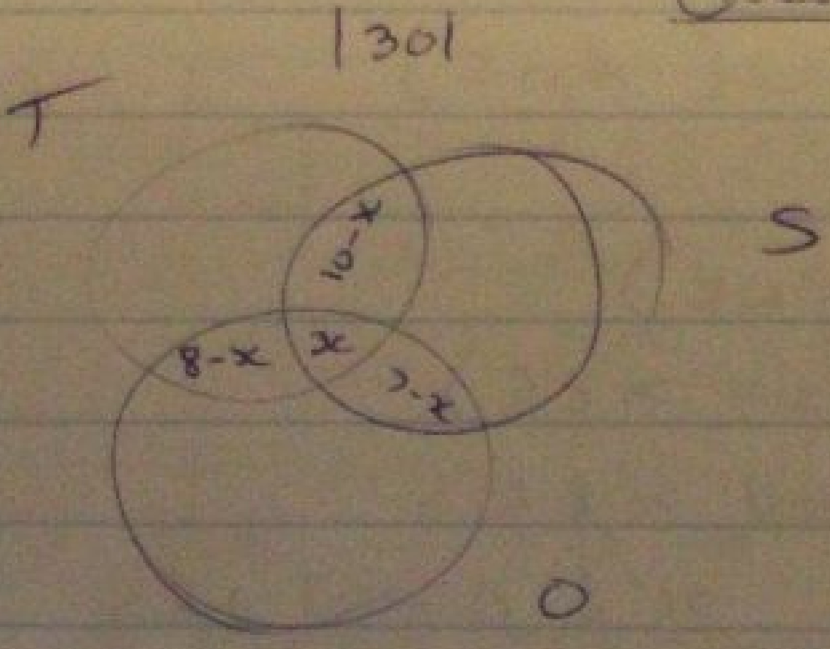
$= \{1\} \cup \{2\} \neq \{2\} - \{2\}$

$= \{1, 2\} \neq \emptyset$

$\therefore$ with the provided sets, you can prove it.

Question 2 (cont)

2.2) i)



$\therefore T = 13 - x - (10-x) - (8-x)$

$= 13 - 10 - 8 - x + x + x$

$= -5 + x$

$S = 20 - x - (10-x) - (7-x)$

$= 20 - 10 - 7 - x + 2x$

$= 3 + x$

$O = 17 - x - (8-x) - (7-x)$

$= 17 - 8 - 7 - x + 2x$

$= 2 + x$

$\therefore 30 = x + (8-x) + (7-x) + (10-x) + (-5+x) + (3+x) + (2+x)$

$= x + 8 + 7 + 10 + 3 + 2 - 5$

$= x + 25$

$\therefore x = 5$

ii) 5 prefer all 3.

iii) $T = -5 + 5$

$= 0$ 0 people prefer T only

iv) Study & Online $= 7 - x = 7 - 5$

$= 2.$

$\therefore$ 2 people would like that.

2.3) $A \cap (B' \cup C) = (A - B) \cup (A \cap C)$

$x \in A \cap (B' \cup C)$

iff $x \in A$ and $x \in (B' \cup C)$

iff $x \in A$ and $x \in (B'$ or $x \in C)$

iff $x \in A$ and $x \notin B$ and $x \in C$

iff $x \in A$ and $x \in B$ or $x \in A$ and $x \in C$

iff $x \in (A - B)$ or $x \in (A \cap C)$

iff $x \in (A - B) \cup (A \cap C)$

Question 3

3.1) a) $T = \{(c,1), (c,c), (a,b), (2,1), (2,c)\}$

$\therefore T \circ T = \{(c,1), (c,c), (2,1), (2,c)\}$

b) (c,c) should be removed.

3.2) a) $A = \{1, 2\}$, $B = \{2, a, b\}$

$\therefore f: A \to B = \{(1,2), (2,a)\}$

b) i) bijective

ii) surjective

iii) injective

3.3) a) for R to be symmetric, $y = -x + 4$ must equal $x = -y + 4$

$\therefore y = -x + 4$

$\therefore$ ~~$y + x \quad y = -x + 4$~~

$\therefore y + x = 4$

$\therefore x = -y + 4$

$\therefore (x, y)$ and (y, x) is $\in R$ ($\therefore$ symmetric)

Question 3 (cont.)

3.3) b) i) $f \circ g = f(g(x))$

$= 3(x^3+1) + 2$

$= 3x^3 + 5$

ii) $\forall a \forall b \,.\, (a \neq b \rightarrow f(a) \neq f(b))$

~~or $\forall a \forall b\, (a = b \rightarrow f(a) = f(b))$~~

or $\forall a \forall b\, (f(a) = f(b) \rightarrow a = b)$

Since $f(x) = 3x+2$ produces a unique value for every unique input, $f(a) = f(b)$ only when $a = b$.

$\therefore f$ is injective.

iii) Since g is not surjective on $\mathbb{Z}^+$, there exists an element in the codomain, not in the range.

$\forall b\, \exists a\, (f(a) \neq b)$

$\therefore$ ~~ran~~ $ran(g) = y = x^3 + 1$

$= x^3 = y - 1$

$= x = \sqrt[3]{y-1}$

$ran(g) = \{ y \mid x = \sqrt[3]{y-1} \wedge x \in \mathbb{Z}^+ \}$

$\therefore x = \sqrt[3]{0-1}$

$= \sqrt[3]{-1}$

$\therefore x = -1$

Since $x \in \mathbb{Z}^+$, this is not possible.

$\therefore g$ is not surjective.

iv) $f(2) = 3(2) + 2$

$= 6 + 2$

$= 8$

$\therefore (2, 8) \in f$.

Question 4

4.1) $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$

a) $A \cdot B = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 2+9 & 4+12 & 10+18 \end{bmatrix}$

$= \underline{\begin{bmatrix} 11 & 16 & 28 \end{bmatrix}}$

b) $2A - B = \begin{bmatrix} -2 & 4 & 0 \\ 0 & 2 & 6 \\ -4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & -2 \\ -3 & 1 & 0 \\ 2 & 2 & -3 \end{bmatrix}$

$= \begin{bmatrix} -2 & 3 & 2 \\ 3 & 1 & 6 \\ -6 & 4 & 5 \end{bmatrix}$

4.2)

a)

*	a	b	c
a	b	a	b
b	a	b	c
c	c	c	c

b) $(a*c)*b = a*(c*b)$

$\therefore b*b = a*(c)$

$\therefore b = b$

no it cannot be used as a counter example, need to check all pairs.

Question 5

i) $(\neg p \to q) \equiv (\neg q \to p)$

$\equiv \neg(\neg p) \vee q \equiv \neg(\neg q) \vee p$

$\equiv p \vee q \equiv q \vee p$

$p \vee q \equiv p \vee q \quad \therefore T.$

Question 5 (cont.)

a) ii) $(p \vee \neg q) \wedge r \equiv p \vee (\neg q \wedge r)$

$= (p \wedge r) \vee (p \wedge \neg q) \overset{?}{\equiv} (p \vee r) \wedge (p \vee \neg q)$

$\therefore$ F.

iii) $\neg(\neg(p \wedge \neg r)) \equiv \neg(r \vee \neg p)$

$= \neg(\neg p \vee r)$

$= \neg(r \vee \neg p) \equiv \neg(r \vee \neg p)$

$\therefore$ T.

b) i) on your own.

5.2) a) $\forall x \in \mathbb{Z}, [(2x+3 > 0) \vee (3x-2 < 0)]$

Statement is false iff $2x+3 < 0$ and $3x-2 > 0$

$\therefore 2x < -3$ and $3x > 2$

$x < -\frac{3}{2}$ and $x > \frac{2}{3}$

Since x cannot be less than $-\frac{3}{2}$ and greater than $\frac{2}{3}$,

Statement is true.

b) $\exists x \in \mathbb{Z}, \neg[(2x+3 > 0) \vee (3x-2 < 0)]$

$\exists x \in \mathbb{Z}, [\neg(2x+3 > 0) \wedge \neg(3x-2 < 0)]$

$\exists x \in \mathbb{Z}, [(2x+3 \leq 0) \wedge (3x-2 \geq 0)]$

c) No, $x = 0$ is a counter example

$3 \leq 0 \wedge -2 \geq 0$

Question 6

6.1) a) if $n^3+1 < 0$, then n is odd

b) if n is even, then $n^3 - 7n^2 + 4$ is odd.

6.2) Assume $x = n+1$ (n is odd)

then $n = x-1$

i.e: $4(x-1)^2 + 2(x-1) + 7$

$= 4(x^2 - 2x + 1) + 2x - 2 + 7$

$= 4x^2 - 8x + 4 + 2x + 5$

$= 4x^2 - 8x + 9$

$= 2(2x^2 - 4x + 4) + 1$ $\quad (9 = 8+1$
$= (2(4)+1)$

$\therefore 4n^2 + 2n + 7$ is odd (form of $n+1$)