Tutorial Letter 202/1/2015

Nonlinear mathematical programming

DSC2606

Semester 1

Department of Decision Sciences

Solutions: Assignment 02

This tutorial letter contains solutions to the questions in Assignment 02.

Bar code

Dear Student

Here are the solutions to the second compulsory assignment.

You are welcome to contact me if you have any queries.

Kind regards

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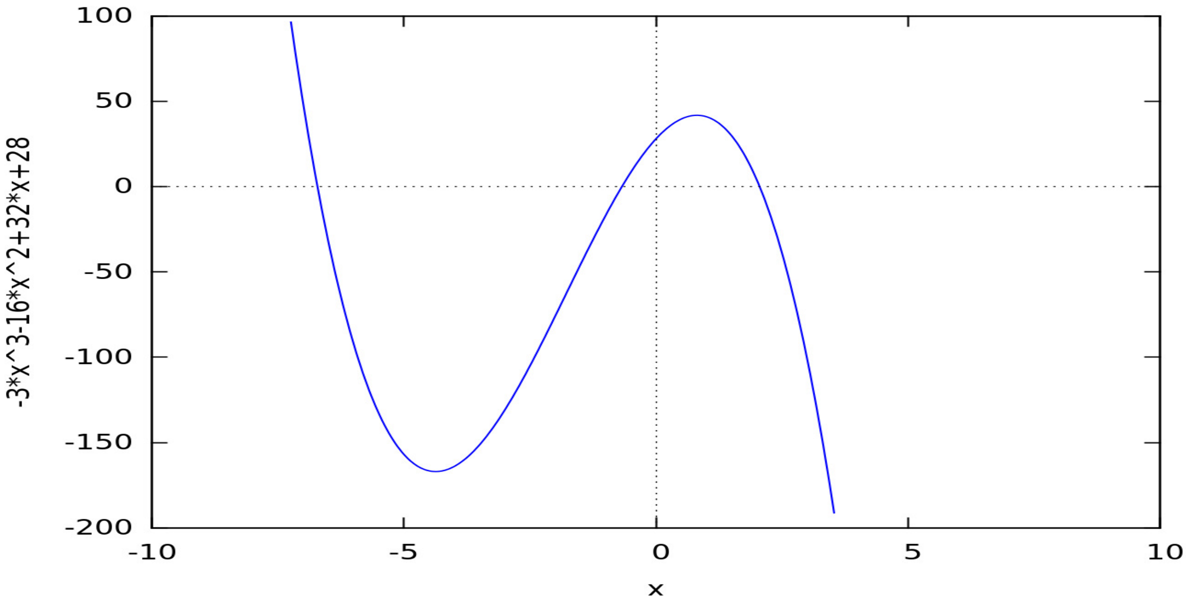
Solutions: Assignment 02

First semester

Answer to question 1

A graph of the cubic function $f(x) = -3x^3 - 16x^2 + 32x + 28$ can be obtained through entering the following input in the program *Maxima*:

```
(%i1) f(x):=-3*x^3 -16*x^2+32*x+28;
(%o1) f(x):=-3*x^3 -16*x^2+32*x+28
(%i2) plot2d([f(x)], [x,-10,10], [y,-200,100]);
```



From the graph it is clear that the function $f(x)$ has three real zeros, one in the interval $(-10; -5)$, one in the interval $(-2{,}0)$ and one in the interval $(0{,}3)$. An accurate value can be determined by implementing the standard function for Newton's method in *Maxima*. We do this three times, once for each root.

```
(%i3) load(newton);
(%o3) C:/PROGRA~1/MAXIMA~1.1-G/share/maxima/5.25.1/share/numeric/newton.mac
(%i4) newton(f(x),-9);
(%o4) - 6.714854483351385b0
```

```
(%i3) load(newton);
(%o3) C:/PROGRA~1/MAXIMA~1.1-G/share/maxima/5.25.1/share/numeric/newton.mac
(%i4) newton(f(x),-1);
(%o4) - 6.756593277601039b-1
```

```
(%i3) load(newton);
(%o3) C:/PROGRA~1/MAXIMA~1.1-G/share/maxima/5.25.1/share/numeric/newton.mac
```

```
(%i4) newton(f(x),1);
(%o4) 2.057180477810444b0
```

The zeros are $x = -6.7149$, $x = -0.06757$ and $x = 2.0572$ (accurate to four decimal places).

The stationary points of the cubic function occur where $f'(x) = 0$:

$$f'(x) = -9x^2 - 32x + 32 = 0,$$

or

$$9x^2 + 32x - 32 = 0.$$

$$\begin{aligned} \Rightarrow x &= \frac{-32 \pm \sqrt{32^2 - 4 \times 9 \times (-32)}}{18} \\ &= \frac{-32 \pm \sqrt{2176}}{18}. \end{aligned}$$

Therefore, we have stationary points at $x = 0.8138$ and $x = -4.3693$.

Second derivative: $f''(x) = -18x - 32$.

$$\begin{aligned} f''(0.8138) &= -46.6448 < 0 \Rightarrow \text{ relative maximum at } x = 0.8138 \\ f''(-4.3693) &= 46.6474 \quad > 0 \Rightarrow \text{ relative minimum at } x = -4.3693 \end{aligned}$$

$$f''(x) = -18x - 32 = 0 \Rightarrow x = -16/9 \text{ is an inflection point.}$$

The function $f(x)$ is concave over the interval $(-\infty; -16/9)$ and convex over the interval $(-16/9; \infty)$. Relative maximum function value at $x = 0.8138$:

$$f(0.8138) = 41.8284$$

Relative minimum function value at $x = -4.3693$:

$$f(-4.3693) = -167.03$$

These values can be calculated in *Maxima* by entering:

```
(%i5) a:0.8138;
(%o5) 0.8138
(%i6) f(a);
(%o6) 41.828405907784
(%i7) b:-4.3693;
(%o7) -4.3693
(%i8) f(b);
(%o8)  - 167.0300520393289
```

Other first derivative values are as follows:

$f'(-6) = -100 < 0$

$f'(-16/9) = 544/9 > 0$ (at the inflection point)

$f'(2) = -68 < 0$

Conclusion:

$f'(x) > 0$ for $x < -4.3693$ $\Rightarrow$ $f(x)$ is decreasing in the interval $(-\infty; -4.3693)$
$f'(x) > 0$ for $-4.3693 < x < 0.8138$ $\Rightarrow$ $f(x)$ is increasing in the interval $(-4.3693; 0.8138)$
$f'(x) < 0$ for $x > 0.8138$ $\Rightarrow$ $f(x)$ is decreasing in the interval $(0.8138; \infty)$