# Tutorial Letter 101/3/2012 <br> Differential Equations <br> APM1513 

Department of Mathematical Sciences

## Semesters 1 and 2

This tutorial letter contains important information about your module

## CONTENTS

1 INTRODUCTION AND WELCOME ..... 4
1.1 Tutorial matter ..... 4
2 PURPOSE OF AND OUTCOMES FOR THE MODULE ..... 4
2.1 Purpose ..... 4
2.2 Outcomes ..... 4
3 LECTURER AND CONTACT DETAILS ..... 5
3.1 Lecturers ..... 5
3.2 Department ..... 5
3.3 University ..... 6
4 MODULE RELATED RESOURCES ..... 6
4.1 Recommended book ..... 6
4.2 Electronic Reserves (e-Reserves) ..... 7
5 STUDENT SUPPORT SERVICES FOR THE MODULE ..... 8
5.1 Contact with fellow students) ..... 8
5.2 myUnisa ..... 8
6 MODULE SPECIFIC STUDY PLAN ..... 8
7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING ..... 8
8 ASSESSMENT ..... 9
8.1 Assessment plan ..... 9
8.1.1 Assessment of assignments ..... 9
8.1.1 Examination admission ..... 10
8.2 General assignment numbers ..... 10
8.2.1 Unique assignment numbers ..... 11
8.2.2 Due dates of assignments ..... 11
8.3 Submission of assignments ..... 11
8.3.1 General remarks ..... 11
9 EXAMINATIONS ..... 12
9.1 Examination period ..... 12
9.2 Examination paper ..... 12
9.3 Previous examination paper ..... 12
10 GETTING STARTED: INSTALLATION OF OCTAVE ..... 14
10.1 Windows ..... 12
10.2 Linux and Mac ..... 12
10.3 Unisa computer laboratories ..... 12
10.3 MATLAB ..... 12
11 ASSIGNMENTS ..... 14

## 1 INTRODUCTION AND WELCOME

We are pleased to welcome you to this module and hope that you will find it both interesting and rewarding. We shall do our best to make your study of this module successful. You will be well on your way to success if you start studying early in the semester and resolve to do the assignments properly.

### 1.1 Tutorial matter

You will receive a number of tutorial letters during the year. A tutorial letter is our way of communicating with you about teaching, learning and assessment. This tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as exam admission. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignment(s), preparing for the examination and addressing questions to your lecturers. In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed study material and other resources and how to obtain it. Please study this information carefully and make sure that you obtain the prescribed material as soon as possible. We have also included certain general and administrative information about this module. Please study this section of the tutorial letter carefully. Right from the start we would like to point out that you must read all the tutorial letters you receive during the semester immediately and carefully, as they always contain important and, sometimes urgent information. We hope that you will enjoy this module and wish you all the best!

## 2 PURPOSE AND OUTCOMES OF THIS MODULE

### 2.1 Purpose

This module will be useful to students interested in developing the basic skills in linear algebra as well as to apply the software package Octave (or MATLAB) for all calculations. Note that we are using the latest version of Octave and this will be part of your study material which you will receive at registration for this module. Students credited with this module will have an understanding of the basic ideas of linear algebra and be able to apply the basic techniques for handling systems of linear equations, matrices, determinants and eigenvectors and linear programming. In all these topics you will be able to apply the software package Octave (or MATLAB) to do all calculations.

### 2.2 Outcomes

The broad outcomes for this module are
(a) To solve systems of linear equations with the use of Octave (or MATLAB)
(b) To perform basic matrix operations.
(c) To use iterative methods to find appropriate solutions for systems
(d) To know what is meant by the eigenvalue equations, to be able to calculate the eigenvalue of a matrics and its corresponding eigenvector and to know the Octave (or MATLAB) code to do it.
(e) To be able to solve linear programming problems by using the software. Specific outcomes are listed in the study guide.

## 3 LECTURER AND CONTACT DETAILS

### 3.1 Lecturers

The lecturers responsible for this module are as follows:
Mr. A. Kubeka
Tel: (012)429 6204
Room no: 6-116
Theo van Wijk Building
e-mail: kubekas@unisa.ac.za

Dr.J. Manale
Tel:
Room no:
Theo van Wijk Building
e-mail:

All queries that are not of a purely administrative nature but are about the content of this module should be directed to us. Please have your study material with you when you contact us. You are always welcome to come and discuss your work please make sure that we are free to help you by making an appointment well before the time. Please come to these appointments well prepared with specific questions that indicate your own efforts to have understood the basic concepts involved. You are also free to write to us about any of the difficulties you encounter with your work. If these difficulties concern exercises which you are unable to solve, you must send us your attempts so that we can see where you are going wrong.

Letters should be sent to:
The Module leader APM2611
Department of Mathematical Sciences
PO Box 392
UNISA
0003
PLEASE NOTE: Letters to lecturers may not be enclosed with or inserted into assignments.

### 3.2 Department

### 3.3 University

If you need to contact the University about matters not related to the content of this module, please consult the publication my Studies @ Unisa that you received with your study material. This booklet contains information on how to contact the University (e.g. to whom you can write for different queries, important telephone and fax numbers, addresses and details of the times certain facilities are open).
Always have your student number at hand when you contact the University.
Please note that all administrative enquiries should be directed to the Unisa Contact Centre. Enquiries will then be channeled to the correct department. The details are as follows:

| Calls (RSA only) | 0861670411 |
| :--- | :--- |
| International Calls | +27116709000 |
| Fax number (RSA) | 0124294150 |
| Fax number (international) | +27124294150 |
| E-mail | study-info@unisa.ac.za |

4 MODULE RELATED RESOURCES

The Department of Despatch will supply you with the following study matter for this module.

- the Study Guide, as Tutorial Letter 501
- a CD containing the Octave software, as well as electronic versions of the study matter
- this tutorial letter, as well as others which may be sent out during the year,
- solutions to the assignments which will be sent out after the relevant closing dates.

Apart from Tutorial letter 101 you will also receive the other tutorial letters during the semester. These tutorial letters will not necessarily be available at the time of registration. Tutorial letters will be despatched to you as soon as they are available or needed (for instance, for feedback on assignments).

Please note that the Study Guide (Tutorial Letter 501) for APM1513 is a complete source of knowledge for you to study this module.
If you have access to the Internet, you can view the study guide and tutorial letters for the modules for which you are registered on the Universitys online campus, myUnisa, at http://my.unisa.ac.za

### 4.1 Recommended book

You may consult the following publication in order to broaden your knowledge of APM1513. A limited number of copies is available in the library

- B.D. Hahn Essential MATLAB for Scientists and Engineers (Pearson Education South Africa, Cape Town, 2002)

A list of recommended titles appears at the end of this tutorial letter. Each title has been allocated a request number which you should supply on the request card when requesting books from the Library.

Recommended books may be requested telephonically from the Main Library by supplying the request numbers and your student number.

NOTE: Do not feel that you should study from this book, simply because we have provided you with it. Sometimes, however, if one really gets bogged down on a particular section or part of the work, a different presentation might just be what is needed to get going again.

## Other recommended books

The following books are also available at the Unisa Library. However, there is a limited number of copies of these books.

- Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974.
- Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972.
- Johnson, Lee W.: Introduction to Linear Algebra (2 $2^{\text {nd }}$ or earlier editions), Addison- Wesley, Reading, MASS., 1989.
- Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.
- Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.
- Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974.
- Nicholson, W.K.: Linear Algebra with Applications ( $3^{r d}$ edition), PWS Publishing Company, Boston.
- Kolman, Bernard \& Hill, David R.: Introductory Linear Algebra; An Applied First Course (8th edition or earlier), Prentice Hall, 2005.
- Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co., Belmont, CA., 1991.


### 4.2 Electronic reserves (e-Reserves)

There are no e-Reserves for this module.

## 5 STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the publication my Studies @ Unisa that you received with your study material.

### 5.1 Contact with fellow students

## Study groups

It is advisable to have contact with fellow students. One way to do this is to form study groups. The addresses of students in your area may be obtained from the following department:

Directorate: Student Administration and Registration
P O Box 392
UNISA
0003

## 5.2 myUnisa

If you have access to a computer that is linked to the internet, you can quickly access resources and information at the University. The myUnisa learning management system is Unisa's online campus that will help you to communicate with your lecturers, with other students and with the administrative departments of Unisa all through the computer and the internet.
To go to the myUnisa website, start at the main Unisa website, www.unisa.ac.za, and then click on the "myUnisa" link below the orange tab labelled "Current students". This should take you to the myUnisa website. You can also go there directly by typing my.unisa.ac.za in the address bar of your browser. Please consult the publication my Studies @ Unisa which you received with your study material for more information on myUnisa

6 MODULE SPECIFIC STUDY PLAN

| Study plan | Semester 1 | Semester 2 |
| :--- | :--- | :--- |
| Outcomes 1.1 to 3.3 to be achieved by | 17 February | 13 August |
| Outcomes 3.4 to 5.6 to be achieved by | 6 March | 28 August |
| Outcomes 6.1 to 6.6 to be achieved by | 4 April | 10 August |
| Revision | 20 April | 25 September |

See the brochure my Studies @ Unisa for general time management and planning skills.

## 7 MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

In each semester there are three written assignments for APM1513. The questions for the assignments are given at the end of this tutorial letter. For each assignment there is a FIXED CLOSING DATE; the date by which the assignment must reach the university. Solutions for each assignment as Tutorial Letter 201, 202, etc. will be posted a few days after the closing date. They will also be made available on myUnisa. Late assignments will be marked, but will be awarded $0 \%$.

Written assignments (Assignments 01, 02 and 03)

Not all the questions in the written assignment will be marked and you will also not be informed beforehand which questions will be marked. The reason for this is that you learn by doing examples, and it is therefore extremely important to do as many problems as possible.

You can self assess the questions that are not marked by comparing your solutions with the printed solutions that will be sent to you.

### 8.1 Assessment plan

### 8.1.1 Assessment of assignments

Please note: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. It is unacceptable for students to submit identical work on the basis that they studied together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

## General guidelines for answering assignments

- The assignments must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
- We will not accept hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.
- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.
- You should use a fixed space font such as courier for computer code and output, and something else for discussion.


### 8.1.2 Examination admission

Please note that lecturers are not responsible for examination admission, and ALL enquiries about examination admission should be directed to the Unisa Contact Centre.

You will be admitted to the examination if and only if Assignment 01 reaches the Assignment Section by 12 March 2012 if you are registered for Semester 1, or by 3 September 2012 if you are registered for Semester 2.

## Semester Exam and Semester Mark

Your semester mark for APM1513 counts 10\% and your exam mark 90\% of your final mark.

The semester mark for this module will be the average of the marks (as percentages) that you obtain for assignments 01,02 and 03 . If you do not submit an assignment, the mark for that assignment is taken to be 0 .

## Example

A student obtains the following marks:

| Assignment 01 | $60 \%$ |
| :--- | ---: |
| Assignment 02 | $50 \%$ |
| Assignment 03 (didn't submit) | $0 \%$ |
| Exam | $60 \%$ |

The semester mark is

$$
\begin{equation*}
\left(\frac{60+50}{3}\right) \% \tag{1}
\end{equation*}
$$

i.e. $38 \%$

The final mark is

$$
\left(\frac{10}{100} \times 38+\frac{90}{100} \times 60\right) \%
$$

i.e. $(3.8+54) \%$
i.e. $59 \%$

### 8.2 General assignment numbers

The assignments are numbered as 01 and 02 for each semester.

### 8.2.1 Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

### 8.2.2 Due dates of assignments

The closing dates for submission of the assignments are:

| SEMESTER 1 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Assignment no | Type | Fixed Closing Date |  |  |  |
| 01 | written | 17 February 2012 |  |  |  |
| 02 | written | 6 March 2012 |  |  |  |
| 03 | written | 4 April 2012 |  |  |  |
| SEMESTER 2 |  |  |  |  |  |
| Assignment no |  |  |  | Type | Fixed Closing Date |
| 01 | written | 13 August 2012 |  |  |  |
| 02 | written | 28 August 2012 |  |  |  |
| 03 | written | 10 September 2012 |  |  |  |

### 8.3 Submission of assignments

### 8.3.1 General remarks

PLEASE NOTE: Enquiries about assignments (e.g. whether or not the University has received your assignment or the date on which an assignment was returned to you) must be addressed to the Unisa Contact Centre at 0861670411 (RSA only), or +27 116709000 (international calls) (also see par. 3 above). You might also find information on myUnisa.

Assignments should be addressed to:
The Registrar
P O Box 392
UNISA
0003

You may submit written assignments and assignments done on mark-reading sheets either by post or electronicaly via myUnisa. Assignments may not be submitted by fax or e-mail. For detailed information and requirements as far as assignments are concerned, see the brochure Your Service Guide @ Unisa that
you received with your study material.

To submit an assignment via myUnisa

- Go to myUnisa.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left-hand menu.
- Click on the assignment number you want to submit.
- Follow the instructions on the screen.


## 9. EXAMINATIONS

### 9.1 Examination period

This module is offered in a semester period of fifteen weeks. This means that if you are registered for the first semester, you will write the examination in May/June 2012 and the supplementary examination will be written in October/November 2012. If you are registered for the second semester you will write the examination in October/November 2012 and the supplementary examination will be written in May/June 2013.

During the semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.

### 9.2 Examination paper

The exam consists of a two hour paper. Note that you are allowed to use a calculator in the exam.

### 9.3 Previous examination paper

We will supply you with a sample exam paper in another tutorial letter to give you an idea of the format of the examination paper that you will write.

## 10. GETTING STARTED STARTED: INSTALLATION OF OCTAVE

This module is mainly about the use of the mathematical software package Octave or MATLAB, and in order to take the module it is a requirement that

- You have regular access to a computer, for example at home, at work, or at a Unisa computer laboratory
- If it is not your own (or Unisa's) computer, you have permission to install the software Octave or MATLAB onto it
- You have had some prior experience in computer programming. If you do not meet the above requirements, then you are wasting your time and ours by trying to take this module.

Your study package contains a CD, and when you open the CD you will see that it contains the following directories, sub-directories and files

- StudyMaterial
- TL501.pdf
- TL101.pdf
- Octave
- Windows
* ocatave-3.0.0-setup.exe
* ocatave-3.0.1-setup.exe
- Linux
* ocatave-3.0.0.tar
* ocatave-3.0.1.tar
- Mac
* ocatave-3.0.0-i386.dmg
* ocatave-3.0.0-ppc.dmg
* ocatave-3.0.1-i386.dmg
* ocatave-3.0.1-ppc.dmg

We include both the latest (in August 2008) available version of Octave as well as the version (3.0.0) that was used in writing the Study Guide. You should install the latest version, but if you notice any discrepancies between the behaviour of this version and that described in the Study Guide, you could re-try with version 3.0.0. If you like, you can also go on the internet to see if there is an even later version Of Octave. The Octave home page is http://www.octave.org, and the repository with various versions of Octave is http://www.gnu.org/software/octave/download.html

### 10.1 Windows

Our experience has been that the installation of Octave is easy and straightforward. Just insert the CD into your computers CD drive, open it using Windows Explorer, and double click the .exe file you want to install. The installation window will open and unless you have experience in systems programming you should
just accept the default options. The only exception is the page that asks you to choose a graphics backend. The default option is a development package based on java, but we have found that on some machines this option is unstable. So we suggest that you click on the stable version based on gnuplot and use that instead. It is as simple as that!

### 10.2 Linux and Mac

Versions of Octave for installation on a Linux or Mac machine are included on the CD, but your lecturers can provide only limited help if you experience system-related problems with these versions.

### 10.3 Unisa computer laboratories

If you are using a Unisa computer laboratory, you should find that Octave has already been installed and the Octave icon will be on the Desktop.

### 10.4 MATLAB

MATLAB is a commercial software product that has to be purchased, whereas Octave is available free of charge. Although there are occasional differences, the syntax of the two programming systems is almost identical. In some advanced, specialized applications we have found that MATLAB was able to solve a problem but Octave was unsuccessful. However, for the introductory purposes of this module, Octave is quite sufficient. If you wish, for example if the computer that you are using for this module already has MATLAB installed, then you are welcome to use MATLAB rather than Octave; but please be aware that there will be minor syntactical and layout differences between MATLAB and the notes in the Study Guide. Otherwise, we would suggest that you work entirely with Octave.

We hope that you will enjoy this module and we wish you success with your studies

Kind regards

## 11. ASSIGNMENTS

## First Semester

| ONLY FOR SEMESTER 1 STUDENTS |
| :---: |
| ASSIGNMENT 01 |
| Getting started with Matlab/Octave. Introduction to programming with |
| Matlab/Octave. Use of Matlab/Octave to solve linear systems of equations. |
| FIXED CLOSING DATE: 17 February 2012 |
| UNIQUE ASSIGNMENT NUMBER: 342368 |

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
- We will not accept hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.
- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.
- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 75 marks distributed as shown, and 75 marks $=100$


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ rand("state",student_number0918);
$>$ rand(1)
where student_number0912 is your student number with "0918" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
$>$ rand("state",12345670918);
$>\operatorname{rand}(1)$

## QUESTION 2

Normally Octave outputs numbers with 5 significant figures. Use the help facility with the keyword format to find how to get output with 15 significant figures. Then evaluate $\sqrt{2}$ to 15 significant figures.

## QUESTION 3

Solve the simultaneous equations

$$
\begin{aligned}
& 2 x_{1}+5 x_{2}=6 \\
& 4 x_{1}+3 x_{2}=2
\end{aligned}
$$

## QUESTION 4

Evaluate the series
$12+22+32+\ldots+1002$

## QUESTION 5

A formula to find a numerical approximation to the second derivative of a function $f(x)$ is

$$
f^{\prime \prime}(x)=\frac{d^{2} f}{d x^{2}}=\frac{f(x+h)-2 f(x)+f(x-h)}{h^{2}}
$$

with the approximation being better and better as $h \rightarrow 0$. Write a function file deriv2.m that implements the formula. The inputs should be the function to be differentiated (remember the function handle construct @), the value of $x$, and the value of $h$. Use your code to estimate the second derivative of $\sin (x)$ at $x=\pi / 4$ with $h=10^{-1}$.

## QUESTION 6

Evaluate the series $\sum_{n=1}^{\infty} u_{n}$ in which $u_{n}$ is not known explicitly but is given in terms of a recurrence relation. You should stop the summation when $|u n|<10^{-8}$

$$
n_{n+1}=\left(u_{n}\right)^{2} \text { with } u_{1}=0.5
$$

## QUESTION 7

In the following formula, $R$ can take any value between 5 and 6 . Work out the value of $I$ for $R=5.005 .01$, $5.02, \ldots, 5.99$, and 6 . Then find the average value of $I$.

$$
I=\frac{25}{\sqrt{R^{2}+20 \pi^{2}}}
$$

## ONLY FOR SEMESTER 1 STUDENTS

ASSIGNMENT 02

## Use of Matlab/Octave to solve linear systems of equations.Overdetermined and underdetermined systems of linear equations.Eigenvalues, eigenvectors and matrix diagonalization <br> FIXED CLOSING DATE: 6 March 2012 <br> UNIQUE ASSIGNMENT NUMBER: 226089

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
- We will not accept hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.
- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.
- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 100 marks distributed as shown, and 100 marks = 100


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ rand("state",student_number0912);
$>\operatorname{rand}(1)$
where student_number0912 is your student number with "0912" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
> rand("state",12345670912);
$>\operatorname{rand}(1)$

## QUESTION 2

Solve the following systems of equations, using the $A \backslash b$ construct, as well as the Gauss-Seidel method with a tolerance of $10-7$ (in some cases convergence may not occur)
(a)

$$
\begin{aligned}
20 x_{1}-x_{2}+x_{3} & =20 \\
2 x_{1}+10 x_{2}-x_{3} & =11 \\
x_{1}+x_{2}-20 x_{3} & =-18
\end{aligned}
$$

(b)

$$
\begin{aligned}
2 x_{1}-x_{2}+3 x_{3} & =8 \\
4 x_{1}+2 x_{2}-5 x_{3} & =-9 \\
6 x_{1}+3 x_{2}+x_{3} & =12
\end{aligned}
$$

## QUESTION 3

Write a function file that takes as input a matrix $A$, and tests whether or not the matrix is (a) square, and (b) diagonally dominant, reporting the answers on the screen. Show that your code is correct by testing it for the matrix in question 2, as well as for cases where the matrix is not diagonally dominant, and not square.

## QUESTION 4

Modify the function file iterative_linear_solve.m to produce a new function file iterative_linear_solve.m, in which the stopping condition is that magnitude of the residual $(A x-b)$ should be less than a given tolerance.

Show that your code works by applying it to the problem in question 2, using the Gauss-Seidel method.

In practice, this alternative stopping method is not often used. Why not?

## QUESTION 5

The Hilbert matrix is a square $n \times n$ matrix defined by

$$
H_{i j}^{n}=\frac{1}{i+j-1}
$$

Define $b^{n}$ to be a column vector of dimension $n$, and with each element 1. Construct $b^{n}$ and, and then solve for $x^{n}, x^{n}=b^{n}$, in the cases $n=4,7,10$ and 13 . Comment on the results.

## QUESTION 6

The sales figures for a business are as follows for the first six months of the year:

R40 000, R44 000, R52 000, R64 000, R80 000, R84 000.

The owner believes that the sales curve can be approximated by a quadratic function. Find the best quadratic fit to the data, and use it to estimate the projected sales for the rest of the year.

## QUESTION 7

Find the best straight line $(y=m x+c)$ fit to the data points
$(x, y)=(0,1),(2,0),(3,1),(3,2),(3,1)$.

Produce a graph showing the line, together with the given data points as discrete points.

## QUESTION 8

Find the eigenvalues and eigenvectors of the following matrices, using both eig and power_method (for the dominant eigenvalue and eigenvector). If the power method fails, discuss why. For those matrices that are diagonalizable, give the diagonalization matrix
(a)

$$
\left(\begin{array}{llll}
-1.6731385 & -3.6381454 & -1.8272855 & -2.7022868 \\
-1.7080530 & -0.0039461 & 0.1019678 & -1.3595453 \\
2.4426950 & 3.4321965 & 3.8982919 & 2.1372124 \\
5.7672696 & 5.4815125 & 1.3344020 & 6.2787928
\end{array}\right)
$$

(b)

$$
\left(\begin{array}{llll}
4.9541 & 6.6650 & 8.1445 & 2.9998 \\
10.1406 & 17.3006 & 14.2773 & 9.3552 \\
8.9200 & 10.6881 & 8.4619 & 7.7253 \\
-25.6960 & -40.6289 & -38.0172 & -21.7166
\end{array}\right)
$$

## QUESTION 9

Modify the power method so that it finds the smallest eigenvalue and corresponding eigenvector. You do this by evaluating

$$
x_{n+1}=A^{-1} x_{n}
$$

rather than $x_{n+1}=A x_{n}$. When convergence occurs, it is to the eigenvector corresponding to the smallest eigenvalue, and to the inverse of the smallest eigenvalue. Demonstrate the validity of your code by running it on a number of test cases.

# ONLY FOR SEMESTER 1 STUDENTS <br> ASSIGNMENT 03 <br> Linear programming <br> FIXED CLOSING DATE: 4 April 2012 <br> UNIQUE ASSIGNMENT NUMBER: 150317 

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
- We will not accept hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.
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- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 75 marks distributed as shown, and 75 marks = 100


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ rand("state",student_number0913);
$>$ rand(1)
where student_number0913 is your student number with "0913" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
$>$ rand("state",12345670913);
$>\operatorname{rand}(1)$

## QUESTION 2

A company receives orders to deliver its goods to three different cities as follows

| City | A | B | C |
| :--- | :--- | :--- | :--- |
| Order | 22 | 21 | 25 |

where the quantity of the order is in truckloads. The company has sufficient stock in its warehouses and a truckload of goods can be delivered from any warehouse to any city. However, there are a limited number
of trucks available at each warehouse

| Warehouse | $P$ | $Q$ | $R$ |
| :--- | :--- | :--- | :--- |
| Trucks | 17 | 31 | 26 |

The variable costs (in Rands) per truckload to deliver goods from each warehouse to each destination are
City A B C
Warehouse

| $P$ | 6000 | 5000 | 4000 |
| :--- | :--- | :--- | :--- |
| Q | 5000 | 5500 | 6000 |
| $R$ | 9000 | 8500 | 8000 |

What is the cheapest delivery schedule and what is its cost?
[Hint: Choose the variables as $x_{1}$ to $x_{9}$ where $x_{1}$ is the number of truckloads from P to $\mathrm{A}, x_{2}$ is the number from P to $\mathrm{B}, \ldots, x_{9}$ is the number from R to C ]

## QUESTION 3

Find the minimum value as well as the point at which the minimum occurs of

$$
L=-3 x 1-4 x 2+x 3
$$

subject to the constraints

$$
\begin{aligned}
-x 1+x 2+2 x 3 & \leq 5 \\
2 x 1+x 2+x 3 & \leq 20 \\
x 1, x 2, x 3 & \geq 0
\end{aligned}
$$

## QUESTION 4

Find the maximum value as well as the point at which the maximum occurs of

$$
L=2 x 1+3 x 2+4 x 3+3 x 4
$$

subject to the constraints

$$
\begin{aligned}
1.5 x 1+2 x 2+1.5 x 3+x 4 & \leq 30 \\
1 x 1+2 x 2+1 x 3+3 x 4 & \leq 45 \\
5 x 1+4 x 2+7 x 3+2 x 4 & \leq 65 \\
6 x 1+3 x 2+7 x 3+4 x 4 & \leq 60 \\
8 x 1+4 x 2+8 x 3+2 x 4 & \leq 70 \\
x 1, x 2, x 3, x 4 & \geq 0
\end{aligned}
$$

## Second semester



- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
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- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.
- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 80 marks distributed as shown, and 80 marks = 100


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ rand("state",student_number0919);
$>$ rand(1)
where student_number0912 is your student number with "0919" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
$>$ rand("state",12345670919);
$>\operatorname{rand}(1)$

## QUESTION 2

The command etime takes as input two 6-dimensional row vectors. Use the help facility for etime and clock to find out more. Then find the number of seconds between 09h 27m and 35s on 17 February 2002, and

17h 49m 02s on 23 August 2006.

## QUESTION 3

Evaluate the series

$$
1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots+\frac{1}{1001}
$$

## QUESTION 4

Evaluate the series $\sum_{n=1}^{\infty} u_{n}$ in which $u_{n}$ is not known explicitly but is given in terms of a recurrence relation. You should stop the summation when $|u n|<10^{-2}$

$$
u_{n+1}=\left(u_{n-1}\right)^{2}+\left(u_{n}\right)^{1.5} \text { with } u_{1}=0.1, u_{2}=0.2
$$

[20]

## QUESTION 5

Plot, on the same graph, the two functions

$$
\begin{array}{r}
x=e^{0.6667 t}-1.5 \\
y_{i+1}=\cos \left(t^{2}\right)
\end{array}
$$

in the range $-2 \leq t \leq 2$. Use the graph to estimate the value of $t$ at which the two functions intersect

## QUESTION 6

Given

$$
x_{k}=\frac{L_{k}}{\sqrt{L_{k}+20 \pi^{2}}}
$$

where

$$
L_{k}=\frac{1}{1+\frac{1}{k}}, \quad k=1 \cdots 47
$$

find

$$
S=\boldsymbol{x} . \boldsymbol{L}=\sum_{K=1}^{47} x_{k} L_{k}
$$

## QUESTION 7

$A$ is a $3 \times 100$ matrix defined as follows

$$
\left(\begin{array}{lllll}
1 & 2 & \cdots & \cdots & 100 \\
101 & 102 & \cdots & \cdots & 200 \\
201 & 202 & \cdots & \cdots & 300
\end{array}\right)
$$

Let $B=A^{\prime} A$ (where $A^{\prime}$ is the transpose of $A$ ), then find $B_{5,6}$.

## ONLY FOR SEMESTER 2 STUDENTS

ASSIGNMENT 02

## Use of Matlab/Octave to solve linear systems of equations.Overdetermined and underdetermined systems of linear equations.Eigenvalues, eigenvectors and matrix diagonalization FIXED CLOSING DATE: 28 August 2012 UNIQUE ASSIGNMENT NUMBER: 286629

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
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- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.
- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 100 marks distributed as shown, and 100 marks = 100


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ rand("state",student_number0914);
$>\operatorname{rand}(1)$
where student_number0913 is your student number with "0914" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
> rand("state",12345670914);
$>$ rand(1)

## QUESTION 2

Solve the following systems of equations, using the $A \backslash b$ construct, as well as the Gauss-Seidel method with a tolerance of $10-7$ (in some cases convergence may not occur)
(a)

$$
\begin{aligned}
0.1 x_{1}+0.05 x_{2}+0.1 x_{3} & =1.3 \\
12 x_{1}+25 x_{2}-3 x_{3} & =10 \\
-7 x_{1}+8 x_{2}+15 x_{3} & =2
\end{aligned}
$$

(b)

$$
\begin{aligned}
12 x_{1}-3 x_{2}+4 x_{3}-2 x_{4} & =12 \\
2 x_{1}+10 x_{2}-x_{3}-20 x_{4} & =15 \\
x_{1}-x_{2}+20 x_{3}+4 x_{4} & =-7 \\
x_{1}+x_{2}-20 x_{3}-3 x_{4} & =-5
\end{aligned}
$$

## [5]

## QUESTION 3

Modify the function file gauss_seidel.m to produce a new function file jacobi.m that implements the Jacobi method. Now use the Jacobi method to solve example 3.2.1, i.e.

$$
\begin{aligned}
20 x_{1}+x_{2}-x_{3} & =17 \\
x_{1}-10 x_{2}+x_{3} & =13 \\
-x_{1}+x_{2}+10 x_{3} & =18
\end{aligned}
$$

## QUESTION 4

Define the $100 \times 100$ square matrix $A$ and the column vector $b$ by

$$
A_{i j}=I_{i j}+\frac{1}{-j^{2}+1}, \quad b_{i}=1+\frac{2}{i}, \quad 1 \leq i, j \leq 100
$$

where $I_{i j}$ is the $100 \times 100$ identity matrix (i.e. 1 on the main diagonal and 0 everywhere else). Solve $A x=b$ for $x$ using both the Gauss-Seidel method and the $A \backslash b$ construct. Do not give the whole vector $x$ in your output, but only $x_{2}, x_{50}$ and $x_{99}$.

## QUESTION 5

A formula for the population of the USA is

$$
P(t)=P_{0}-a e^{-0.02(t-1800)}
$$

where $t$ is the date in years. Some actual data is as follows

| Date | Population |
| :--- | :--- |
| 1800 | 5308000 |
| 1820 | 9638000 |
| 1840 | 17069000 |
| 1870 | 38558000 |
| 1900 | 75995000 |
| 1930 | 122775000 |
| 1950 | 150697000 |

Find values of $P_{0}$ and $a$ that give a best fit of the formula to the data. Produce a graph showing the function $P(t)$ against time as a continuous line, together with the given data points as discrete points

## QUESTION 6

Find the cubic polynomial that best fits the data points
$(x, y)=(-1,14),(0,-5),(1,-4),(2,1),(3,22)$.

Produce a graph showing the polynomial, together with the given data points as discrete points.

## QUESTION 7

Find the eigenvalues and eigenvectors of the following matrices, using both eig and power_method (for the dominant eigenvalue and eigenvector). If the power method fails, discuss why. For those matrices that are diagonalizable, give the diagonalization matrix
(a)

$$
\left(\begin{array}{lllll}
2.781344 & -1.921334 & 0.493612 & 1.367198 & -1.014289 \\
0.015050 & -0.205731 & 0.903377 & 1.780261 & -0.824057 \\
-0.087144 & 0.606003 & 2.977860 & -0.140473 & -0.750938 \\
0.212440 & -2.477599 & 0.980236 & 4.233562 & -1.207581 \\
-0.136646 & -1.168924 & 0.453692 & 0.915245 & 1.712964
\end{array}\right)
$$

(b)

$$
\left(\begin{array}{llll}
-1.54575 & -3.47002 & -1.70112 & -2.58917 \\
-3.28104 & -2.07998 & -1.45597 & -2.75629 \\
0.55497 & 0.94078 & 2.02863 & 0.46100 \\
8.94120 & 9.67047 & 4.47796 & 9.09710
\end{array}\right)
$$

## QUESTION 8

Consider a fictional species, and suppose that the population can be divided into three different age groups: babies, juveniles and adults. Let the population in year $n$ in each of these groups be

$$
x_{(n)}=\left(\begin{array}{c}
x_{b(n)} \\
x_{j(n)} \\
x_{a(n)}
\end{array}\right)
$$

The population changes from one year to the next according to $x_{(n+1)}=A x_{(n)}$, where the matrix $A$ is

$$
A=\left(\begin{array}{lll}
1 / 2 & 5 & 3 \\
1 / 2 & 0 & 0 \\
0 & 2 / 3 & 0
\end{array}\right)
$$

In the long term, what will be the relative distribution of the population amongst the age groups?

# ONLY FOR SEMESTER 2 STUDENTS ASSIGNMENT 03 <br> Linear programming. <br> FIXED CLOSING DATE: 10 September 2012 <br> UNIQUE ASSIGNMENT NUMBER: 240835 

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via MyUnisa.
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- You should use a fixed space font such as courier for computer code and output, and something else for discussion.
- Question 1 does not carry any marks, but is compulsory as it enables us to check that your student number was actually used in an Octave (or MATLAB) session
- There are 80 marks distributed as shown, and 80 marks $=100$


## QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
rand("'state",student_number0915);
rand(1)
where student_number0913 is your student number with "0915" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
> rand("state",12345670915);
$>$ rand(1)

## QUESTION 2

The Suitcase Manufacturing Company produces a number of different types of suitcase of varying qualities, which are called S1, S2, S3, S4 and S5. The manufacturing process involves different departments in the factory, and we call these departments D1 to D6. Each suitcase requires time (in minutes) in the various departments as follows

|  | D1 | D2 | D3 | D4 | D5 | D6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | 10 | 15 | 10 | 12 | 5 | 5 |
| S2 | 15 | 20 | 16 | 20 | 5 | 5 |
| S3 | 21 | 25 | 20 | 20 | 8 | 8 |
| S4 | 26 | 21 | 28 | 25 | 10 | 10 |
| S5 | 33 | 28 | 30 | 29 | 15 | 15 |

The contribution to gross profit (i.e., the selling price less the cost of raw materials) of each type of suitcase is given in the following table, which also shows the minimum number of each type of suitcase that must be produced together with the maximum number (in terms of contracts with retail stores)

|  | S1 | S2 | S3 | S4 | S5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Profit (Rands) | 120 | 150 | 235 | 300 | 350 |
| Minimum number | 200 | 100 | 100 | 100 | 100 |
| Maximum number | 500 | 300 | 300 | 300 | 300 |

In addition, there is, this month, a supply limitation on the locks used on the higher quality suitcases (S3, S4, and S5), and the total production of these suitcases cannot exceed 600.

Each department can provide 24000 minutes per month, except Department D6, which can only offer 15000 minutes. How much of each product line should be produced so as to maximize the company's
trading profit?

## QUESTION 3

Find the minimum value as well as the point at which the minimum occurs of

$$
L=-2 x_{1}-5 x_{2}+x_{3}
$$

subject to the constraints

$$
\begin{aligned}
x_{1}+2 x_{2}-x_{3} & \leq 6, \\
x_{2}+2 x_{3} & \leq 6, \\
2 x_{2}+x_{3} & \leq 4, \\
x_{1}, x_{2}, x_{3} & \geq 0 .
\end{aligned}
$$

## QUESTION 4

Find the minimum value as well as the point at which the minimum occurs of

$$
L=-2 x_{1}-10 x_{2}+27 x_{3}+50 x_{4}+32 x_{5}
$$

subject to the constraints

$$
\begin{aligned}
x_{1}+2 x_{2}+x_{3}+1 x_{4}+2 x_{5} & \leq 6, \\
x_{2}+2 x_{3}+7 x_{4}-3 x_{5} & \leq 6, \\
2 x_{2}+x_{3}+1 x_{4}-2 x_{5} & \leq 4, \\
6 x_{1}+x_{2}+x_{3}+x_{5} & \leq 16, \\
-2 x_{3}+4 x_{4}+9 x_{5} & \leq 30, \\
x_{1}, x_{2}, x_{3}, x_{4}, x_{5} & \geq 0 .
\end{aligned}
$$

