Tutorial Letter 101/3/2014

Applied Linear Algebra
APM1513

Semesters 1 & 2

Department of Mathematical Sciences

IMPORTANT INFORMATION:

This tutorial letter contains important
information about your module.

BAR CODE

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CONTENTS

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1   INTRODUCTION

We are pleased to welcome you to this module and hope that you will find it both interesting and rewarding. We shall do our best to make your study of this module successful. You will be well on your way to success if you start studying early in the semester and resolve to do the assignments properly.

You will receive a number of tutorial letters during the year. A tutorial letter is our way of communicating with you about teaching, learning and assessment. This tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as exam admission. We urge you to read it carefully and to keep it at hand when working through the study material, preparing the assignment(s), preparing for the examination and addressing questions to your lecturers. In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed study material and other resources and how to obtain it. Please study this information carefully and make sure that you obtain the prescribed material as soon as possible. We have also included certain general and administrative information about this module. Please study this section of the tutorial letter carefully. Right from the start we would like to point out that **you must read all the tutorial letters** you receive during the semester **immediately and carefully**, as they always contain important and, sometimes urgent information.

We hope that you will enjoy this module and wish you all the best!

2   PURPOSE OF AND OUTCOMES FOR THE MODULE

2.1   Purpose

This module will be useful to students interested in developing the basic skills in linear algebra as well as to apply the software package "Octave" for all calculations. Note that we are using the latest version of "Octave" and this will be part of your study material which you will receive at registration for this module. Students credited with this module will have an understanding of the basic ideas of linear algebra and be able to apply the basic techniques for handling systems of linear equations, matrices, determinants and eigenvectors and linear programming. In all these topics you will be able to apply the software package Octave or Matlab to do all calculations.

2.2   Outcomes

The broad outcomes for this module are

- To solve systems of linear equations with the use of Octave or Matlab.
- To perform basic matrix operations.
- To use iterative methods to find appropriate solutions for systems
- To know what is meant by the eigenvalue equation, to be able to calculate the eigenvalue of a matric and its corresponding eigenvector and to be able to write the Octave or Matlab code to do so.

- To be able to solve linear programming problems and to give a geometric interpretation by means of an illustration in the two or more dimensional cases by using your software.

Specific outcomes are listed in the study guide.

3   LECTURER(S) AND CONTACT DETAILS

3.1   Lecturer(s)

The lecturers responsible for this module are as follows:

Mr. A. Kubeka
Tel: (012) 429 6204
Room no: 6-111
Theo van Wijk Building
Pretoria Campus
e-mail: kubekas@unisa.ac.za

Dr. J. Manale
Tel: (011) 471 2912
Room no: 329
Block C
Florida Campus
e-mail: manaljm@unisa.ac.za

3.2   Department

The contact details of the department are as follows:

Department of Mathematical Sciences
Office: Theo van Wijk Building, Room 6-128
Telephone: (012) 429 6202
Fax: 012 429 6064
E-mail: mathsciences@unisa.ac.za

3.3   University

If you need to contact the University about matters not related to the content of this module, please consult the *my Studies @ Unisa* brochure. Always use your student number when you contact the University.

4 MODULE RELATED RESOURCES

4.1 Prescribed books

There are no prescribed books for this module.

4.2 Recommended books

You may consult the following publication in order to broaden your knowledge of APM1513. A **limited** number of copies is available in the Library.

- B.D. Hahn *Essential MATLAB for Scientists and Engineers* (Pearson Education South Africa, Cape Town, 2002)

Recommended books may be requested telephonically from the Main Library in Pretoria by supplying the **request numbers** and your **student number**.

- Ayres, Frank: Schaum's Outline of Theory and problems of Matrices, McGraw-Hill, New York, 1974.

- Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972.

- Johnson, Lee W.: Introduction to Linear Algebra (2nd or earlier editions), Addison-Wesley, Reading, MASS., 1989.

- Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.

- Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.

- Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974.

- Nicholson,W.K.: Linear Algebra with Applications, (3rd edition), PWS Publishing Company, Boston.

- Kolman, Bernard & Hill, David R.: Introductory Linear Algebra; An Applied First Course (8th edition or earlier), Prentice Hall, 2005.

- Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co., Belmont, CA., 1991.

NOTE: Do not feel that you **should** study from these books, simply because we have provided you with this list. Sometimes, however, if one really gets bogged down on a particular section or part of the work, a different presentation might just be what is needed to get going again.

4.3 Electronic Reserves (e-Reserves)

There are no e-Reserves for this module.

5   STUDENT SUPPORT SERVICES FOR THE MODULE

For information on the various student support systems and services available at Unisa (e.g. student counselling, tutorial classes, language support), please consult the *my Studies @ Unisa* that you received with your study material.

6   MODULE SPECIFIC STUDY PLAN

Study plan	Semester 1	Semester 2
Outcomes 1.1 to 3.3 to be achieved by	15 February	14 August
Outcomes 3.4 to 5.6 to be achieved by	6 March	28 August
Outcomes 6.1 to 6.6 to be achieved by	4 April	9 August
Revision	20 April	25 September

See the brochure *my Studies @ Unisa* for general time management and planning skills.

7   MODULE PRACTICAL WORK AND WORK INTEGRATED LEARNING

There are no practicals for this module.

8   ASSESSMENT

8.1   Assessment plan

In each semester there are three assignments for APM1513. For each assignment there is a **FIXED CLOSING DATE**; the date by which the assignment **must reach** the university. Solutions for each assignment (communicated as Tutorial Letter 201, 202, etc). will be posted a few days after the closing date. They will also be made available on *myUnisa*.

Late assignments will be marked, but will be awarded 0%.

Written assignments (Assignments 01, 02 and 03)
Not all the questions in the written assignments will be marked and you will also not be informed beforehand which questions will be marked. The reason for this is that you learn by doing examples, and it is therefore extremely important to do as many problems as possible. You can self assess the questions that are not marked by comparing your solutions with the printed solutions that will be sent to you.

8.2   General assignment numbers

The assignments are numbered as 01, 02 and so on for each semester.

8.2.1 Unique assignment numbers

Please note that each assignment has a unique assignment number which must be written on the cover of your assignment.

8.2.2 Due dates for assignments

The closing dates for submission of the assignments are:

SEMESTER 1	

Assignment no	Fixed closing date
01	14 February 2014
02	14 March 2014
03	09 April 2014

SEMESTER 2	

Assignment no	Fixed closing date
01	15 August 2014
02	15 September 2014
03	25 September 2014

8.3   Submission of assignments

Please note: Although students may work together when preparing assignments, each student must write and submit his or her own individual assignment. It is unacceptable for students to submit identical assignments on the basis that they worked together. That is copying (a form of plagiarism) and none of these assignments will be marked. Furthermore, you may be penalised or subjected to disciplinary proceedings by the University.

Assignments should be addressed to:

The Registrar
P O Box 392
UNISA
0003

You may submit written assignments and assignments done on mark-reading sheets either by post or electronicaly via *myUnisa*. Assignments may **not** be submitted by fax or e–mail. For detailed information and requirements as far as assignments are concerned, see the brochure *my Studies @ Unisa* that you received with your study material.

To submit an assignment via myUnisa:

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on assignments in the left-hand menu.
- Click on the assignment number you wish to submit.
- Follow the instructions on the screen.

8.4 Assignments

ONLY FOR SEMESTER 1 STUDENTS
ASSIGNMENT 01
Getting started with MATLAB/Octave.
Introduction to programming with MATLAB/Octave.
Use of MATLAB/Octave to solve linear systems of equations.
FIXED CLOSING DATE: 14 February 2014
UNIQUE ASSIGNMENT NUMBER: 810965

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 75 marks distributed as shown, and 75 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
> **rand("state",student_number0917);**
> **rand(1)**
where student_number0917 is your student number with "0917" at the end and with "-" removed.
For example, if your student number is 123-456-7, you would enter
> **rand("state",12345670917);**
> **rand(1)**

QUESTION 2

Normally Octave outputs numbers with 5 significant figures. Use the help facility with the keyword format to find how to get output with 15 significant figures. Then evaluate $\sqrt{2}$ to 15 significant figures. [5]

QUESTION 3

Solve the simultaneous equations
$2x_1 + 3x_2 = 10$
$4x_1 + 5x_2 = 8$ [10]

QUESTION 4

Evaluate the following series
$1^2 + 2^2 + 3^2 + ... + 100^2$ [10]

QUESTION 5

A formula to find a numerical approximation to the second derivative of a function $f(x)$ is

$$f''(x) = \frac{d^2f}{dx^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

with the approximation being better and better as $h \to 0$. Write a function file deriv2.m that implements the formula. The inputs should be the function to be differentiated (remember the function handle construct @), the value of x, and the value of h. Use your code to estimate the second derivative of $\sin(x)$ at $x = \pi/4$ with $h = 10^{-1}$. [25]

QUESTION 6

Evaluate the following series $\sum_{n=1}^{\infty} u_n$ in which u_n is not known explicitly but is given in terms of a recurrence relation. You should stop the summation when $|u_n| < 10^{-8}$

$$u_{n+1} = (U_{n-1})^2 + (u_n)^2 \text{ with } u_1 = 0.5, \; u_2 = 0.6$$

[20]

QUESTION 7

In the following, R can take any value between 5 and 6. Work out the value of I for R = 5.00, 5.01, 5.02, ..., and 6. Then find the average value of I.

$$I = \frac{30}{R\sqrt{R^2 + 15\pi^2}}$$

[5]

Total [75]

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 100 marks distributed as shown, and 100 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
$>$ **rand("state",student_number0913);**
$>$ **rand(1)**
where student_number0913 is your student number with "0913" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
$>$ **rand("state",12345670913);**
$>$ **rand(1)**

QUESTION 2

Solve the following systems of equations, using the $A\backslash b$ construct, as well as the Gauss-Seidel method with a tolerance of $10-7$ (in some cases convergence may not occur).

(a) $2x_1 - x_2 + 3x_3 = 8$

  $4x_1 + 2x_2 - 5x_3 = -9$

  $6x_1 + 3x_2 + x_3 = 12$ [5]

(b) $10x_1 + x_2 + 2x_3 = 3$

  $x_1 + 10x_2 - x_3 = 1.5$

  $2x_1 + x_2 + 10x_3 = -9$ [5]

QUESTION 3

Write a function file that takes as input a matrix A, and tests whether or not the matrix is (*a*) square, and (*b*) diagonally dominant, reporting the answers on the screen. Show that your code is correct by testing it for the matrix in question 2, as well as for cases where the matrix is not diagonally dominant, and not square.   [20]

QUESTION 4

Define the 100×100 square matrix A and the column vector b by

$$A_{ij} = I_{ij} + \frac{1}{(i-j)^2 + 1}, \quad b_i = 1 + \frac{2}{i}, \quad 1 \le i,j \le 100$$

where I_{ij} is the 100×100 identity matrix (i.e. 1 on the main diagonal and 0 everywhere else). Solve $Ax = b$ for x using both the Gauss-Seidel method and the $A \backslash b$ construct. Do not give the whole vector x in your output, but only x_2, x_{50} and x_{99}.   [10]

QUESTION 5

The Hilbert matrix is a square $n \times n$ matrix defined by

$$H_{ij}^n = \frac{1}{i+j-1}.$$

Define b^n to be a column vector of dimension n, and with each element 1. Construct b^n and, then solve for x^n, $H^n x^n = b^n$, in the cases $n = 10, 15, 20$ and 30. Comment on the results.   [20]

QUESTION 6

The sales figures for a business are as follows for the first six months of the year:
R40 000, R44 000, R52 000, R64 000, R80 000, R84 000.
The owner believes that the sales curve can be approximated by a quadratic function. Find the best quadratic fit to the data, and use it to estimate the projected sales for the rest of the year.   [5]

QUESTION 7

Find the cubic polynomial that best fits the data points

$$(x, y) = (-1, 14),\ (0, -5),\ (1, -4),\ (2, 1),\ (3, 22)$$

Produce a graph showing the polynomial, together with the given data points as discrete points.[5]

QUESTION 8

Find the eigenvalues and eigenvectors of the following matrices, using both *eig* and *power_method* (for the dominant eigenvalue and eigenvector). If the power method fails, discuss why. For those matrices that are diagonalizable, give the diagonalization matrix.

(a)

$$\begin{pmatrix} 2.6731385 & 2.6381454 & 3.8272855 & 4.7022868 \\ 2.7080530 & -0.00.9461 & 0.1019678 & 2.3595453 \\ 5.4426950 & 2.4321965 & 4.8982919 & 7.1372124 \\ 2.7672696 & 3.4815125 & 2.3344020 & 8.2787928 \end{pmatrix}$$

[5]

(b)

$$\begin{pmatrix} 2.9541 & 3.6650 & 7.1445 & 1.9998 \\ 9.1406 & 15.3006 & 12.2773 & 10.3552 \\ 7.9200 & 9.6881 & 7.4619 & 6.7253 \\ 21.6960 & 35.6289 & 30.0172 & 20.7166 \end{pmatrix}$$

[5]

QUESTION 9

Modify the power method so that if finds the smallest eigenvalue and corresponding eigenvector. You do this by evaluating

$$x_{n+1} = A^{-1}x_n$$

rather than $x_{n+1} = Ax_n$. When convergence occurs, it is to the eigenvector corresponding to the inverse of the smallest eigenvalue. Demonstrate the validity of your code by running it on a number of test cases. [20]

Total [100]

> ONLY FOR SEMESTER 1 STUDENTS
> ASSIGNMENT 03
> Linear programming
> FIXED CLOSING DATE: 9 April 2014
> UNIQUE ASSIGNMENT NUMBER: 843478

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 75 marks distributed as shown, and 75 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
> **rand("state",student_number0914);**
> **rand(1)**
where student_number0914 is your student number with "0914" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
> **rand("state",12345670914);**
> **rand(1)**

QUESTION 2

Find the maximum value as well as the point at which the maximum occurs of

$$L = x_1 + 2x_2 + 3x_3$$

subject to the constraints

$$x_1 + x_2 + 2x_3 \leq 8$$
$$3x_1 + 3x_2 + x_3 \leq 9$$
$$x_1, x_2, x_3 \geq 0$$

[15]

QUESTION 3

Find the minimum value as well as the point at which the minimum occurs of

$$L = -4x_1 - 5x_2 + 2x_3$$

subject to the constraints

$$-x_1 + 2x_2 + 2x_3 \leq 10$$
$$2x_1 + 4x_2 + 5x_3 \leq 25$$
$$x_1, x_2, x_3 \geq 0$$

[15]

QUESTION 4

Find the maximum value as well as the point at which the maximum occurs of

$$L = 2x_1 + 3x_2 + 4x_3 + 3x_4$$

subject to the constraints

$$1.5x_1 + 2x_2 + 1.5x_3 + x_4 \leq 30$$
$$1x_1 + 2x_2 + 1x_3 + 3x_4 \leq 45$$
$$5x_1 + 4x_2 + 7x_3 + 2x_4 \leq 65$$
$$6x_1 + 3x_2 + 7x_3 + 4x_4 \leq 60$$
$$8x_1 + 4x_2 + 8x_3 + 2x_4 \leq 70$$
$$x_1, x_2, x_3 \geq 0$$

[20]

Total [50]

16

> **ONLY FOR SEMESTER 2 STUDENTS**
> **ASSIGNMENT 01**
> **Getting started with MATLAB/Octave.**
> **Introduction to programming with MATLAB/Octave.**
> **Use of MATLAB/Octave to solve linear systems of equations.**
> **FIXED CLOSING DATE: 15 August 2014**
> **UNIQUE ASSIGNMENT NUMBER: 869256**

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 60 marks distributed as shown, and 60 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
> **rand("state",student_number0918);**
> **rand(1)**
where student_number0918 is your student number with "0918" at the end and with "-" removed.
For example, if your student number is 123-456-7, you would enter
> **rand("state",12345670918);**
> **rand(1)**

QUESTION 2

The command *etime* takes as input two 6-dimensional row vectors. Use the help facility for *etime* and emphasize to find out more. Then find the number of seconds between 08h 27m 30s on 18 February 2003, and 19h 35m 09s on 13 August 2008. [10]

QUESTION 3

Evaluate the series

$$1 - 3 + 5 - 7 + 9 - ... + 1001.$$

[10]

QUESTION 4

Evaluate the series $\sum_{n=1}^{\infty} u_n$ in which u_n is not known explicitly but is given in terms of a recurrence relation. You should stop the summation when $|u_n| < 10^2$

$$u_{n+1} = (u_{n-1})^2 + (u_n)^{1.5} \text{ with } u_1 = 0.1, \ u_2 = 0.2$$

[20]

QUESTION 5

Evaluate the series $\sum_{n=1}^{\infty} u_n$ in which u_n is not known explicitly but is given in terms of a recurrence relation. You should stop the summation when $|u_n| < 10^{-18}$

$$u_{n+1} = (u_n)^2 \text{ with } u_1 = 0.5$$

[10]

QUESTION 6

Given

$$x_k = \frac{L_k}{L_k \sqrt{L_k + 20\pi^2}}$$

where

$$L_k = \frac{1}{1 + \frac{1}{k}}, \ k = 1...50$$

find

$$S = \mathbf{x}.\mathbf{L} = \sum_{K=1}^{50} x_k L_k$$

[10]

Total [60]

> ONLY FOR SEMESTER 2 STUDENTS
> **ASSIGNMENT 02**
> **Use of MATLAB/Octave to solve linear systems of equations.**
> **Overdetermined and underdetermined systems of linear equations.**
> **Eigenvalues, eigenvectors and matrix diagonalization.**
> **FIXED CLOSING DATE: 15 September 2014**
> **UNIQUE ASSIGNMENT NUMBER: 896382**

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 100 marks distributed as shown, and 100 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
> **rand("state",student_number0915);**
> **rand(1)**
where student_number0915 is your student number with "0915" at the end and with "-" removed. For example, if your student number is 123-456-7, you would enter
> **rand("state",12345670915);**
> **rand(1)**

QUESTION 2

Solve the following systems of equations, using the $A \backslash b$ construct, as well as the Gauss-Seidel method with a tolerance of $10 - 7$ (in some cases convergence may not occur).

(a) $2x_1 - x_2 + 3x_3 = 8$

$4x_1 + 2x_2 - 5x_3 = -9$

$6x_1 + 3x_2 + x_3 = 12$ [5]

(b) $10x_1 + x_2 + 2x_3 = 3$

$x_1 + 10x_2 - x_3 = 1.5$

$2x_1 + x_2 + 10x_3 = -9$ [5]

QUESTION 3

Modify the function file gauss_seidel.m to produce a new function file Jacobi.m that implements the jacobi method. Now use the Jacobi method to solve example 3.2.1, i.e.

$12x_1 - 3x_2 + 4x_3 - 2x_4 = 12$

$2x_1 + 10x_2 - x_3 - 20x_4 = 15$

$x_1 - x_2 + 20x_3 + 4x_4 = -7$

$x_1 + x_2 - 20x_3 - 3x_4 = -5$ [20]

QUESTION 4

Define the 100×100 square matrix A and the column vector b by

$$A_{ij} = I_{ij} + \frac{1}{-j^2 + 2}, \quad b_i = 1 + \frac{1}{i}, \quad 1 \le i,j \le 100$$

where I_{ij} is the 100×100 identity matrix (i.e. 1 on the main diagonal and 0 everywhere else). Solve $Ax = b$ for x using both the Gauss-Seidel method and the $A \backslash b$ construct. Do not give the whole vector x in your output, but only x_2, x_{50} and x_{99}. [20]

QUESTION 5

Find the best straight line ($y = mx + c$) fit to the data points

$$(x, y) = (0, 1), \ (2, 0), \ (3, 1), \ (3, 2), \ (3, 1).$$

Produce a graph showing the line, together with the given data points as discrete points. [5]

QUESTION 6

The sales figures for a business are as follows for the first six months of the year:
R40 000, R44 000, R52 000, R64 000, R80 000, R84 000.
The owner believes that the sales curve can be approximated by a quadratic function. Find the best quadratic fit to the data, and use it to estimate the projected sales for the rest of the year. [20]

QUESTION 7

Find the eigenvalues and eigenvectors of the following matrices, using both *eig* and *power_method* (for the dominant eigenvalue and eigenvector). If the power method fails, discuss why. For those matrices that are diagonalizable, give the diagonalized matrix.

20

(a)

$$\begin{pmatrix} 2.781344 & -1.921334 & 0.493612 & 1.367198 & -1.014289 \\ 0.015050 & -0.205731 & 0.903377 & 1.780261 & -0.824057 \\ -0.087144 & 0.606003 & 2.977860 & -0.140473 & -0.750938 \\ 0.212440 & -2.477599 & 0.980236 & 4.233562 & -1.207581 \\ -0.136646 & -1.168924 & 0.453692 & 0.915245 & 1.712964 \end{pmatrix}$$

[5]

(b)

$$\begin{pmatrix} -1.54575 & -3.47002 & -1.70112 & -2.58917 \\ -3.28104 & -2.07998 & -1.45597 & -2.75629 \\ 0.55497 & 0.94078 & 2.02863 & 0.46100 \\ 8.94120 & 9.67047 & 4.47796 & 9.09710 \end{pmatrix}$$

[5]

QUESTION 8

Consider the fictional species, and suppose that the population can be divided into three different age groups: babies, juveniles and adults. Let the population in year n in each of these groups be

$$x_{(n)} = \begin{pmatrix} x_{b(n)} \\ x_{j(n)} \\ x_{a(n)} \end{pmatrix}$$

The population changes from one year to the next according to $x_{(n+1)} = Ax_{(n)}$, where the matrix A is

$$A = \begin{pmatrix} 1/2 & 5 & 3 \\ 1/2 & 0 & 0 \\ 0 & 2/3 & 0 \end{pmatrix}$$

In the long term, what will be the relative distribution of the population amongst the age groups?

[15]

Total [100]

- The assignment must be answered using Octave (or MATLAB), and for each question you must include your computer code including any .m files used, as well as the output. These should be copied and pasted into a word processing system, and you should produce a single file containing all the answers to the questions in the assignment, which can then be printed out, for hard copy submission, or submitted electronically via *MyUnisa*.

- We **will not accept** hand-written solutions, or anything that does not contain Octave (or MATLAB) code and output.

- In many questions, as well as the computer code and output, you will need to include some form of comment in your answer. This should be in the form of complete sentences that make sense to the reader.

- You should use a fixed space font such as courier for computer code and output, and something else for discussion.

- Question 1 does not carry any marks, but is **compulsory** as it enables us to check that your student number was actually used in an Octave (or MATLAB) session.

- There are 65 marks distributed as shown, and 65 marks = 100%.

QUESTION 1

Enter the following two commands, and copy and paste the output as your answer
> **rand("state",student_number0916);**
> **rand(1)**
where student_number0916 is your student number with "0916" at the end and with "-" removed.
For example, if your student number is 123-456-7, you would enter
> **rand("state",12345670916);**
> **rand(1)**

QUESTION 2

Find the minimum value as well as the point at which the minimum occurs of

$$L = -3x_1 - 4x_2 + x_3$$

subject to the constraints

$$-x_1 + x_2 + 2x_3 \leq 5$$
$$2x_1 + x_2 + x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

[20]

QUESTION 3

Find the minimum value as well as the point at which the minimum occurs of

$$L = -2x_1 - 5x_2 + x_3$$

subject to the constraints

$$x_1 + 2x_2 - x_3 \leq 6$$
$$x_2 + 2x_3 \leq 6$$
$$2x_2 + x_3 \leq 4$$
$$x_1, x_2, x_3 \geq 0$$

[20]

QUESTION 4

Find the minimum value as well as the point at which the minimum occurs of

$$L = -2x_1 - 10x_2 + 27x_3 + 50x_4 + 32x_5$$

subject to the constraints

$$x_1 + 2x_2 + x_3 + 1x_4 + 2x_5 \leq 6$$
$$x_2 + 2x_3 + 7x_4 - 3x_5 \leq 6$$
$$2x_2 + x_3 + 1x_4 - 2x_5 \leq 4$$
$$6x_1 + x_2 + x_3 + x_5 \leq 16$$
$$-2x_3 + 4x_4 + 9x_5 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

[25]

Total [65]

9 OTHER ASSESSMENT METHODS

There are no other assessment methods for this module.

10 EXAMINATION

Examination period
This module is offered in a semester period of fifteen weeks. This means that if you are registered for the first semester, you will write the examination in May/June 2014 and the supplementary examination will be written in October/November 2014. If you are registered for the second semester you will write the examination in October/November 2013 and the supplementary examination will be written in May/June 2015.

During the semester, the Examination Section will provide you with information regarding the examination in general, examination venues, examination dates and examination times.

Examination paper
The exam consists of a two hour paper. Note that you are **not allowed** to use a calculator in the exam.

Previous examination paper
We will supply you with a sample exam paper in another tutorial letter to give you an idea of the format of the examination paper that you will write.

11 FREQUENTLY ASKED QUESTIONS

The *my Studies @ Unisa* brochure contains an A-Z guide of the most relevant study information.

12 SOURCES CONSULTED

No other sources were consulted in preparing this tutorial letter.

13 GETTING STARTED: INSTALLATION OF OCTAVE

This module is mainly about the use of the mathematical software package Octave or MATLAB, and in order to take the module it is a requirement that

- You have regular access to a computer, for example at home, at work, or at a Unisa computer laboratory

- If it is not your own (or Unisa's) computer, you have permission to install the software Octave or MATLAB onto it

- You have had some prior experience in computer programming.

Your study package contains a CD, and when you open the CD you will see that it contains the following directories, sub-directories and files

- StudyMaterial

    - TL501.pdf
    - TL101.pdf

- Octave

    - Windows

        * octave-3.0.0-setup.exe
        * octave-3.0.1-setup.exe

    - Linux

        * octave-3.0.0.tar
        * octave-3.0.1.tar

    - Mac

        * octave-3.0.0-i386.dmg
        * octave-3.0.1-i386.dmg
        * octave-3.0.1-ppc.dmg

We include both the latest (in August 2008) available version of Octave as well as the version (3.0.0) that was used in writing the Study Guide. You should install the latest version, but if you notice any discrepancies between the behaviour of this version and that described in the Study Guide, you could re-try with version 3.0.0. If you like, you can also go on the internet to see if there is an even later version of Octave. The Octave home page is http://www.octave.org, and the repository with various versions of Octave is http://www.gnu.org/software/octave/download.html

13.1 Windows

Our experience has been that the installation of Octave is easy and straightforward. Just insert the CD into your computer's CD drive, open it using Windows Explorer, and double click the .exe file you want to install. The installation window will open and unless you have experience in systems programming you should just accept the default options. The only exception is the page that asks you to choose a graphics backend. The default option is a development package based on java, but we have found that on some machines this option is unstable. So we suggest that you click on the stable version based on gnuplot and use that instead. It is as simple as that!

13.2 Linux and Mac

Versions of Octave for installation on a Linux or Mac machine are included on the CD, but your lecturers can provide only limited help if you experience system-related problems with these versions.

13.3   Unisa computer laboratories

If you are using a Unisa computer laboratory, you should find that Octave has already been installed and the Octave icon will be on the Desktop.

13.4   MATLAB

MATLAB is a commercial software product that has to be purchased, whereas Octave is available free of charge. Although there are occasional differences, the syntax of the two programming systems is almost identical. In some advanced, specialized applications we have found that MATLAB was able to solve a problem but Octave was unsuccessful. However, for the introductory purposes of this module, Octave is quite sufficient. If you wish, for example if the computer that you are using for this module already has MATLAB installed, then you are welcome to use MATLAB rather than Octave; but please be aware that there will be minor syntactical and layout differences between MATLAB and the notes in the Study Guide. Otherwise, we would suggest that you work entirely with Octave.

14   CONCLUSION

We hope that you will enjoy this module and we wish you success with your studies.

Your APM1513 lecturers