Solutions for Assignment 4, Semester 1, 2018

Chapters 1 & 2

Question 1

As part of its search for extrasolar planets, NASA discovers a planet that appears to be very much like Earth orbiting a star 40 lightyears from our Solar System. An expedition is planned to send astronauts to the planet. NASA would like the astronauts to age no more than 30 years during the journey. In this problem, neglect any issues related to the acceleration of the astronauts' spaceship. (Hint: A lightyear is the distance traveled by light in one year, which is just c multiplied by one year, or 9.46×10^{12} km. In many problems it is simpler to write it as $1c \cdot \text{year}$, since c often cancels out.)

(a) At what velocity must the astronauts' spaceship travel in Earth's reference frame so that the astronauts age 30 years during the journey?

(b) According to the astronauts in the spaceship, what will be the distance of their journey?

(c) Exactly half way to the planet, two of the astronauts get homesick and set off in a space module to return to Earth. According to the astronauts who remain on the spaceship, the module travels at a velocity of $5c/6$ in the direction toward Earth. Find the total amount of time that the two astronauts will have been away according to people on Earth.

Solution

Part A

Method 1: According to the people on Earth, the distance between Earth and the planet d_E is 40 lightyears, or $40\, c \cdot \text{year}$. This distance will be contracted for the astronauts, and they would measure the distance to be

$$d_A = \frac{d_E}{\gamma}$$

The astronauts will say that the trip to the planet will last

$$\begin{aligned} \Delta t_A &= \frac{d_A}{V} \\ \therefore d_A &= V t_A \end{aligned}$$

where V is the speed that the spaceship is travelling at. Combining the two equations above gives

$$\begin{aligned} \frac{d_E}{\gamma} &= V \Delta t_A \\ d_E \sqrt{1 - V^2/c^2} &= V \Delta t_A \end{aligned}$$

We want to know what the speed V will be if we set $\Delta t_A = 30$ years. So we substitute this and $d_E = 40\, c \cdot \text{year}$ and solve for V.

$$\begin{aligned} (40\, c \cdot \text{year}) \sqrt{1 - V^2/c^2} &= V\, (30\, \text{year}) \\ 1 - V^2/c^2 &= V^2 \left(\frac{3}{4c}\right)^2 \\ 1 &= V^2 \left(\frac{9}{16c^2} + \frac{1}{c^2}\right) \\ V^2 &= \left(\frac{9}{16c^2} + \frac{16}{16c^2}\right)^{-1} \\ &= \left(\frac{25}{16c^2}\right)^{-1} \\ &= \frac{16c^2}{25} \\ V &= \frac{4}{5} c \end{aligned}$$

Therefore, the spaceship must travel at a speed of $V = \frac{4}{5}c$ if the astronauts are to only age 30 years.

Method 2: The time for the astronauts to reach the plane according to an observer on Earth is

$$\Delta t_E = \frac{d_E}{V}$$

The time that the people on Earth measure will be related to the time that the astronauts measure by the time dilation formula so that

$$\Delta t_E = \gamma \Delta t_A$$

Combining these two equations we get

$$\begin{aligned} \gamma \Delta t_A &= \frac{d_E}{V} \\ \frac{\Delta t_A}{\sqrt{1 - V^2/c^2}} &= \frac{d_E}{V} \end{aligned}$$

We want to know what the speed V will be if we set $\Delta t_A = 30$ years. So we substitute this and $d_E = 40\, c \cdot \text{year}$ and solve for V.

$$\begin{aligned} \frac{(30\,\text{year})}{\sqrt{1 - V^2/c^2}} &= \frac{(40\, c \cdot \text{year})}{V} \\ V^2 \left(\frac{3}{4c} \right)^2 &= 1 - V^2/c^2 \\ V^2 \left(\frac{9}{16c^2} + \frac{1}{c^2} \right) &= 1 \\ V^2 &= \left(\frac{9}{16c^2} + \frac{16}{16c^2} \right)^{-1} \\ &= \left(\frac{25}{16c^2} \right)^{-1} \\ &= \frac{16c^2}{25} \\ &= \frac{4}{5}c \end{aligned}$$

Therefore, the spaceship must travel at a speed of $V = \frac{4}{5}c$ if the astronauts are to only age 30 years.

Part B

Now that we know the speed, we can calculate the Lorentz factor between the two frames. We get

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{1-\frac{16}{25}}} \\
&= \frac{1}{\sqrt{0.36}} \\
&= \frac{1}{0.6} \\
&= \frac{5}{3}
\end{aligned}
$$

The distance of the journey, as measured by the astronauts will be

$$
\begin{aligned}
d_A &= \frac{d_E}{\gamma} \\
&= \frac{3}{5}\,(40\,c\cdot\text{year}) \\
&= 24\,c\cdot\text{year}
\end{aligned}
$$

So according to the astronauts, they will travel 24 lightyears to the planet.

Part C

According to the astronauts on the spaceship, the astronauts on the module is travelling at $5c/6$ in the *opposite direction* to them. We have implicitly taken their speed to be in the positive x-direction, so the module is travelling in the negative x-direction and we take the modules speed to be $v_M = -5c/6$. To determine the speed of the module according to the people on Earth, we use the velocity transformation equation.

Note that the velocity transformation equations given in the textbook on p30 has been derived for transforming the velocity of an object to a frame that is moving with speed V in the positive x-direction relative to the frame the speed was initially measured in. We want to transform the speed to a frame moving in the negative x-direction (speed $-V$) with respect the the frame that we have the measurement in, since, according to the astronauts on the spaceship, Earth is moving at $4c/5$ in the negative x-direction. We therefore just substitute V with $-V$ in the given velocity transformation equations to get

$$
\begin{aligned}
v_E &= \frac{v_M + V}{1 + v_M V/c^2} \\
&= \frac{-\frac{5c}{6} + \frac{4c}{5}}{1 + \left(-\frac{5c}{6}\right)\left(\frac{4c}{5}\right)/c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{-\frac{1}{30}c}{\frac{1}{3}} \\
&= -\frac{c}{10}
\end{aligned}
$$

So, according to the people on Earth, the module is approaching them at $c/10$.

The total amount of time that the astronauts in the module would have been away according to the people on Earth will be the sum of the time spent on the outward journey plus the time spent on the journey back. In both cases, the distance (according to the people on Earth) will be $40/2 = 20$ lightyears. The time for the outward journey, where the astronauts were travelling on the spaceship is

$$\frac{d_E/2}{V} = \frac{20\,c \cdot \text{year}}{4c/5} = 25\,\text{years}$$

The time for the journey back to Earth, where the astronauts were travelling on the module is

$$\frac{d_E/2}{v_E} = \frac{20\,c \cdot \text{year}}{c/10} = 200\,\text{years}$$

So the total journey, according to the people on Earth, would be $200 + 25 = 225\,$years.

> **Note:** If you want to use the equation
>
> $$\Delta x = \frac{\Delta t}{v}$$
>
> all the quantities need to be measured in the same frame of reference. That is why we transforms the speed of the module to Earth's frame.

Question 2

Tshepo travels to work on the Gautrain every day. He travels between the Hatfield and Park stations, which is a 62 km distance, one way. Suppose the train travels at a constant speed of $160\,\text{kmh}^{-1}$ every trip to and from work, and that he works 250 day per year. How many years would it take for him to be $1\,\mu\text{s} = 10^{-6}\,$s younger due to travelling on the Gautrain? (Hints: For speeds that are $V \ll c$, it is necessary to use the binomial theorem when calculating γ. Also, leave all your equations in symbolic form right until the end for the best accuracy.)

Solution

Let's call the frame in which the ground is stationary S and the frame moving with the train S'. For a single one-way journey we have the following quantities for the situation:

$V = 160\,\text{kmh}^{-1}$	Time	Distance
Ground (S)	$\Delta T = ?$	$L = 62\,\text{km}$
Train (S')	$\Delta T' = ?$	$L' = ?$

What we need to calculate is the time he gains on each trip, $\Delta t = \Delta T - \Delta T'$ and figure out what number it needs to be multiplied with to give $1\,\mu$s.

From the time dilation formula we know that

$$\begin{aligned}\Delta t &= \Delta T - \Delta T' \\ &= \Delta T - \Delta T/\gamma \\ &= \Delta T\,(1 - 1/\gamma)\end{aligned}$$

Since we have $V \ll c$, we use the binomial theorem

$$1 - \frac{1}{\gamma} = 1 - \left[1 - \left(\frac{V}{c}\right)^2\right]^{1/2} \approx \frac{1}{2}\left(\frac{V}{c}\right)^2$$

The time Tshepo spends on the train per day as measured in S is

$$\Delta T = \frac{2L}{V}$$

So we have

$$\begin{aligned}\Delta t &= \Delta T\,(1 - 1/\gamma) \\ &\approx \frac{1}{2}\left(\frac{2L}{V}\right)\left(\frac{V}{c}\right)^2 \\ &= \frac{VL}{c^2}\end{aligned}$$

Converting L and V to SI units gives

$$62\,\text{km} = 62\,000\,\text{m} = 6.2 \times 10^4\,\text{m}$$

and

$$\begin{aligned} 160\,\text{kmh}^{-1} &= 160 \times \frac{\text{km}}{\text{h}} \times \frac{1000\,\text{m}}{1\,\text{km}} \times \frac{1\,\text{h}}{60 \times 60\,\text{s}} \\ &= 4.44\,\text{ms}^{-1}\,. \end{aligned}$$

So for a day we have

$$\begin{aligned} \Delta t &= \frac{\left(4.44\,\text{ms}^{-1}\right)\left(6.2 \times 10^{4}\,\text{m}\right)}{\left(3 \times 10^{8}\,\text{ms}^{-1}\right)^{2}} \\ &= 3.06 \times 10^{-12}\,\text{s} \end{aligned}$$

For Tshepo to gain 1 μs, he would have to take the Gautrain for

$$\frac{1 \times 10^{-6}}{3.06 \times 10^{-12}} = 3.27 \times 10^{5}\,\text{days}\,.$$

He works 250 days per year, so it would take him

$$\frac{3.27 \times 10^{5}}{250} = 1308\,\text{years}$$

of travelling on the Gautrain before he is 1 μs younger than someone who didn't travel.

Question 3

Maxwell's wave equation for an electric field propagating in the x-direction is

$$\frac{\partial^{2} E}{\partial x^{2}} = \frac{1}{c^{2}} \frac{\partial^{2} E}{\partial t^{2}}\,,$$

where $E\,(x,\,t)$ is the amplitude of the electric field. Show that this equation is *not* invariant under a Galilean transformation to a reference frame moving with relative speed v along the$x$-axis.

Solution

The relevant Galilean transformations are given by

$$x' = x - vt$$

$$t' = t$$

We use the chain rule to obtain

$$\begin{aligned}
\frac{\partial E}{\partial x} &= \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial x} \\
&= \frac{\partial E}{\partial x'} \times 1 + \frac{\partial E}{\partial t'} \times 0 \\
&= \frac{\partial E}{\partial x'}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E}{\partial t} &= \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} \\
&= \frac{\partial E}{\partial x'} \times (-v) + \frac{\partial E}{\partial t'} \times 1 \\
&= -v\frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}
\end{aligned}$$

Therefore we have

$$\begin{aligned}
\frac{\partial^2 E}{\partial x^2} &= \frac{\partial^2 E}{\partial x'^2} \\
\frac{\partial^2 E}{\partial t^2} &= \left(-v\frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}\right)\left(-v\frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}\right) \\
&= v^2\frac{\partial^2 E}{\partial x'^2} - 2v\frac{\partial E}{\partial x'}\frac{\partial E}{\partial t'} + \frac{\partial^2 E}{\partial t'^2}
\end{aligned}$$

Substituting this into the wave equation

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}\,,$$

and rearranging gives

$$\frac{\partial^2 E}{\partial x'^2} = \frac{1}{c^2}\left(v^2\frac{\partial^2 E}{\partial x'^2} - 2v\frac{\partial E}{\partial x'}\frac{\partial E}{\partial t'} + \frac{\partial^2 E}{\partial t'^2}\right)$$

This does not have the same form as the electromagnetic wave equation. Therefore, the electromagnetic wave equation is not invariant under a Galilean transformation.

Note: If the wave equation were invariant under a Galilean transformation, we would have gotten

$$\frac{\partial^2 E}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2}$$

for the transformed coordinates.

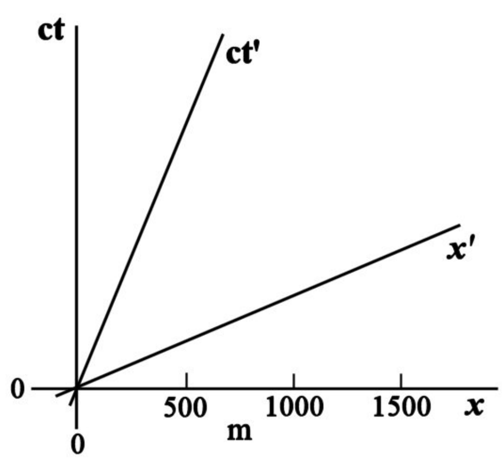
Question 4

In frame S, event B occurs $2\,\mu$s (2×10^{-6} s) after event A and at $x_B = 1.5$ km from event A. Take event A to occur at time $t_A = 0$ and position $x_A = 0$ in frame S.

(a) How fast must an observer in frame S' be moving along the positive x-axis so that events A and B occur simultaneously in his frame?

(b) Is it possible for event B to precede event A for some observer?

(c) Roughly copy the Minkowski diagram below for frames S and S'. Indicate events A and B on your diagram. If you answered "yes" to part (b) indicate the axes ct'' and x'' of an inertial frame S'' for which event B occurs before event A. If you answered "no" to part (b), use the diagram to explain why.



(d) Compute the spacetime separation $(\Delta s)^2$ between the events.

(e) Are the two events causally related? Explain your answer.

Solution

Part A

These kinds of questions may seem a bit confusing on the first read, but they are usually very simple. Always start by writing down what you know and work from there. From the question, we know that $\Delta t = t_B - t_A = 2\,\mu\text{s}$ and $\Delta x = x_B - x_A = 1.5\,\text{km}$.

Part A of the question introduces an observer in frame S' that moves past frame S at an unknown speed V.

The origin of the frames are always arbitrary, so we can choose that to be anything. It is almost always simplest to choose the origins to be at the an event, or Event A in this case. Remember that an "event" is just something that happens that we can assign a specific set of coordinates to. With these kinds of problems that have only one spatial dimension, the sets of coordinates will consist of one spatial and one temporal coordinate. In this case we will choose the origin of both frames to coincide with Event A. This means that we assign $(x_A,\, t_A) = (x'_A,\, t'_A) = (0,\, 0)$.

Part A of this question mentions that in S', Events A and B occur simultaneously so that $t'_A = t'_B$.

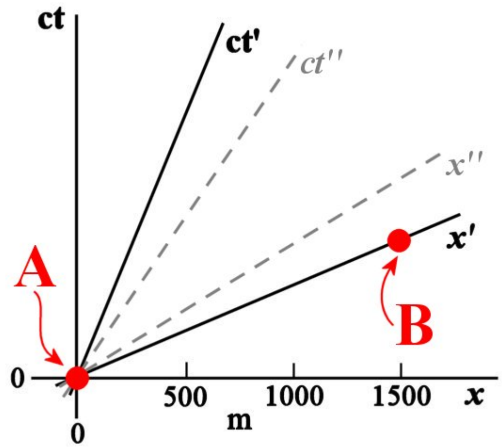
We can now construct a table with all the known spacetime coordinates.

	Event A	Event B
Frame S	$x_A = 0\,\text{km}$; $t_A = 0\,\text{s}$	$x_B = 1.5\,\text{km}$; $t_B = 2\,\mu\text{s}$
Frame S'	$x'_A = 0\,\text{km}$; $t'_A = 0\,\text{s}$	$x'_B =?$; $t'_B = 0\,\text{s}$

With some problems, it will also make sense to draw a picture of the situation to help visualize it.

This part of the question requires us to determine the speed V of S' relative to S. From the transformation rules for intervals, we have

$$\begin{aligned}
\Delta\bar{t} &= \gamma\left(\Delta t - V\Delta x/c^2\right)\\
\bar{t}_B - \bar{t}_A &= \gamma\left((t_B - t_A) - V\left(x_B - x_A\right)/c^2\right)\\
0 &= \gamma\left(2\times 10^{-6}\,\text{s} - V\left(1500\,\text{m}\right)/c^2\right)
\end{aligned}$$



$$
\begin{aligned}
V\,(1500\,\mathrm{m})\,/c^2 &= 2\times 10^{-6}\,\mathrm{s} \\
V &= \frac{(2\times 10^{-6}\,\mathrm{s})\,c^2}{1500\,\mathrm{m}} \\
&= \frac{(2\times 10^{-6}\,\mathrm{s})\,(3\times 10^{8}\,\mathrm{ms}^{-1})^2}{1500\,\mathrm{m}} \\
&= 1.2\times 10^{8}\,\mathrm{ms}^{-1} \\
&= 0.4c
\end{aligned}
$$

Note that the coordinates were converted to SI units (seconds and metres) so that we get an answer in ms^{-1} (meters per seconds). If you kept them in the original units, you had to convert $3\times 10^{8}\,\mathrm{ms}^{-1}$ to kilometers per millisecond first.

So $\bar{S}$ is moving with a speed $V = 0.4c$ relative to S.

Part B

Yes.

Part C

Event A should be at the origin of both sets of coordinates and Event B should be above 1500 km on the x axis and on the x' axis, as it occurs at $t' = 0$.

The axes of the frame S'' has to "inside" the axes of S', indicating that S'' travels at a speed greater that $0.4c$ relative to S. Event A is still on the origin of S'', so that occurs at time $t'' = 0\,\mathrm{s}$. Event B is below the x''axis, so that it will occur at a negative time, i.e. *before* $t'' = 0\,\mathrm{s}$. (Remember that the x'' axis indicates the line for which $t'' = 0\,\mathrm{s}$)

Part D

The spacetime separation is given by

$$\begin{aligned}(\Delta s)^2 &= (c\Delta t)^2 - (\Delta x)^2 \\ &= \left[\left(3\times 10^8\,\mathrm{ms}^{-1}\right)\left(2\times 10^{-6}\,\mathrm{s}\right)\right]^2 - (1500\,\mathrm{m})^2 \\ &= 3.6\times 10^5\,\mathrm{m}^2 - 2.25\times 10^6\,\mathrm{m}^2 \\ &= -1.89\times 10^6\,\mathrm{m}^2\end{aligned}$$

Part E

No. The spacetime separation between the two events is negative. You can also argue that in the calculations above we have shown that there exists a frame where Event A precedes Event B (S) and a frame where Event B precedes Event A (S''). So the one event could not have caused the other one.

Question 5

Consider a particle with mass $m = 10^{-25}$ kg that is moving at a constant velocity described by the vector $\mathbf{v} = (0.3c,\, 0.7c\,, -0.4c)$ relative to an observer in S.

(a) What is the contravariant four-momentum $[P^\mu]$ of the particle?

(b) Assuming Minkowski spacetime, determine the covariant counterpart of the four-momentum $[P_\mu]$.

Solution

Part A

The speed of the particle with respect to the S frame is given by the magnitude of the velocity vector. Therefore, the speed is

$$v = \sqrt{(0.3c)^2 + (0.7c)^2 + (-0.4c)^2}$$

$$\begin{aligned} &= \sqrt{0.74}c \\ &= 0.86c \end{aligned}$$

the negative sign of v_z indicates that the particle is moving in the negative z-direction.

The Lorentz factor between the S frame and the particle's rest frame is

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-0.74}} \\ &= 1.96 \end{aligned}$$

To determine $[P^\mu]$, we need to know the total energy of the particle. This is given by

$$\begin{aligned} E &= \gamma mc^2 \\ &= (1.96)\left(10^{-25}\right)\left(3\times10^8\right)^2 \\ &= 1.76\times10^{-8}\text{ J} \end{aligned}$$

The momentum vector is given by

$$\begin{aligned} \mathbf{p} &= (p_x,\, p_y\, p_z) \\ &= \gamma m\mathbf{v} \\ &= \gamma m\,(v_x,\, v_y\, v_z) \\ &= \gamma m\,(0.3c,\, 0.7c\, -0.4c) \\ &= (1.96)\left(10^{-25}\right)\left(3\times10^8\right)[0.3,\, 0.7\, -0.4] \\ &= 5.88\times10^{-17}\,(0.3,\, 0.7\, -0.4) \\ &= \left(1.76\times10^{-17},\, 4.12\times10^{-17}\, -2.35\times10^{-17}\right)\text{ kgms}^{-1} \end{aligned}$$

Now we have

$$\begin{aligned} [P^\mu] &= (E/c,\, \mathbf{p}) \\ &= (E/c,\, p_x,\, p_y\, p_z) \\ &= \left(\frac{1.76\times10^{-8}}{3\times10^8},\, 1.34\times10^{-17},\, 3.13\times10^{-17}\, -1.79\times10^{-17}\right) \\ &= \left(5.87\times10^{-17},\, 1.76\times10^{-17},\, 4.12\times10^{-17}\, -2.35\times10^{-17}\right) \end{aligned}$$

$$= \quad 10^{-17}\,(5.87,\ 1.76,\ 4.12\ -2.35)$$

Part B

We can determine the covariant covariant counterpart of the four-momentum $[P_\mu]$ by lowering the index of $[P^\mu]$.

We use

$$P_\mu = \sum_{\nu=0}^{3} \eta_{\mu\nu} P^\mu$$

to get

$$P_\mu \quad = \quad \eta_{\mu 0} P^\mu + \eta_{\mu 1} P^\mu + \eta_{\mu 2} P^\mu + \eta_{\mu 3} P^\mu$$

We use the fact that $\eta_{\mu\nu} = 0$ if $\mu \neq \nu$ and $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$ to get

$$\begin{aligned} P_0 &= \eta_{00} P^0 = P^0 = 5.87 \times 10^{-17} \\ P_1 &= \eta_{11} P^1 = -P^1 = -1.76 \times 10^{-17} \\ P_2 &= \eta_{22} P^2 = -P^2 = -4.12 \times 10^{-17} \\ P_3 &= \eta_{33} P^3 = -P^3 = 2.35 \times 10^{-17} \end{aligned}$$

So that we have

$$[P_\mu] = 10^{-17}\,(5.87,\ -1.76,\ -4.12\,2.35)$$

Question 6

A particle is measured in an inertial frame S to have a total energy of $E = 5\,\text{GeV}$ ($1\,\text{GeV} = 10^9\,\text{eV}$) and momentum of $p = 3\,\text{GeV}/c$.

(a) What is the mass of the particle, in GeV/c^2?

(b) What is the speed of the particle?

(c) What is the energy E' of the particle in another inertial frame S' in which the particle's momentum is $p' = 4\,\text{GeV}/c$?

(d) What is the kinetic energy of the particle in S'?

(e) What is the maximum momentum this particle can have, according to the limits set by special relativity?

Solution

Part A

Solution 1 Rearranging the equation

$$E^2 = p^2c^2 + m^2c^4$$

gives

$$m^2 = \frac{E^2}{c^4} - \frac{p^2}{c^2}$$

Substituting the given values for E and p gives

$$\begin{aligned} m^2 &= \frac{(5)^2}{c^4} - \frac{(3\,\mathrm{GeV}/c)^2}{c^2} \\ &= \frac{25\,\mathrm{GeV}^2}{c^4} - \frac{9\,\mathrm{GeV}^2}{c^4} \\ &= \frac{16\,\mathrm{GeV}^2}{c^4} \\ m &= 4\,\mathrm{GeV}/c^2 \end{aligned}$$

Solution 2 If you prefer, you can convert all the quantities to SI units, but this tends to be unnecessarily tedious as the c's don't cancel. We use the conversion factor $1\,\mathrm{eV} = 1.60 \times 10^{-19}\,\mathrm{J}$

$$\begin{aligned} m^2 &= \frac{E^2}{c^4} - \frac{p^2}{c^2} \\ &= \frac{(5 \times 10^9 \times 1.60 \times 10^{-19})^2}{c^4} - \frac{(3 \times 10^9 \times 1.60 \times 10^{-19}/c)^2}{c^2} \\ &= \frac{(8 \times 10^{-10})^2}{c^4} - \frac{(4.8 \times 10^{-10})^2}{c^4} \end{aligned}$$

$$
\begin{aligned}
&= \frac{(8\times 10^{-10})^2}{(3\times 10^8)^4} - \frac{(4.8\times 10^{-10})^2}{(3\times 10^8)^4} \\
&\quad \frac{6.4\times 10^{-19}}{8.1\times 10^{33}} - \frac{2.3\times 10^{-19}}{8.1\times 10^{33}} \\
&= 7.90\times 10^{-53} - 2.84\times 10^{-53} \\
&= 5.06\times 10^{-53} \\
m &= 7.11\times 10^{-27}\,\text{kg}
\end{aligned}
$$

Part B

Solution 1 The Lorentz factor γ depends only on the speed, so if we can calculate γ, we can get V.

$$
\begin{aligned}
E &= \gamma m c^2 \\
\gamma &= \frac{E}{mc^2} \\
&= \frac{5\,\text{GeV}}{(4\,\text{GeV}/c^2)\,c^2} \\
&= \frac{5}{4} \\
\frac{1}{\sqrt{1 - V^2/c^2}} &= \frac{5}{4} \\
1 - V^2/c^2 &= \frac{16}{25} \\
V^2/c^2 &= 1 - \frac{16}{25} \\
&= \frac{9}{25} \\
V &= \frac{3}{5}c
\end{aligned}
$$

Solution 2 Or, if you insist on using SI units, you can get the Lorentz factor as follows

$$
\begin{aligned}
\gamma &= \frac{E}{mc^2} \\
&= \frac{8\times 10^{-10}}{(7.11\times 10^{-27})\,(3\times 10^8)^2} \\
&= \frac{8\times 10^{-10}}{6.4\times 10^{-10}} \\
&= \frac{5}{4}
\end{aligned}
$$

The rest of the solution is the same as for Solution 1.

Part C

In any single inertial frame, the equations of special relativity hold, so we can calculate the energy E' in the frame where the momentum is equal to p'as follows.

$$\begin{aligned} E'^2 &= p'^2c^2 + m^2c^4 \\ &= \left(4\,\mathrm{GeV}/c\right)^2 c^2 + \left(4\,\mathrm{GeV}/c^2\right)^2 c^4 \\ &= 32\,\mathrm{GeV}^2 \\ E' &= 4\sqrt{2}\,\mathrm{GeV} \end{aligned}$$

Remember that the mass of the particle is invariant, so it is the same in all inertial frames.

Part D

The kinetic energy of the particle is given by

$$E'_K = \left(\gamma' - 1\right) mc^2$$

At this point *we do not know the speed of the particle in S', so we do not know the value of γ'*. The Lorentz factor is not invariant. You can calculate the value of γ' for S'using a similar method as we did in Part B, and substitute it into the above equation. Or you can so it like this:

$$\begin{aligned} E'_K &= \left(\gamma' - 1\right) mc^2 \\ &= \gamma' mc^2 - mc^2 \\ &= E' - mc^2 \\ &= 4\sqrt{2}\,\mathrm{GeV} - \left(4\,\mathrm{GeV}/c^2\right) c^2 \\ &= 4\left(\sqrt{2} - 1\right)\,\mathrm{GeV} \\ &= 1.66\,\mathrm{GeV} \end{aligned}$$

Part E

Special relativity places no upper limit on momentum. Below is a graph showing the classical (blue) and relativistic (red) momenta for a object at different speeds. In classical (Newtonian) mechanics, the momentum increases linearly with the speed. In special relativity, the Lorentz factor ensures that the momentum goes to infinity as the speed approaches c. Form the graph you can also see that the relativistic momentum approaches the Newtonian momentum at speeds much smaller than c.