Tutorial letter 201/2/2017

Quantitative Modelling 1
DSC1520

Semesters 2

Department of Decision Sciences

Solutions to Assignment 1
Dear Student

This tutorial letter contains the solutions to the first compulsory assignment. Please contact me if you have any questions or need any help with the next assignment.

Kind regards

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**Question 1**

We find

\[ t = \frac{V + 1}{V - 5} \]

and when, for example \( V = 5,6 \), then \( t = 11 \). [Option 4]

**Question 2**

We need to find the consumer surplus for demand function

\[ P = 58 - 0.4Q \]

when the market price \( P = 10 \).

From the textbook page 128, we know that the consumer surplus is calculated as

\[ CS = \text{Amount willing to pay} - \text{Amount actually paid}. \]

This is determined by calculating the area of the triangle \( P_0E_0a \) in the following graph which is equal to

\[ \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times Q_0 (a - P_0) \]

with

\( P_0 \) the market price,

\( Q_0 \) the number of units demanded at price \( P_0 \), and

\( a \) the \( y \)-intercept of the demand function \( P = a - bQ \).
In general we can summarise the steps of determining the consumer surplus as follows:

**Method:**

1. Calculate $Q_0$ if $P_0$ is given.
2. Draw a rough graph of the demand function.
3. Read the value of $a$ from the demand function, that is the $y$-intercept of the demand function.
4. Calculate the area of $CS = \frac{1}{2} \times Q_0 \times (a - P_0)$.

First we need to determine $Q$ from the demand function

$$P = 58 - 0.4Q$$

if $P = 10$. That is

$$10 = 58 - 0.4Q$$

Giving

$$Q = 120.$$ 

Next we draw the demand function.

The value of $a$ is found by substituting $Q = 0$ into the demand function, that is

$$P = 58 - bQ = 58.$$ 

The consumer surplus is the area of the shaded triangle in the sketch, that is

$$CS = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 120 \times (58 - 10) = 2880.$$ 

The consumer surplus is equal to 2880 if the price $P$ is equal to 10.

[Option 3]
Question 3

Demand function: \[ P_d = 50 - 0.6Q \]
Supply function: \[ P_s = 20 + 0.4Q \]

Equilibrium is the price and quantity where the demand and supply functions are equal, or the point where the lines of the demand and supply function intersect.

Therefore we need to determine the value of \( P \) and \( Q \) for which

\[ P_d = P_s \]

or

\[ Q_d = Q_s. \]

Now given are

\[ Q = 50 - 0.5P \text{ and } Q = 10 + 0.5P. \]

Thus:

\[ P_d = P_s \]

\[ 50 - 0.6Q = 20 + 0.4Q \]
\[ -0.6Q - 0.4Q = 20 - 50 \]
\[ Q = 30. \]

To calculate the quantity at equilibrium, we substitute the value of \( Q \) into the demand or supply function and calculate \( P \). Say we use the demand function, then:

\[ P_d = 50 - 0.6Q \]
\[ = 50 - 0.6(30) \]
\[ = 32. \]

The equilibrium price is equal to 32 and the quantity is 30.

[Option 2]

Question 4

If subsidy is R4, then the price that the supplier receives, increases by 4, that is

\[ P_s + 4 = 20 + 0.5Q \]

so that

\[ P_s = 16 + 0.4Q. \]

Again set \( P_d = P_s \) and solve \( Q \):

\[ 50 - 0.6Q = 16 + 0.4Q \]

gives \( Q = 34 \). Substituting this gives

\[ P = 16 + 0.4(34) = 29.60. \]

[Option 2]
Question 5

From

\[ Q = 80 - 2.5P \]

it follows that

\[ P = 32 - 0.4Q \]

giving \( a = 32 \) and \( b = 2.5 \). At \( P = 20 \),

\[ \varepsilon_d = \frac{P}{P - a} = \frac{20}{20 - 32} = -1.667. \]

\[ | - 1.7 | = 1.7 > 1 \]

Therefore, demand is elastic at \( P = 20 \). [Option 1]

Question 6 [Option 2]

Question 7

(1) + (2): \( 3x + y = 7 \) \hspace{1cm} (4)

(2) − (3): \( 2x - 2y = 0 \) and therefore \( x = y \)

\[ 2x - 14 + 6x = 0 \]

giving \( x = 1.75 \)

substitute \( x = y \) into (4):

\[ 4y = 7, y = \frac{7}{4} = 1.75 = x \]

From (3):

\[ z = 2 - y = 2 - 1.75 = 0.25 \]

Hence:

\[ x + y + z = 3.75 \] [Option 4]

Question 8 [Option 3]

Question 9 [Option 2]
Question 10

\[
PS = \frac{1}{2} \times \text{base} \times \text{height} \\
= \frac{1}{2} \times 90 \times (230 - 50) \\
= 8100.
\]

[Option 2]

Question 11

[Option 4]

Question 12

[Option 3]

Question 13

Total cost \( TC = 15X + 25000 \), \( TR = 45X \) and profit \( \pi = 45X - (15X + 25000) = 30X - 25000 \). [Option 1]

Question 14

Slope

\[
m = \frac{5000 - 3000}{250 - 300} = \frac{2000}{-50} = -4000.
\]

Using (300; 3000):

From \( Q = mP + c \) we find

\[
3000 = -4000(300) + c
\]

giving

\[
c = 15000.
\]

Therefore

\[
Q = -4000P + 15000.
\]

[Option 2]

Question 15

Let \( Q \) represent bracelets. \( TR = 60Q \), \( TC = 30Q + 900 \). At break-even, \( TR = TC \), therefore,

\[
60Q = 30Q + 900 \text{ giving } 30Q = 900 \text{ or } Q = 30.
\]

[Option 3]