Tutorial letter 202/2/2018

Theoretical Computer Science 1

COS1501

Semester 2

School of Computing

Discussion of Assignment 02

Dear Student,

The solutions to the second assignment questions are discussed in this tutorial letter.

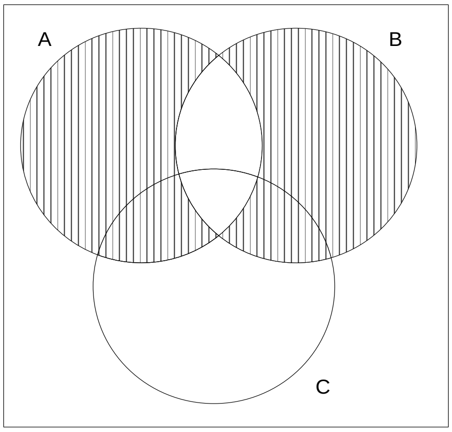
Regards,
COS1501 Team

Discussion of assignment 02 semester 2

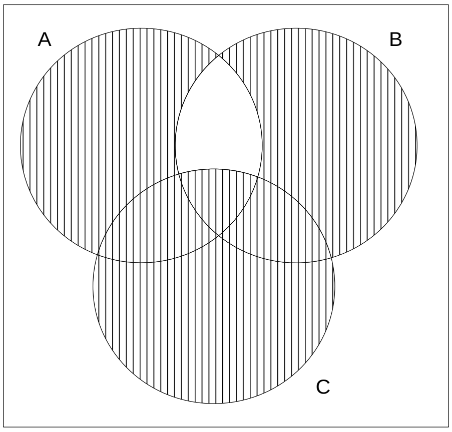
Question 1 **Alternative 2**

We determine the Venn diagram for the set ((A + B) ∪ C)) − (B ∩ C) step by step:

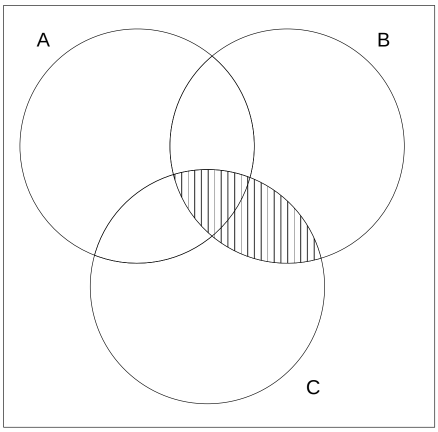
A + B



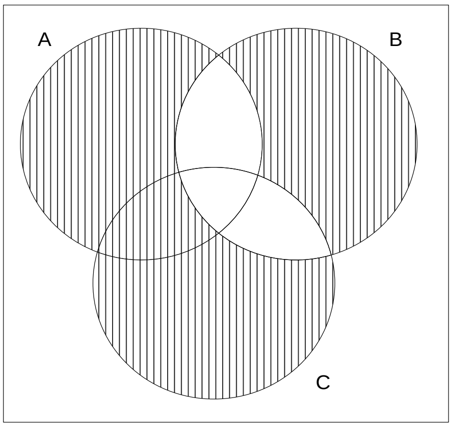
(A + B) ∪ C



B ∩ C



((A + B) ∪ C) − (B ∩ C)



Refer to study guide, pp 41-43, 50, 51

Question 2 **Alternative 1**

Let A, B and C be subsets of a universal set U. The statement $A \cup (B' \cap C) = (A - B) \cup C$ is NOT an identity. Which of the following sets A, B and C with U = {1, 2, 3} provides a *counterexample* that can be used to show that the given statement is not an identity?

1. A = {1}, B = {2, 3} & C = {3}
2. A = {1}, B = {2} & C = {1}
3. A = {1}, B = {1} & C = {1}
4. A = {1, 2, 3}, B = {1} & C = {1}

Discussion

Given: A, B, C $\subseteq$ U = {1, 2, 3}, and $A \cup (B' \cap C) = (A - B) \cup C$ is not an identity.

*We do **not** start our counterexample solution with $A \cup (B' \cap C) \neq (A - B) \cup C$.*
***First** we **determine $A \cup (B' \cap C)$, then** we **determine $(A - B) \cup C$** by using the sets provided in the different alternatives, then we **compare the answers** and get to a conclusion.*

We consider the different alternatives:

1. We use the sets A = {1}, B = {2, 3} & C = {3} to determine $A \cup (B' \cap C)$ and $(A - B) \cup C$ then we compare the answers. Note that we use **curly brackets** to indicate **sets**.

$A \cup (B' \cap C) = \{1\} \cup (\{1\} \cap \{3\})$
$= \{1\} \cup \{\ \}$
$= \{1\}$
$(A - B) \cup C = (\{1\} - \{2, 3\}) \cup \{3\} = \{1\} \cup \{3\}$
$= \{1, 3\}$

Clearly $\{1\} \neq \{1, 3\}$, so $A \cup (B' \cap C) \neq (A - B) \cup C$.

2. We use the sets A = {1}, B = {2} & C = {1} to determine $A \cup (B' \cap C)$ and $(A - B) \cup C$ then we compare the answers. Note that we use **curly brackets** to indicate **sets**.

$A \cup (B' \cap C) = \{1\} \cup (\{1, 3\} \cap \{1\})$
$= \{1\} \cup \{1\}$
$= \{1\}$
$(A - B) \cup C = (\{1\} - \{2\}) \cup \{1\} = \{1\} \cup \{1\}$
$= \{1\}$

Thus $A \cup (B' \cap C) = (A - B) \cup C$.

3. We use the sets A = {1}, B = {1} & C = {1} to determine $A \cup (B' \cap C)$ and $(A - B) \cup C$ then we compare the answers. Note that we use **curly brackets** to indicate **sets**.

$A \cup (B' \cap C) = \{1\} \cup (\{2, 3\} \cap \{1\})$
$= \{1\} \cup \{\ \}$
$= \{1\}$
$(A - B) \cup C = (\{1\} - \{1\}) \cup \{1\} = \{\ \} \cup \{1\}$
$= \{1\}$

Thus $A \cup (B' \cap C) = (A - B) \cup C$.

4. We use the sets $A = \{1, 2, 3\}$, $B = \{1\}$ & $C = \{1\}$ to determine $A \cup (B' \cap C)$ and $(A - B) \cup C$ then we compare the answers. Note that we use **curly brackets** to indicate **sets**.

$A \cup (B' \cap C) = \{1, 2, 3\} \cup (\{2, 3\} \cap \{1\})$
$= \{1, 2, 3\} \cup \{\ \}$
$= \{1, 2, 3\}$
$(A - B) \cup C = (\{1, 2, 3\} - \{1\}) \cup \{1\} = \{2, 3\} \cup \{1\}$
$= \{1, 2, 3\}$

Thus $A \cup (B' \cap C) = (A - B) \cup C$.

Alternatives 2, 3 and 4 do not provide counterexamples, but a counterexample is provided in alternative 1, thus this alternative should be selected.

Refer to study guide, pp 41-43, 60, 61.

Question 3 **Alternative 4**

We want to prove that, for all $X, Y \subseteq U$,

$$(X - Y) \times (X \cup Y) = ((X - Y) \times X) \cup ((X - Y) \times Y) \text{ is an identity.}$$

Discussion

Venn diagrams cannot be drawn for Cartesian products. In the proof we apply definitions of Cartesian product, union and difference of sets. The notation should be correct and all the necessary steps should appear in the proof. ***Only when a Cartesian product is involved, do we start a proof with (u,v) ∈ …***

Proof:

$(u, v) \in (X - Y) \times (X \cup Y)$

iff $u \in (X - Y)$ and $v \in (X \cup Y)$

iff $(u \in X$ and $u \notin Y)$ and $(v \in X$ or $v \in Y)$

iff $(u \in X$ and $u \notin Y$ and $v \in X)$ or $(u \in X$ and $u \notin Y$ and $v \in Y)$

iff $(u \in (X - Y)$ and $v \in X)$ or $(u \in (X - Y)$ and $v \in Y)$

iff $(u,v) \in (X - Y) \times X$ or $(u,v) \in (X - Y) \times Y$

iff $(u,v) \in ((X - Y) \times X) \cup ((X - Y) \times Y)$

Alternative 4 provides the required steps to complete the given proof.
Refer to study guide, pp 41, 42, 73

Question 4 **Alternative 2**

A newly built old age home has 40 townhouses. Residents may only plant clivias, rose bushes and lavenders in their gardens. Of the 50 gardens

18 grow clivias,
30 grow roses, and
17 grow lavenders.

(Residents do not necessarily plant only one of the plants.)

Furthermore, some gardens have the following kinds of plants:

10 grow clivias and rose bushes
5 grow clivias and lavenders,
15 grow rose bushes and lavenders, and

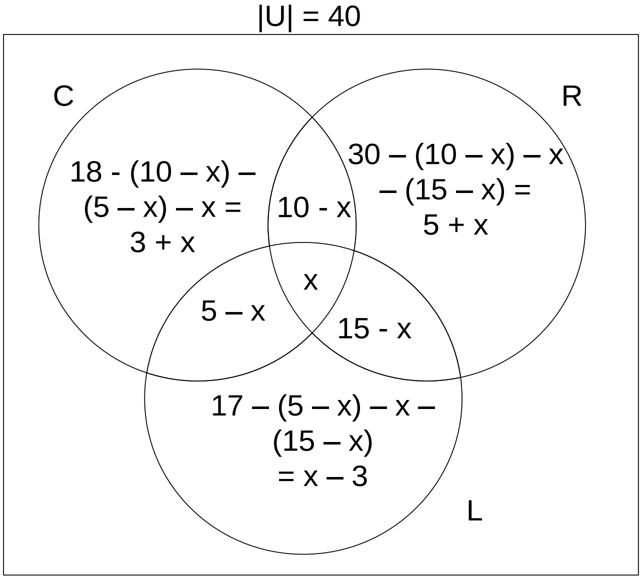
How many gardens grow clivias, rose bushes and lavenders?

1. 0
2. 5
3. 19
4. 40

Solution:

$|U| = 40$, $|C| = 18$, $|R| = 30$, $|L| = 17$,
$|C \cap R| = 10$,
$|C \cap L| = 5$,
$|R \cap L| = 15$,
$|C \cap R \cap L| = x$ the unknown we want to solve.

Now we can fill in the various regions. We initially fill in x for the value of $|C \cap R \cap L|$.



$|C \cup R \cup L| = 3 + x + 10 – x + 5 + x + 5 – x + x + 15 – x + x – 3 = 40$
i.e. $35 + x = 40$
i.e. $x = 5$, i.e. 5 gardens have clivias, rose bushes and lavenders.
Alternative 2 should therefore be selected.

Refer to study guide, pp 64-66

Question 5 **Alternative 3**
Which one of the following sets is equal to the set $\{u \in \mathbb{Z} \mid 2u^2 + 4u - 30 < 0\}$?

1. $\{u \in \mathbb{Z} \mid u > -5 \text{ or } u < 3\}$
2. $\{w \in \mathbb{Z} \mid -5 > w > 3\}$
3. $\{w \in \mathbb{Z} \mid -5 < w < 3\}$
4. $\{u \in \mathbb{Z} \mid -5 > u < 3\}$

$\{u \in \mathbb{Z} \mid 2u^2 + 4u - 30 < 0\}$
$= \{u \in \mathbb{Z} \mid (2u – 6)(u + 5) < 0\}$
$= \{u \in \mathbb{Z} \mid ((2u – 6) > 0$ and $(u + 5) < 0)$ or $((2u – 6) < 0$ and $(u + 5) > 0)\}$ (minus x plus = minus)
$= \{u \in \mathbb{Z} \mid (u > 3$ and $u < -5$, ie impossible) or $(u < 3$ and $u > -5)\}$
$= \{u \in \mathbb{Z} \mid -5 < u < 3\}$
$= \{w \in \mathbb{Z} \mid -5 < w < 3\}$ (It does not matter which variable we use to describe a set.)
Alternative 3 should therefore be selected.

Refer to study guide p 67

Consider the following relations on the set B = {1, 4, a, c, f}:

P = {(1, 4), (4, 1), (a, 4), (c, 4), (f, 1)} and R = {(1, c), (4, 1), (a, 4)}.

Answer questions 6 to 8 by using the given relations P and R defined on the set B.

Question 6 **Alternative 3**
Which one of the following alternatives represents the **range** of P?

1. {1, 4, a, c, f}
2. {1, a, c, f}
3. {1, 4}
4. 1, 4

Discussion
First we look at the definition for the range of a given relation S on T:
*ran(S) = {y / for some x $\in$ T, (x, y) $\in$ S}. (Note: The range is a **set** with all the **second co-ordinates** of the ordered pairs of S as its **elements**.)*

Ran(P) is the **set** of elements appearing as second co-ordinates in the ordered pairs of P. The elements **1** and **4** are the only elements that appear as second co-ordinates in the ordered pairs of P, thus ran(P) = {1, 4}. From this discussion one can conclude that alternative 3 should be selected.

Refer to study guide, p 74.

Question 7 **Alternative 1**

Which one of the following relations represents the composition relation P; R (i.e R ○ P)?

1. { (1, 1), (4, c), (a, 1), (c, 1), (f, c) }
2. { (1, 4), (1, c), (4, 1), (a, 4), (c, 4), (f, 1) }
3. { (1, 4), (4, 4), (a, 1) }
4. { (4, 1), (a, 4) }

Discussion

P = {(1, 4), (4, 1), (a, 4), (c, 4), (f, 1)} and R = {(1, c), (4, 1), (a, 4)} is defined on set B = (1, 4, a, c, f}.

According to the definition of composition:

Given relations R from A to B and S from B to C, the composition of R followed by S (S ○ R or R; S) is the relation from A to C defined by

S ○ R = R; S = {(a, c) | there is some b ∈ B such that (a, b) ∈ R and (b, c) ∈ S}.

To determine P; R we start with the pair (1, **4**) of P, and then we look for a pair in R that has as first co-ordinate a **4**, and then see where it takes us.

Link (1, **4**) of P with (**4**, 1) of R, then (1, 1) ∈ P; R,
link (a, **4**) of P with (**4**, 1) of R, then (a, 1) ∈ P; R,
link (c, **4**) of P with (**4**, 1) of R, then (c, 1) ∈ P; R,
link (4, **1**) of P with (**1**, c) of R, then (4, c) ∈ P; R, and
link (f, **1**) of P with (**1**, c) of R, then (f, c) ∈ P; R.

No other pairs can be linked, so R ○ P = {(1, 1), (4, c), (a, 1), (c, 1), (f, c)}. Alternative 1 is therefore the correct alternative.

Refer to study guide, pp 108, 109

Question 8 **Alternative 4**

The relation R = {(1, c), (4, 1), (a, 4)} is not transitive. Which ordered pair(s) should be included in R so that the extended relation would satisfy transitivity?

1. only (4, c)
2. only (a, 1)

3. only (4, c) & (a, 1)
4. (4, c), (a, 1), & (a, c)

Discussion
A relation R $\subseteq$ *A* $\times$ *A is* transitive *iff R has the property that for all x, y, z* $\in$ *A,*
whenever (x, y) $\in$ *R and (y, z)* $\in$ *R, then (x, z)* $\in$ *R. Let's say the y in the ordered pairs (x, y) and (y, z) plays the role of an "intermediary".*

R = {(1, c), (4, 1), (a, 4)} *is defined on B = {1, 4, a, c, f}. We identify the ordered pairs that have an "intermediary".* We name the new transitive relation 'R_1'.

For transitivity:
(4, 1), (1, c) $\in$ R_1 thus (4, c) should also be an element of R_1; and
(a, 4), (4, 1) $\in$ R_1 then (a, 1) should also be an element of R_1.
We now have $R_1 \supseteq$ {(1, c), (4, 1), (4, c), (a, 1), (a, 4)}, but we investigate which other elements should also be in R_1:
since (a, 4), (4, c) $\in$ R_1 then (a, c) should also be an element of R_1; and
since (a, 1), (1,c) $\in$ R_1 then (a, c) should also be an element of R_1.

We now have R_1 = {(1, c), (4, 1), **(4, c)**, **(a, 1)**, (a, 4), **(a, c)**} which is a transitive relation. The ordered pairs in bold, are the ones that we added. If you follow which ordered pairs we added in the discussion above, you will see that alternative 4 contains all these ordered pairs, and is therefore the correct alternative.

Refer to study guide p 77.

Let A = {1, 2, {2}, 3, {3, 4}}.

Answer questions 9 and 10 by using the given set A.

Question 9 **Alternative 2**
Which one of the following sets is NOT a partition of **A**?

1. { {{3, 4}}, {1, 2}, {{2}, 3} }
2. { {1, {2}}, 2, 3, {{3, 4}} }
3. { {1, 3, {3, 4}}, {2}, {{2}} }
4. { {1, {2}, 3}, {2, {3, 4}} }

Discussion
First we look at the definition of a partition:
For a nonempty set A, a partition of A is a set S = {S_1, S_2, S_3, …}. The members of S are subsets of A (each set S_i is called a part of S) such that
a. for all i, $S_i \neq \emptyset$ (that is, each part is nonempty),

b. *for all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$ (that is, different parts have nothing in common), and*

c. *$S_1 \cup S_2 \cup S_3 \cup \ldots = A$ (that is, every element in A is in some part S_i).*

We consider the sets provided in the different alternatives:

1. Let P = { {{3, 4}}, {1, 2}, {{2}, 3} } (say).

We test whether P is a partition of A = {1, 2, {2}, 3, {3, 4}}:

{{3, 4}}, {1, 2} and {{2}, 3} are three **non-empty** subsets of A,
{{3, 4}} ∩ {1, 2} ∩ {{2}, 3} = ∅ and
{{3, 4}} ∪ {1, 2} ∪ {{2}, 3} = A.
Because P has the above properties, it is a partition of A.

2. Let P = {{1, {2}}, 2, 3, {{3, 4}}} (say).

We test whether P is a partition of A = {1, 2, {2}, 3, {3, 4}}:

{1, {2}} and {{3, 4}} are two **non-empty** subsets of A, but 2 and 3 are not subsets, so P is not a partition of A. Thus alternative 2 should be selected.

3. Let P = { {1, 3, {3, 4}}, {2}, {{2}} } (say).

We test whether P is a partition of A = {1, 2, {2}, 3, {3, 4}}:

{1, 3, {3, 4}}, {2} and {{2}} are three **non-empty** subsets of A,
{1, 3, {3, 4}} ∩ {2} ∩ {{2}} = ∅ and
{1, 3, {3, 4}} ∪ {2} ∪ {{2}} = A.

Because P has the above properties, it is a partition of A.

4. Let P = { {1, {2}, 3}, {2, {3, 4}} } (say).

We test whether P is a partition of A = {1, 2, {2}, 3, {3, 4}}:

{1, {2}, 3} and {2, {3, 4}} are two **non-empty** subsets of A,
{1, {2}, 3} ∩ {2, {3, 4}} = ∅ and
{1, {2}, 3} ∪ {2, {3, 4}} = A.

Because P has the above properties, it is a partition of A.

Refer to study guide, pp 94, 95.

Question 10 **Alternative 3**

Consider the following relation on A:

R = {(1, 1), (1, {2}), (1, {3, 4}), ({2}, 1), ({2}, {2}), ({2}, {3, 4}), ({3, 4}, {2}) }.

Which one of the following statements regarding the relation R is TRUE?

1. R is reflexive, transitive and anti-symmetric.
2. R is irreflexive, transitive and symmetric.
3. R is neither reflexive nor irreflexive, and is not transitive and not symmetric.
4. R is neither reflexive nor irreflexive, and is transitive and not symmetric.

Reflexive:
We ask the question: Is it true that for all x ∈ A that we have (x, x) ∈ R?
Irreflexive:
We ask the question: is it true that there is no x ∈A such that (x, x) ∈ R. In other words, for any x ∈ A, (x, x) ∉ R?
Symmetric:
We ask the question: Is it true that R has the property that, for all x, y ∈A, if (x, y) ∈ R, then (y, x) ∈ R?
Antisymmetric:
We ask the question: Is it true that for all x, y ∈ A, if x ≠ y and (x, y) ∈ R then (y, x) ∉ R?
Transitive:
We ask the question: Is it true that for all x, y, z ∈ A, if (x, ***y****) ∈ R and (****y****, z) ∈ R then (x, z) ∈ R?*

Reflexivity:
In the relation R = {(1, 1), (1, {2}), (1, {3, 4}), ({2}, 1), ({2}, {2}), ({2}, {3, 4}), ({3, 4}, {2}) } we do not have ordered pairs (x, x) ∈ R for every x ∈ A. Thus R is not reflexive. We provide a counterexample: {3, 4} ∈ A, but ({3, 4},{3, 4}) ∉ R.

Irreflexivity:
To be irreflexive, it should be true that for all x ∈ A, we do not have any ordered pair (x, x) ∈ R. R is not irreflexive. We provide a counterexample: (1, 1) ∈ R.

Antisymmetry:
R is not antisymmetric. We give a counterexample:
({2}, {3, 4}) as well as ({3, 4}, {2}) are ordered pairs in R. Therefore R is not antisymmetric.
But R is also not symmetric. We give a counterexample:
(1, {3, 4}) ∈ R, but ({3, 4}, 1) ∉ R. R therefore is not symmetric.

Transitivity:
R is not transitive. We give a counterexample:
({3, 4}, **{2}**) ∈ R and (**{2}**, 1) ∈ R, but ({3, 4}, 1} ∉ R.
1.
As shown above, R is not reflexive, nor transitive and anti-symmetric, so alternative 1 is false.
2.
As shown above, R is not irreflexive, nor transitive and symmetric, so alternative 2 is false.
3.
We can conclude that R is neither reflexive nor irreflexive, neither symmetric nor anti-symmetric and also not transitive, thus alternative 3 should be selected.
4.

We have shown that R is not transitive, so alternative 4 is false.

Refer to study guide pp 73 -77.

Let S be some relation from A = {1, 3, 4, 6} to B = {1, 2, 4} defined by

(a, b) $\in$ S iff 3a – b is odd. (S $\subseteq$ A $\times$ B, and A, B $\subseteq$ U = $\mathbb{Z}$.)

Answer questions 11 and 12 by using the given relation S.

Hint: First determine set A $\times$ B (a Cartesian product) and relation S.

Question 11 **Alternative 4**

Which one of the following alternatives provides some elements belonging to S?

1. (1, 2), (3, 1), (6, 1)
2. (4, 1), (6, 1), (6, 4)
3. (1, 1), (3, 1), (4, 2)
4. (1, 4), (3, 2), (4, 1)

Discussion

A $\times$ B = {(1, 1), (1, 2), (1, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 4), (6, 1), (6, 2), (6, 4)}

S = {(1, 2), (1, 4), (3, 2), (3, 4), (4, 1), (6, 1)}

Let's look at the different alternatives:

1.

In this alternative (3, 1) $\notin$ S, because **3a - b** = (3)(3) - 1 = 8 which is not odd.

2.

In this alternative (6, 4) $\notin$ S, because **3a - b** = (3)(6) - 4 = 14 which is not odd.

3.

In this alternative (1, 1) $\notin$ S, because **3a - b** = (3)(1) - 1 = 2 which is not odd.

4.

In this alternative (1, 4) $\in$ S, because **3a - b** = (3)(1) – 4 = -1, which is odd;

(3, 2) $\in$ S, because **3a - b** = (3)(3) – 2 = 7, which is odd; and

(4, 1) $\in$ S, because **3a - b** = (3)(4) – 1 = 11, which is odd.

(1, 4), (3, 2) and (4, 1) are all elements of S, therefore this alternative should be selected.

Refer study guide p 73.

Question 12 **Alternative 1**

Which one of the following statements is TRUE?

1. (4, 2) $\in$ A $\times$ B but (4, 2) $\notin$ S.
2. (3, 1) $\in$ A $\times$ B and (3, 1) $\in$ S.
3. (2, 3) $\notin$ A $\times$ B and (2, 3) $\in$ S.
4. (2, 6) $\in$ A $\times$ B and (6, 2) $\in$ S.

By inspecting the relations $A \times B$ and S we see that $(4, 2) \in A \times B$ but $(4, 2) \notin S$ is a true statement thus alternative 1 should be selected.

Let us look at the alternatives:

1. The statements $(4, 2) \in A \times B$ and $(4, 2) \notin S$ are both true. (3)(4) - 2 = 10 which is not odd, so it is true that $(4, 2) \notin S$. This alternative should therefore be selected

2. The statements $(3, 1) \in A \times B$ and $(3, 1) \in S$ are not both true. (3)(3) - 1 = 8 which is not odd, so it is not true that $(3, 1) \in S$.

3. The statement $(2, 3) \notin A \times B$ is true because $2 \notin A = \{1, 3, 4, 6\}$, but $(2, 3) \in S$ is false, because $2 \notin A = \{1, 3, 4, 6\}$.

4. The statement $(2, 6) \in A \times B$ is false because $2 \notin A = \{1, 3, 4, 6\}$ and $6 \notin B = \{1, 2, 4\}$. Furthermore $(6, 2) \in S$ is also false because (6)(3) - 2 = 16 which is not odd.

(Refer to study guide, p 73)

Let R be the relation on $\mathbb{Z}$ defined by

$$(x, y) \in R \text{ iff } y > 1 - x.$$

Answer questions 13 to 15 by using the given relation R.

Question 13 **Alternative 2**

Which one of the following is **not** an ordered pair belonging to R?

1. (5, 0)
2. (0, 1)
3. (0, 5)
4. (5, -3)

Discussion

y > 1 – x, ie x + y > 1. We use the expression x + y > 1 to substitute values for x and y in the alternatives:

1. Let x = 5 and y = 0, then
5 + 0 = 5 and 5 > 1
so $(5, 0) \in R$.

2. Let x = 0 and y = 1, then
0 + 1 = 1 but $1 \not> 1$

ie $y \ngtr 1 - x$

so $(0, 1) \notin R$. Alternative 2 is therefore the correct choice.

3. Let x = 0 and y = 5, then
0 + 5 = 5 and 5 > 1
so $(0, 5) \in R$.

4. Let x = 5 and y = -3, then
5 + (-3) = 2 and 2 > 1
so $(5, -3) \in R$.

Refer to study guide, pp 70 – 73.

Question 14 **Alternative 2**

Which of the following alternatives can be used to prove that R is a symmetric relation?

1. Let $x, y \in \mathbb{Z}$ be given such that $(x, y) \in R$ and $(y, x) \in R$
   then $y > 1 - x = x > 1 - y$
   ie $y > 1 - x = x - 1 > -y$
   ie $y > 1 - x = 1 - x < y$ (multiply by -1, which changes > to < as well)

2. Let $x, y \in \mathbb{Z}$ be given such that $(x, y) \in R$. We have to prove that $(y, x) \in R$, ie. $x > 1 - y$
   If $(x, y) \in \mathbb{Z}$ then
   $y > 1 - x$
   ie $y - 1 > -x$
   ie $1 - y < x$ (multiply by -1, which changes < to >)
   ie $x > 1 - y$
   thus $(y, x) \in R$.

3. $(5, -3) \in R$ and $(-3, 5) \in R$, therefore R is symmetric.
   We substitute the ordered pairs (5, -3) and (-3, 5) in $y > 1 - x$ and $1 - x < y$ for the proof:
   $-3 > 1 - 5$ and $5 > 1 - (-3)$
   ie -3 > -4, which is true ie 5 > 4, which is also true

4. R is not a symmetric relation. We give a counterexample:
   The ordered pairs (5, -4) and (-4, 5) are not ordered pairs in R.
   We substitute the ordered pairs (5, -4) and (-4, 5) in $y > 1 - x$ and $1 - x < y$ for the proof:
   $-4 > 1 - 5$ and $5 > 1 - (-4)$
   ie -4 > -4, which is false ie 5 > 5, which is also false
   We have proved that R is not symmetric.

Discussion:

Let's look at the definition for symmetry:

A relation $R \subseteq A \times A$ is symmetric iff R has the property that, for all x, y $\in$ A, if (x, y) $\in$ R, then (y, x) $\in$ R.

Let us look at the different alternatives to see which one provides a valid proof:

1. Alternative 1 states that for x, y $\in \mathbb{Z}$, (x, y) $\in$ R and (y, x) $\in$ R. We should prove <u>if</u> (x, y) $\in$ R <u>then</u> (y, x) $\in$ R. This, however, is what we have to prove. We also cannot assign the expression y > 1 – x equal to the expression x > 1 – y as was done here. It is therefore not a valid proof.

2. This alternative provides the proof, showing that R is symmetric. We assume (x, y) $\in$ R and then prove that (y, x) $\in$ R.

3. Although it is true that (5, -3) $\in$ R and (-3, 5) $\in$ R, substituting these values for x and y in the equation does not constitute a general formal proof for the statement, ie "**if** … **then** …"

4. The definition for symmetry states that if (x, y) $\in$ R, then it must be true that (y, x) $\in$ R. Neither of the ordered pairs provided in the counterexample are ordered pairs in R. If we want to use a counterexample, we have to find an ordered pair (x, y) $\in$ R, and then prove that (y, x) $\notin$ R, but this is not possible since R is actually a symmetric relation.

Refer to study guide, p 76.

Question 15 **Alternative 2**

Which one of the following statements regarding the relation R is TRUE?

1. R satisfies trichotomy.
2. R is not transitive.
3. (1, 0), (0, 1) $\in$ R.
4. R is an equivalence relation.

Discussion

1. Let's look at the definition for trichotomy:
   A relation R on A satisfies the requirement for trichotomy iff, for every x and y chosen from A such that x ≠ y, we have that x and y are comparable, i.e. for all x, y $\in$ A such that x ≠ y, x R y or y R x (ie (x, y) $\in$ R or (y, x) $\in$ R).

   Relation R does not satisfy trichotomy. We give a counterexample:

   (0, 1) $\notin$ R since $1 \ngtr 1 - 0$, and

   (1, 0) $\notin$ R since $0 \ngtr 1 - 1$.

Thus 0 and 1 are not comparable, ie neither (0, 1) $\in$ R nor (1, 0) $\in$ R.

2. Let's look at the definition for transitivity: *R is transitive iff, for all x, y, z $\in$ X, if (x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R. (Goal: to show that whenever (x, y) $\in$ R and (y, z) $\in$ R then (x, z) $\in$ R.)*

The relation R is not transitive as stated in the question. A counterexample proves that it is indeed not transitive:

(1, **9**) $\in$ R and (**9**, 0) $\in$ R but (1, 0) $\notin$ R

ie 9 > 1 – 1 and 0 > 1 – 9, but $0 \ngtr 1 - 1$.

3. Neither (0, 1) nor (1, 0) are ordered pairs in R since 1 $\ngtr$ 1 – 0, and 0 $\ngtr$ 1 – 1.

4. To be an equivalence relation R should be symmetric, transitive and reflexive. We have shown in the discussion of alternative 2 that R is not transitive. R is therefore not an equivalence relation.

Refer to study guide, pp 73-75, 77-78, 90-92.

Note: A discussion of the self-assessment questions is provided in tutorial letter 102.