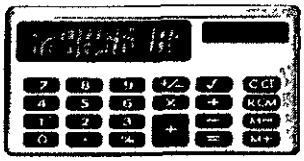
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1513

May/June 2011

APPLIED LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MR AS KUBEKA
SECOND DR JM MANALE

Use of a non-programmable pocket calculator is permissible.

This paper consists of 5 pages

INSTRUCTIONS TO CANDIDATES

Answer all questions

Question 1

(a) Given the following **code**

$n = 10$

fact $= 1 \cdot 5$

for k 1 n fact $= k*$ fact,

factorials(k,) $= [k$ fact];

end

factorials

Answer the following questions,

(i) What does the *for loop* in the above code do? (Write a short paragraph detailing all the computations involved) (6)

(ii) List the final computed result (4)

(iii) Write a simple *for loop* code to evaluate the following series

$$1^2 + 2^2 + 3^2 + \quad + 100^2$$

(5)

(b) (i) What is the difference between Octave and MatLab? (1)

(ii) What does the following statement do?

>cd "c /DocumentsandSettings/Users/MyDocuments/apm1513"

(2)

(iii) What is a Home directory? (1)

(iv) What is an M-file? (1)

(c) What does the following statement do?

$$B1 = \text{ones } (2, 3)$$

(2)

(d) Use the concept in (c) to explicitly construct

(i) A row vector with entries

$$-1, \ 0,6, \ 10, \ a^2$$

(1)

(ii) A row vector where the values change by equal increment of $2a$, a being a variable (2)

[25]

Question 2

(a) A car is undergoing uniform acceleration Measurements have been made of the car's displacement (s) from the origin against time (t) as follows

t	s
1	6 7118
2	10 3049
3	16 1415
4	23 9482
5	33 8839
6	45 9405
7	60 1468
8	76 3638
9	94 6527
10	115 1413

Since the car is undergoing uniform acceleration, s and t are related by

$$s = s_0 + u_0 t + \frac{1}{2}at^2$$

Use the above data to find the best estimate of the constants s_0, u_0 and a Write only the Octave/MatLab program that solves the system and hypothetically plot the graph of the estimate (10)

(b) Write an Octave/MatLab code that finds the best straight line ($y = mx + c$) fit to the data points

x	y
0	1
2	0
3	1
3	2
3	1

Your code must plot the above points together with smooth graph for the estimate (3)

(c) The sales figures for a business are as follows for the first six months of the year

$$R40\,000,\quad R44\,000,\quad R52\,000,\quad R64\,000,\quad R80\,000,\quad R84\,000$$

The owner believes that the sales curve can be approximated by a quadratic function

Write an Octave/MatLab code that finds the best quadratic fit to the data (3)

(d) A formula for the population of the USA is

$$P(t) = P_0 - a\,e^{-0.02(t-1800)}$$

where t is the date in years Some actual data is as follows

Date	Population
1800	5308000
1820	9638000
1840	17069000
1870	38558000
1900	75995000
1930	122775000
1950	150697000

Write an Octave/MatLab code that finds values of P_0 and a that give a best fit of the formula to the data Your code must plot a graph showing the function $P(t)$ against time as a smooth line, together with the given data points as discrete points (9)

[25]

[TURN OVER]

Question 3

(a) Compose a function file GaussSeidel m that implements the Gauss-Seidel method to solve a linear system of equations (5)

(b) Suppose the system $A\,X = b$, has

$$A = [20\ 1\ -1, 1\ -10\ 1, -1\ 1\ 10],$$

and

$$b = [17\ 13\ 18]',$$

What is the value of X? (5)

(c) (i) State an Octave/MatLab command for determining eigenvalues and eigenvector of the matrix A (2)

(ii) Consider the characteristic equation

$$\lambda^2 + 1 = 0$$

Are the subsequent eigenvalues repeated, complex or real? (2)

(d) Compose a function file PowerMethod m that implements the power method, using a conditional loop that exits once the relative error is below a given tolerance, or the maximum number of iterations has been exceeded The inputs to the function are the matrix A, the tolerance TOL, and the maximum number of iterations MaxIt (10)

(e) Modify the power method so that the stopping condition is changed to

$$\left|\frac{Ax_n - \mu_n x_n}{|x_n|}\right|$$

(2)

[26]

Question 4

(a) A MatLab/Octave code for optimising a linear programming problem is

[xmax,Lmax]=glpk(C,A,b,lb,ub,ctype,vartype,s)

Explain the role of each of the parameters lb, ub, ctype, vartype and s (5)

(b) Use the *simplex* method to solve the following linear programming problem

Maximize

$$L = 5x_1 + 8x_2$$

subject to the constraints

$$\begin{aligned} x_1 + 3x_2 &< 12 \\ 3x_1 + 2x_2 &< 15 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(19)

[24]

TOTAL MARKS: [100]