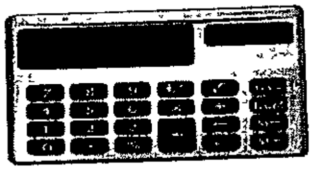
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1513

October/November 2011

APPLIED LINEAR ALGEBRA

100 Marks

Duration 2 Hours

EXAMINERS:
FIRST MR AS KUBEKA
SECOND DR JM MANALE

Use of a non-programmable pocket calculator is permissible

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue.

This paper consists of 4 pages

Answer **ALL** the questions

QUESTION 1

(a) What does the following statements do? (1)

(i) > edit (2)

(ii) >ex1 (3)

(iii) >help fplot (1)

(b) A bank wants to calculate the interest due to its customers and add to that accounts. The interest rate applicable depends on the initial balance, as follows

Initial balance	**Interest rate**
Less than R1000	0%
R1000 to R5000	5%
R5000 to R10 000	8%
R10000 to R20000	9%
Above R20 000	10%

Write an Octave/Matlab code that impliment and solve the above problem. (Hint Use the following psoudo-if lop structure

if (*condition*)

statement

[TURN OVER]

else ıf (*condıtıon*)

else

statement

end ıf (4)

(c) Gıve the mathematıcal mean of the followıng Octave/Matlab operatıons

(ı) < (1)

(ıı) <= (1)

(ııı) > (1)

(ıv) >= (1)

(v) == (1)

(d) Wrıte an Octave/Matlab code that solve the followıng problems

Evalulate the followıng serıes $\sum_{n=1}^{\infty} U_n$ ın whıch U_n ıs not known explıcıty but ıs gıven ın terms of a recurrence relatıon You should stop the summatıon when $|U_n| < 10^{-8}$

$U_{n+1} + U_n^2$, wıth $U_1 = 0\ 5$ (10)

[25]

QUESTION 2

(a) Spread of an ınfectıous dısease ın a town can be modeled (when the number of ınfected people ıs much less than the total populatıon as $x_{(n+1} = Ax_{(n)}$, where n refers to the month, and

$$x_{(n)} = \begin{bmatrix} x_{s(n)} \\ x_{m(s)} \\ x_{r(n)} \\ x_{d(n)} \end{bmatrix}$$

The dısease ıs spread by mosquıto bıtes, and $x_{m(n)}$ refers to the number of mosquıtoes that carry the dısease, $x_{s(n)}$ ıs the number of sıck people, $x_{r(n)}$ ıs the number of people who recovered, and $x_{d(n)}$ ıs the number of people who dıe The matrıx A ıs

$$A = \begin{bmatrix} 0\,7 & 0\,8 & 0 & 0 \\ 1\,6 & 0 & 0\,1 & 0 \\ 0\,2 & 0 & 0 & 0 \\ 0\,1 & 0 & 0 & 0 \end{bmatrix}$$

[TURN OVER]

In the long term, at what rate will the incidence of the disease increase every month? Consequently as a public health initiative, the mosquito breeding areas are being sprayed with insecticide, and the matrix A changes to B where

$$B = \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0.6 & 0 & 0.1 & 0 \\ 0.2 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 \end{bmatrix}$$

Show that the effect will be that the disease will be eradicated by writting an Octave/Matlab code that implement and solve the problem. (10)

(b) Consider a infictious species and suppose that the population can be divided into three different age groups babies, juveniles and adults Let the population in year n in each of these groups be

$$x_{(n)} = \begin{bmatrix} x_{b(n)} \\ x_{j(n)} \\ x_{a(n)} \end{bmatrix}$$

The population changes from one year to the next accoding to $x_{(n+1)} = Ax_{(n)}$, where the matrix A is

$$A = \begin{bmatrix} \frac{1}{2} & 5 & 3 \\ \frac{1}{2} & 0 & 0 \\ 0.2 & \frac{2}{3} & 0 \end{bmatrix}$$

In the long term, what will be the relative distribution of the population amongst the age group? Write an Octave/Matlab code that implement and solve the problem (10)

(c) Find the cubic polynomial that best fits the data points

$$(x, y) = (-1, 14), (0, -5), (1, -4), (2, 1), (3, 22).$$

Produce a graph showing the polynomial, together with the given data points as discrete points by writing an octave/Matlab code that implement and solve the problem (5)

[25]

QUESTION 3

(a) Compose a function file Gauss m that implements the Gauss method to solve a linear system of equations.

(5)

(b) Consider the system

$$2x - y + z = 4,\ x + y + z = 3,\ 3x - y - z = 1$$

(i) Determine A and b for $AX = b$ (2)

(ii) determine det (A) (2)

(iii) From the value of det (A), is the matrix invertible? (2)

(iv) Determine $X = (x, y, z)$ using Gaussian elimation (15)

[26]

QUESTION 4

(a) A MatLab/Octave code for optimising a linear programming problem is

```
[xmax,Lmax]=glpk(C,A,b,lb,ub,ctype,vartype,s)
```

Explain the role of each of the parameters lb,ub,ctype,vartype and s

(14)

(b) Use the *simplex* method to solve the following linear programming problem

Maximize

$$L = x_1 + 1\ 6x_2$$

subject to the constraints

$$\begin{aligned} 0\ 33x_1 + x_2 &< 4 \\ x_1 + 0\ 66x_2 &< 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(10)

[24]

TOTAL: [100]

END OF PAPER

©
UNISA 2011