Tutorial Letter 203/2/2015

Quantitative Modelling
DSC1520

Semester 2

Department of Decision Sciences

Important Information:

This tutorial letter contains the solutions for Assignment 03.

Bar code

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UNISA university of south africa

Dear student

You have completed the assignments for the course and it is now time to start your revision for the examination. Work through all the assignments, the evaluation exercises and the previous examination paper in preparation for the examination. The questions in the Oct/Nov examination paper are similar to the problems in the previous paper. Do as many examples as possible – the more examples you work through, the better you will be able to recognise a problem and solve it.

Please contact me if you need any help. My contact details are as follows:

Office: Hazelwood Campus, Room 4-37, Unisa

Tel: +27 12 433 4602

E-mail: mabemgv@unisa.ac.za

I wish you everything of the best with your preparation for the examination.

Victoria Mabe-Madisa

ASSIGNMENT 3: SOLUTIONS

Question 1

The only logs your calculator can handle are log to the base 10 and ln which is log to the base e. We therefore need to change the base 3 to a base of either 10 or e.

$$\begin{aligned}\log_3\left(\tfrac{3}{\sqrt{3}}\right) &= \tfrac{\ln\left(\frac{3}{\sqrt{3}}\right)}{\ln 3}\\ &= \tfrac{0{,}54931}{1{,}09861} \qquad \text{Rounded to 5 decimal places}\\ &= 0{,}5000.\end{aligned}$$

[Option 4]

Question 2

First we write the inequality in the standard quadratic format $y = ax^2 + bx + c$:

$$\begin{aligned}x^2 - 3x &\geq 6 - 2x\\ x^2 - x - 6 &\geq 0.\end{aligned}$$

The x values for which

$$x^2 - x - 6 = 0$$

are where

$$(x + 2)(x - 3) = 0,$$

giving

$$x = -2 \quad \text{and} \quad x = 3.$$

Now, to determine the area where the inequality

$$x^2 - x - 6 \geq 0$$

is true, we substitute a point in the region smaller than -2, the region between -2 and 3 and a value in the region greater than 3, into the inequality. The inequality region is then the area in which the selected point chosen, makes the inequality true. You can choose any values in the different regions.

First we choose a value smaller than -2, for example $x = -3$. The left hand side of the inequality is

$$\begin{aligned}\text{LHS} &= x^2 - x - 6\\ &= (-3)^2 - (-3) - 6 = 6,\end{aligned}$$

which is larger than zero. The inequality is therefore true for values smaller than -2.

Next we choose a value between -2 and 3 for example $x = 0$, where the left hand side of the inequality is

$$\begin{aligned} \text{LHS} &= x^2 - x - 6 \\ &= (0)^2 - (0) - 6 \\ &= -6, \end{aligned}$$

which is smaller than zero.

The inequality

$$x^2 - x - 6 \geq 0$$

therefore doesn't hold for values between -2 and 3.

Finally, when we choose a value bigger than 3, for example $x = 4$, the left hand side of the inequality is

$$\begin{aligned} \text{LHS} &= x^2 - x - 6 \\ &= (4)^2 - (4) - 6 \\ &= 14, \end{aligned}$$

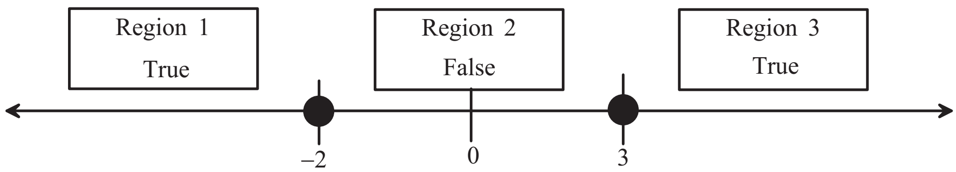
which is larger than zero.

The inequality is therefore true for values greater than 3.

Graphing the solution of the equation

$$x^2 - x - 6 = 0$$

on a number line gives



The area of the inequality is therefore true if $x \leq -2$ and $x \geq 3$. **[Option 3]**

Question 3

To find the roots of the expression

$$6x^2 + 5x - 1,$$

we need to solve the equation

$$6x^2 + 5x - 1 = 0.$$

Comparing this equation with the general form of a quadratic function

$$y = ax^2 + bx + c,$$

we conclude that $a = 6, b = 5$ and $c = -1$. Substituting a, b and c into the quadratic formula gives

$$\begin{aligned} x &= \tfrac{-5 \pm \sqrt{(5)^2 - 4(6)(-1)}}{2(6)} \\ &= \tfrac{-5+7}{12} \quad \text{and} \quad x = \tfrac{-5-7}{12} \\ &= \tfrac{1}{6} \qquad\qquad\quad\; = -1. \end{aligned}$$

Alternatively:

By using factorisation,

$$6x^2 + 5x - 1 = 0$$

can be written as

$$(6x - 1)(x + 1) = 0.$$

The roots are therefore where

$$6x - 1 = 0 \quad \text{and} \quad x + 1 = 0,$$

that is

$$x = \frac{1}{6} \quad \text{and} \quad x = -1.$$

[Option 4]

Question 4

We need to solve for x in

$$0{,}0625 = 2^{-x},$$

that is

$$\begin{aligned} \ln 0{,}0625 &= \ln(2^{-x}) \\ \ln 0{,}0625 &= -x \ln(2) \\ \tfrac{\ln 0{,}0625}{\ln 2} &= -x \\ x &= 4. \end{aligned}$$

The value of x if $y = 0{,}0625$ is equal to 4.

[Option 3]

Question 5

To integrate the function, we use of the power rule of integration.

$$\begin{aligned} \int_{-2}^{2} (x^2 - 3)\ \mathrm{d}x &= \int_{-2}^{2} x^2 \mathrm{d}x - \int_{-2}^{2} 3\mathrm{d}x \\ &= \tfrac{x^3}{3}\Big|_{-2}^{2} - \tfrac{3x}{1}\Big|_{-2}^{2} \\ &= \left[\tfrac{2^3}{3} - \tfrac{-2^3}{3}\right] - [3(2) - 3(-2)] \\ &= \tfrac{16}{3} - 12 \\ &= \tfrac{-20}{3} = -6\tfrac{2}{3}. \end{aligned}$$

[Option 2]

Question 6

Before we can use the power rule, we need to simplify the expression, that is

$$x^2\left(1+\frac{1}{x^2}\right)=x^2+1.$$

Therefore,

$$\begin{aligned}\int x^2\left(1+\tfrac{1}{x^2}\right)\ \mathrm{d}x &= \int (x^2+1)\ \mathrm{d}x\\ &= \tfrac{x^3}{3}+\tfrac{x}{1}+c\\ &= \tfrac{1}{3}x^3+x+c.\end{aligned}$$

[Option 2]

Question 7

To apply the power rule for differentiation, we need to simplify the given expression, to find

$$\begin{aligned}\tfrac{x-x^2}{\sqrt{x}} &= \tfrac{x-x^2}{x^{\frac{1}{2}}}\\ &= x^{\frac{1}{2}}-x^{\frac{3}{2}}.\end{aligned}$$

Now, differentiating the simplified expression, gives

$$\begin{aligned}\tfrac{\mathrm{d}}{\mathrm{d}x}(x^{\frac{1}{2}}-x^{\frac{3}{2}}) &= \tfrac{1}{2}x^{-\frac{1}{2}}-\tfrac{3}{2}x^{\frac{1}{2}}\\ &= \tfrac{1}{2x^{\frac{1}{2}}}-\tfrac{3}{2}x^{\frac{1}{2}}\\ &= \tfrac{1}{2\sqrt{x}}-\tfrac{3}{2}\sqrt{x}.\end{aligned}$$

[Option 2]

Question 8

Revenue is price $\times$ demand or

$$R=P\times Q.$$

It is given that price is P and demand is

$$Q=150-0{,}5P.$$

Substituting the demand function into the formula for R gives

$$\begin{aligned}R &= P\times(150-0{,}5P)\\ &= 150P-0{,}5P^2.\end{aligned}$$

To determine the marginal revenue we need to differentiate the revenue function, that is

$$\begin{aligned} MR &= \tfrac{\mathrm{d}}{\mathrm{d}P}\left(150P - 0{,}5P^2\right) \\ &= 150 - (2 \times 0{,}5)\,P \\ &= 150 - P. \end{aligned}$$

We need to find P if MR is equal to 0, that is

$$0 = 150 - P$$

or

$$P = 150.$$

The quantity when MR is equal to zero is

$$\begin{aligned} Q &= 150 - 0{,}5 \times (150) \\ &= 150 - 75 \\ &= 75. \end{aligned}$$

When demand is 75, marginal revenue is equal to 0.

[Option 2]

Question 9

There are different ways to determine the x-coordinates of the maximum point of a quadratic function.

Method 1:

The x-coordinate of the maximum point can be calculated by using the formula

$$x = \frac{-b}{2a}.$$

Comparing the given function

$$y = x^2 - 2x + 3$$

with the standard form of the quadratic function

$$y = ax^2 + bx + c,$$

we can conclude that

$$a = 1, b = -2 \ \text{ and } \ c = 3.$$

Therefore the x-value of the turning point is

$$\begin{aligned} x &= \tfrac{-b}{2a} \\ &= \tfrac{-(-2)}{2\times 1} \\ &= 1. \end{aligned}$$

Substitute the calculated x-value into the given equation, to find

$$\begin{aligned} y &= x^2 - 2x + 3 \\ &= (1)^2 - 2(1) + 3 \\ &= 1 - 2 + 3 \\ &= 2. \end{aligned}$$

The coordinates of the turning point are $(1;\ 2)$.

Method 2:

We can also use differentiation, where the derivative of a function is set equal to zero to find the turning points of a function.

Differentiating the function,

$$f(x) = x^2 - 2x + 3$$

results in

$$\begin{aligned} f'(x) &= (2)x^{2-1} - 2x^{1-1} + 0 \\ &= 2x - 2. \end{aligned}$$

To find the turning points of f, we set $f'(x) = 0$. Thus,

$$\begin{aligned} 2x - 2 &= 0 \\ x &= 1. \end{aligned}$$

The coordinates of the turning point are (1; 2).

[Option 2]

Question 10

The total cost function TC is given. We need to determine the marginal cost if $Q = 10$. The marginal cost function MC is the derivative of

$$TC = Q^4 - 30Q^2 + 300Q + 500,$$

that is

$$\begin{aligned} MC &= \tfrac{\mathrm{d}}{\mathrm{d}Q}(Q^4 - 30Q^2 + 300Q + 500) \\ &= 4Q^3 - 30 \times 2Q + 300. \end{aligned}$$

Now, the marginal cost when $Q = 10$ is

$$\begin{aligned} MC &= 4(10)^3 - 60(10) + 300 \\ &= 4\,000 - 600 + 300 \\ &= 3\,700. \end{aligned}$$

The marginal cost is equal to 3 700 when Q equals 10.

[Option 1]