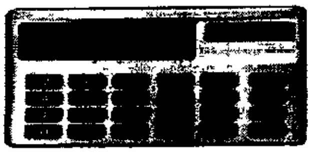
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA

APM1513

October/November 2010

APPLIED LINEAR ALGEBRA

Duration : 2 Hours

100 Marks

EXAMINERS :
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Use of a non-programmable pocket calculator is permissible.

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This paper consists of 4 pages

THE USE OF A NON-PROGRAMMABLE POCKET CALCULATOR IS PERMISSIBLE.

Answer ALL the questions

QUESTION 1

(a) How does one create a variable in Octave/Matlab? (1)

(b) What does the following statement do?

$$B1 = \text{ones } (2,3)$$

(3)

(c) Use the concept in (b) to explicitly construct

(i) A row vector with entries 3.3, 1.7, 2.1, (2)

(ii) A row vector where the values change by equal increament, (2)

[TURN OVER]

(d) Given U2, V2, V3,and A2 as

$$V2 = \begin{bmatrix} 1.00000 \\ 0.80000 \\ 0.60000 \\ 0.40000 \end{bmatrix}$$

$$V2 = [1.00000 \quad 0.80000 \quad 0\ 60000 \quad 0.40000],$$

$$V3 = [1.00000 \quad 2.00000 \quad 3.00000 \quad 4\ 00000],$$

$$A2 = \begin{bmatrix} 1.00000 & 2.00000 & 3.00000 & 4.00000 \\ 5.00000 & 6.00000 & 7.00000 & 8.00000 \\ 9.50000 & 11\ 50000 & 13.50000 & 15.50000 \\ 1.00000 & 0.00000 & 0.00000 & 0.00000 \end{bmatrix}.$$

(i) What is $A4 = [A2\ \mathrm{U2}]$ (List entries), (3)

(ii) What is $A4 = [A2; V3]$ (List entries), (3)

(iii) What is $A5 = A4$ (List entries), (2)

(iv) Find the values of the following elements of a vector or a matrix

(1) In a row vector, $V3(2)$, (1)

(2) In a column vector, $U2(4)$, (1)

(3) In a matrix, $A2(2,3)$, (2)

(4) In a matrix, $A2(3,2)$, (2)

(v) What does the following notations do?

A2 (2; :),

A2 (: ;3),

and write down the matrix entries in each. (3)

[25]

QUESTION 2

(a) Write an Octave/Matlab code to solve a general Gauss-Seidel problem

Hint Your inputs should be matrix A and b and **xold** as an initial solution to a system of equation (say four equations for example) (10)

[TURN OVER]

(b) Given the following systems of equations

$$
\begin{aligned}
20x_1 - x_2 + x_3 &= 20, \\
2x_1 + 10x_2 - x_3 &= 11, \\
x_1 + x_2 - 20x_3 &= -18,
\end{aligned}
$$

write an $M-$ file called "iterative_linear_solver.m" to solve this system with maximum permitted value of the relative error **TOL** $<$ **0.005**, the initial estimate of the solution **xinitial** $=$ [0000] ; the maximum number of permitted iteration **MAX_it = 50**, and the method to be used for obtaining an iterative solution **method**, which is the **Gauss_Seidel** method (code) you have written in (a) Your M-file should state A, b as initial matrix inputs from the equations above. (15)

[25]

QUESTION 3

Consider the system of equations

$$
\begin{aligned}
6x_1 - 2x_2 + x_3 &= 11, \\
x_1 + 2x_2 - 5x_3 &= -1, \\
-2x_1 + 7x_2 + 2x_3 &= 5.
\end{aligned}
$$

(a) Use the Jacobi iterative technique to solve the system. Begin with the solution

$$\left(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}\right) = (0, 0, 0)$$

and round to three *decimal places*. Iterate five times (14)

(b) Solve the system by Gaussian elimination (7)

(c) Write a brief formula that Octave/Matlab can use to solve the formula. (2)

(d) Why is convergence of the Jacobi iterative technique guaranteed? (3)

[26]

QUESTION 4

(a) Either compose an Octave/Matlab code, or use Tableaus to apply the *simplex* method to solve the linear programming problem:

Maximize

$$L = 5x_1 + 8x_2$$

subject to the constraints

$$\begin{aligned} x_1 + 3x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 15 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(14)

(b) Give a full *geometrical* interpretation of your solution (10)

[24]

TOTAL: [100]

END OF PAPER