Tutorial letter 201/2/2018

Theoretical Computer Science 1

COS1501

Semester 2

School of Computing

This tutorial letter contains

a discussion of assignment 01, and
examination information.

Dear Student,

In this tutorial letter the solutions to the first assignment questions are discussed and examination information is provided.

Regards,
COS1501 Team

ASSIGNMENT 01 (SEMESTER 2)

Consider the following sets, where U represents a universal set:
U = {1, 2, {1}, {1, 2}, b, {c}} , A = {1, 2, {c}} , B = {{1, 2}, b}, C = {2, {1}, {c}}.
Questions 1 to 10 are based on the sets defined above.
Note: The Venn diagrams in study unit 4 will help you to understand the definitions in study unit 3.

Question 1 **Alternative 3**

A ∪ B
= {1, 2, {c}} ∪ {{1, 2}, b}
= {1, 2, {1, 2}, b, {c}}.

Set union: The elements 1, 2, {1, 2}, b and {c} belong to A or B.

Refer to study guide, p 41.

Question 2 **Alternative 1**

A ∩ C = {1, 2, {c}} ∩ {2, {1}, {c}}
= {2, {c}}

Intersection: The elements 2 and {c} belong to A and C.

Refer to study guide, p 42.

Question 3 **Alternative 3**

B – C = {{1, 2}, b} – {2, {1}, {c}}
= {{1, 2}, b}

Set difference: The elements {1, 2} and b belong to B but not to C.

Refer to study guide, pp 42, 43.

Question 4 **Alternative 2**

B′ = U − B = {1, 2, {1}, {1, 2}, b, {c}} − {{1, 2}, b} = {1, 2, {1}, {c}}

Set complement: The elements 1, 2, {1} and {c} belong to U but not to B.

Refer to study guide, p 43.

Question 5 **Alternative 4**

A + C
= {1, 2, {c}} + {2, {1}, {c}}
= {1, {1}}.

Set symmetric difference: The elements 1 and {1} belong to A or to C, but not both.
It is also the case that A + C = (A ∪ C) – (A ∩ C), so

A + C = (A ∪ C) − (A ∩ C) *Include elements belonging to A ∪ C but not to A ∩ C.*
= ({1, 2, {c}} ∪ {2, {1}, {c}}) − ({1, 2, {c}} ∩ {2, {1}, {c}})
= {1, 2, {1}, {c}} − {2, {c}}
= {1, {1}}

Refer to study guide, pp 43, 44.

Question 6 **Alternative 2**

The number of elements in a set is called the cardinality of the set.
The set U = {1, 2, {1}, {1, 2}, b, {c}} has 6 elements namely 1, 2, {1}, {1, 2}, b and {c}, so |U| = 6; and the set C = {2, {1}, {c}} has 3 elements namely 2, {1} and {c}, so |C| = 3.

Thus |U - C| = 6 – 3 = 3.

Refer to study guide, p 44.

Question 7 **Alternative 4**

Let's look at the definition of a subset:
For all sets F and G, F is a subset of G if and only if every element of F is also an element of G.
Subsets of G can be formed by ***keeping the outside brackets*** *of G and then throwing away* ***none***, ***one*** *or* ***more*** *elements of G.*

Consider U = {1, 2, {1}, {1, 2}, b, {c}}. We investigate whether or not the sets in the different alternatives are subsets of U:

Alternative 1: {{1, 2}, {2, {1}}} is **not** a subset of U. We provide a counterexample:

{2, {1}} ∈ {{1, 2}, {2, {1}}} but {2, {1}} ∉ U, therefore {{1, 2}, {2, {1}}} ⊈ U.

Alternative 2: {{b, {c}}, {1, 2}} is **not** a subset of U. We provide a counterexample:

$\{$b, {c}$\}$ ∈ $\{\{$b, {c}$\}$, {1, 2}$\}$ but $\{$b, {c}$\}$ ∉ U, therefore $\{\{$b, {c}$\}$, {1, 2}$\}$ ⊈ U.

Alternative 3: {1, 2, b, c} is **not** a subset of U. We provide a counterexample:

c ∈ {1, 2, b, c} but c ∉ U, therefore {1, 2, b, c} ⊈ U.

Alternative 4: If we throw away elements 2, b and {c} of set U, we are left with elements 1, {1} and {1, 2} that belong to the subset $\{$1, {1}, {1, 2}$\}$ of U, thus $\{$1, {1}, {1, 2}$\}$ ⊆ U.

From the arguments provided we can deduce that alternative 4 should be selected.

Refer to study guide, p 40.

Question 8 **Alternative 3**

We first determine the cardinality of Ƥ(B).

The number of elements in a set is called the cardinality of the set.
The set B = {{1, 2}, b} has 2 elements namely {1, 2} and b, i.e. |B| = 2.

We determine the cardinality of Ƥ(B):

|Ƥ(B)| = 2^n (|Ƥ(B)| is the cardinality of Ƥ(B), and n is the number of elements in B.)
= 2^2
= 4

Thus the cardinality of Ƥ(B) is 4.

The elements of Ƥ(B) are all the subsets of B.

We form the subsets of B:

Throw away no element of set B, then {{1, 2}, b} ⊆ B;
throw away the element {1, 2} of set B, then {b} ⊆ B;
throw away the element b of set B, then {{1, 2}} ⊆ B;
throw away all the elements of set B, then { } ⊆ B.

*If B = {{1, 2}, b} then the **subsets** of B, namely*
{{1, 2}, b};
{b};
{{1, 2}}; and
*{ } are all the **elements** of Ƥ(B), thus*

Ƥ(B) = { {{1, 2}, b}, {b}, {{1, 2}}, { } }.

Alternatively, if we remove the outer brackets of Ƥ(B), we are left with all the elements of Ƥ(B).

We now consider the alternatives:

Alternative 1: Are {1, 2} and b elements of *P(B)*? No. {1, 2} and b are elements of the subset {{1, 2}, b} of B, which is an element of *P(B).* Remember that an element of any set can in turn be a set containing elements. So it is true that {1, 2} ∈ {{1, 2}, b} and b ∈ {{1, 2}, b}, but {1, 2} ∉ *P(B)* and b ∉ *P(B)*.

Alternative 2: If we remove the outer brackets of *P(B)*, is {{{1, 2}, b}}, one of the *elements* of *P(B)*? No. This alternative provides a set containing the element {{1, 2}, b} which happen to be one of the elements of *P(B)*, ie {{1, 2}, b} ∈ *P(B)*, but {{{1, 2}, b}} ∉ *P(B)*.

Alternative 3: If we remove the outer brackets of *P(B)*, we find that the elements {{1, 2}} and {b} are indeed elements of *P(B)*, ie {{1, 2}} ∈ *P(B)* and {b} ∈ *P(B)*.

Alternative 4: {∅} is a set containing the empty set ∅ as its only element. Remember that ∅ = { }. Although it is true that ∅ ∈ *P(B)* (or { } ∈ *P(B)*), {∅} ∉ *P(B).*

From the arguments provided we can deduce that alternative 3 should be selected.

Refer to study guide, pp 36, 38, 40, 45.

Question 9 **Alternative 1**

Let's first discuss ***proper subsets****:*

If we throw away ***one*** *or* ***more*** *elements of some set G, then the resulting set (let's call it F) is a proper subset of G. So every element of F is also an element of G, but set F has fewer elements than set G. We may write F ⊂ G.*

We determined in Question 8 that if B = {{1, 2}, b} then the ***subsets*** *of B, namely*

{{1, 2}, b};

{b};

{{1, 2}};

{ } are all the ***elements*** *of P(B), thus*

P(B) = { {{1, 2}, b}, {b}, {1, 2}, { } }.

We consider the sets in the different alternatives:

Alternative 1: If we throw away the elements {b}, {1, 2} and { } from P(B), we are left with the subset {{{1, 2}, b}}, that is provided in this alternative. It is indeed true that {{{1, 2}, b}} ⊂ *P(B)*, because the only element in $\{\{\{1, 2\}, b\}\}$ is also an element of $\mathcal{P}(B)$, and $\{\{\{1, 2\}, b\}\}$ has fewer elements than $\mathcal{P}(B)$.

Alternative 2: $\{\{1, 2, b\}\}$ is not a subset of $\mathcal{P}(B)$. We provide a counterexample:
$\{1, 2, b\} \in \{\{1, 2, b\}\}$ but $\{1, 2, b\} \notin \mathcal{P}(B)$, therefore $\{\{1, 2, b\}\} \not\subset \mathcal{P}(B)$.

Alternative 3: The set $\{\{\ \}, \{\{1, 2\}\}, \{b\}, \{\{1, 2\}, b\}\}$ provided in this alternative is exactly the powerset of B, $\mathcal{P}(B)$, that we determined in Question 8. Although $\mathcal{P}(B) \subseteq \mathcal{P}(B)$, *ie* $\mathcal{P}(B)$ is a subset of itself, is not true that $\mathcal{P}(B)$ is a proper subset of itself. Remember that a proper subset of some set has *fewer* elements than the set. Therefore $\mathcal{P}(B) \not\subset \mathcal{P}(B)$.

Alternative 4: {b} is not a *subset* of $\mathcal{P}(B)$ but {b} is an *element* of $\mathcal{P}(B)$. We can provide a counterexample: $b \in \{b\}$, but $b \notin \mathcal{P}(B)$. Therefore $\{b\} \not\subset \mathcal{P}(B)$. Can you see that {{b}}, containing the element {b}, would indeed be a proper subset of $\mathcal{P}(B)$?

From the arguments provided we can deduce that alternative 1 should be selected.

Refer to study guide, pp 40-41, 44, 45; Tutorial Letter 102, p 9.

Question 10 **Alternative 4**

We have U = {1, 2, {1}, {1, 2}, b, {c}}. We determine which alternative provides a set D such that D + U = {{1, 2}, {c}}:

Alternative 1:
By the definition of symmetric difference, the elements of D + U = {{1, 2}, {c}}, namely {1, 2} and {c} belong to D or to U but not both.

In the question it is stated that D + U = D + {1, 2, {1}, {1, 2}, b, {c}} = {{1, 2}, {c}}.
Let D = {{1, 2, {1}, {1, 2}, b, {c}},
then D + U = {{1, 2, {1}, {1, 2}, b, {c}} + {1, 2, {1}, {1, 2}, b, {c}} = { },
but { } ≠ {{1, 2}, {c}}.
All the elements that belong to D, also belong to U, therefore no element belongs to only one of these sets. Thus alternative 1 does not provide the required result.

Alternative 2:
We should determine whether D + U = {{1, 2}, {c}}.
Let D = {1, {1, 2}, {c}},

then D + U = {1, {1, 2}, {c}} + {1, 2, {1}, {1, 2}, b, {c}} = {2, {1}, b}. The elements 2, {1} and b belong to either D or U, but not both,
but {2, {1}, b} ≠ {{1, 2}, {c}}, thus alternative 2 does not provide the required result.

Alternative 3:
We should determine whether D + U = {{1, 2}, {c}}.
Let D = {{1, 2}, {c}},
then D + U = {{1, 2}, {c}} + {1, 2, {1}, {1, 2}, b, {c}} = {1, 2, {1}, b}. The elements 1, 2, {1} and b belong to either D or U, but not both,
but {1, 2, {1}, b} ≠ {{1, 2}, {c}}, thus alternative 3 does not provide the required result.

Alternative 4:
We should determine whether D + U = {{1, 2}, {c}}.
Let D = {1, 2, {1}, b},
then D + U = {1, 2, {1}, b} + {1, 2, {1}, {1, 2}, b, {c}} = {{1, 2}, {c}},
so {{1, 2}, {c}} is the required set for D + U.

It is also the case that D + U = (D ∪ U) – (D ∩ U), so

D + U = (D ∪ U) − (D ∩ U) *Include elements belonging to D ∪ U but not to D ∩ U.*
= ({1, 2, {1}, b} ∪ {1, 2, {1}, {1, 2}, b, {c}}) –
({1, 2, {1}, b } ∩ {1, 2, {1}, {1, 2}, b, {c}})
= {1, 2, {1}, {1, 2}, b, {c}} − {1, 2, {1}, b}
= {{1, 2}, {c}}

From the arguments provided we can deduce that alternative 4 should be selected.

Refer to study guide, p 43.

EXAMINATION INFORMATION

The examination paper will test you on **study units 3 – 10** of the study guide, and the material in **all** the tutorial matter. **Study units 1 – 2** of the study guide provide background material.

Bear in mind that the assignment questions do not cover all the work that we test in the examination paper. You have to prepare **all the work** prescribed above.

It will be expected of you to write down the answers to all the examination questions and provide proofs where required.

PLEASE NOTE: The examination paper will be a fill-in paper as from 2017 onwards. You will receive a notification when a practice fill-in paper with solutions is posted on myUnisa. Please work through this paper, especially if you have never written a fill-in exam.

How to prepare:
Cover all the study units in the study guide thoroughly and **test yourself** by doing the activities before you look at the solutions that are supplied as part of the learning units, as well as under *Additional Resources* on myUnisa.

Work through the 2009 examination paper along with the solutions provided in the MO001 tutorial letter. It is very important that you also do all the self-assessment questions provided in assignments 02 and 03, and take note of the hints provided since these hints will help you to avoid making common errors in the examination. Make sure that you understand the model solutions to **all assignment questions** provided in tutorial letters.

Take note of the **structure and notation of solutions** provided in all tutorial matter. For example, when a proof is required and connectives such as "**iff**" or "**if…then**" are left out, the proof is not convincing. Also, **symbols** (e.g. "$\cap$") should be used as **connectives** for **sets** (e.g. $Y \cap W$), and **words** (e.g. "and") should be used as **connectives** in **sentences** (e.g. $x \in Y$ **and** $x \in W$). Some proofs should start with the word "**assume**" and then logic reasoning should follow. These and other notation issues are mentioned in the hints provided in the assignments.

The examination, and the supplementary examination that follows in the examination period of the following semester, will have a structure and format similar to the practice exam paper, and similar type of questions as in the previous years' exam papers available on myUnisa and the 2009 exam paper for which the solution is given as part of the learning units. The order of the sections will be in line with the order in which the concepts appear in the study guide. We do not repeat questions from previous exam papers, so please make sure that you understand the content. Your e-tutor is available to assist with a discussion of any old exam paper if you post your answer for a specific question that you have problems with, on the discussion forum.

Additional advice:

Venn diagrams: Draw your diagrams in stages as described on page 51 of the study guide. Also remember to draw the sets within the context of a universal set, name the sets and provide subscripts for the diagrams.

Relations and functions: You need to **apply** the definitions, not merely give them. For example, when you want to prove that a relation is functional and you only write the statement "for every *x* there is only one *y*", you will receive no marks since it is neither a properly formulated definition nor a proof. **Relate each answer to the actual definition of the specific relation or function** given in the question.

When solving problems, don't forget the useful **shortcut notations** that help you to express an English sentence in precise mathematical notation. For example, an even number can be expressed as $2k$, an odd number as $2k + 1$, a multiple of three as $3k$ and so on, and two consecutive numbers could be k and $k + 1$, with $k \in Z$.

Truth tables: Use the notations T and F for *true* and *false*, and when three declarative statements (e.g. p, q and r) are involved, use the same order for the combinations of T and F that is used in the table on page 144 of the study guide.

Mathematical proofs and counterexamples: If you are required to provide a mathematical proof and you give an example instead of a general proof, you will receive no marks for that answer. In other words, **never** attempt to prove that something is true by using an example. However, you **should use a counterexample** to prove that something is not true. If you are required to give a **counterexample** and you attempt to give some mathematical proof instead of a **counterexample**, you will receive no marks for that answer.

Note: Read all the hints provided in tutorial letter 101. These hints will help you to avoid making general mistakes in the examination.