

APM1513

May/June 2012

APPLIED LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS .

FIRST

MR AS KUBEKA

SECOND

DR S FALEYE

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This paper consists of 4 pages

Answer all questions

QUESTION 1

(a) Write an m-file that uses Octave to solve the following problems

(i) Plot a graph of $\cos(x)$ in the range $-1 < x < 5$, (4)

(ii) Evaluate $9 \cdot 2^{-0.5}$, (3)

(iii) Evaluate $\frac{\sqrt{7-3}}{2 \cdot 4^3 + 3 \cdot 1}$, (3)

(iv) Solve the simultaneous equations

$$2x_1 + 3x_2 = 10,$$

$$4x_1 + 5x_2 = 8$$

(5)

(b) Explicitly construct

(i) a row vector with 3 3, 1 7, 2 1, (2)

(ii) a column vector with 3 3, 1 7, 2 1, (2)

(iii) a matrix, where the entries are known explicitly, (2)

(iv) a row vector, when the values change by equal increments, (2)

(v) a row vector, when the increment is 1 (2)

[25]**{TURN OVER}**

QUESTION 2

(a) Given $V3 = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$,

$$U2 = \begin{bmatrix} 1 \\ 0 & 8 \\ 0 & 6 \\ 0 & 4 \end{bmatrix},$$

$$A2 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9.5 & 11.5 & 13.5 & 15.5 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

find the values of the following elements of a vector or matrix

- (i) In a row vector, $V3(2)$, (2)
- (ii) In a row vector, $V3(3)$, (2)
- (iii) In a column vector, $U2(4)$, (2)
- (iv) In a column vector, $U2(3)$, (2)
- (v) In a matrix, $A2(2, 3)$, (2)
- (vi) In a matrix, $A2(3, 2)$. (2)

(b) Write an Octave/Matlab code that calculate $n!$ (i.e. n – factorial), with

$$n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$$

n goes from 1 to 10

(5)

(c) Write an Octave/Matlab code that evaluates the following series

(i) $1^2 + 2^2 + 3^2 + \dots + 100^2$, (3)

(ii) $0 \times 1^2 + 1 \times 2^2 + 2 \times 3^2 + \dots + 99 \times 100^2$ (5)

[25]**[TURN OVER]**

QUESTION 3

(a) Write an Octave/Matlab code that evaluates

$$\int_1^3 \frac{dv}{1+v} \approx 0.8, \text{ where}$$

v is taken to be the 0.3 after the integration (5)

(b) Calculate the average Slope of a function in (a) above over an interval $a < x < b$, defined

$$\text{as } \frac{f(b) - f(a)}{b - a}, \text{ where}$$

$$a = 1 \text{ and } b = 3$$

(10)

(c) Use Octave to draw the following graphs

$$(i) f(x) = x \sin(x), \quad -3 \leq x \leq 8, \quad (3)$$

$$(ii) f(x) = \cos(x^2), \quad 0 \leq x \leq 5, \quad (2)$$

(d) Write an Octave/Matlab code that plots the following graph

$$Z = \frac{\sin \sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}},$$

in the range $-7 \leq x \leq 7$, $-11 \leq y \leq 11$, with a contour map (5)

[25]

QUESTION 4

(a) Consider the equation

$$AX = B$$

where $X^T = (x_1, x_2, \dots, x_n)$ and $B^T = (b_1, b_2, \dots, b_n)$ are $1 \times n$ vectors, and A an $n \times n$ matrix

(i) Write Octave/Matlab commands that can be used to capture A and B into the computer memory, (5)

(ii) Write Octave/Matlab command(s) that replaces the i^{th} column of A with B , (5)

(iii) Express the ratio $\frac{B}{A}$ in Octave/Matlab using \ and / , (2)

[TURN OVER]

(b) Consider the system of equations

$$3x_1 + 4x_2 + 5x_3 = 4,$$

$$6x_1 + 2x_2 + 3x_3 = 7,$$

$$x_1 + 3x_2 + 3x_3 = 1$$

(i) Solve the system by writing an Octave/Matlab code using the Gauss–seidal iterative method (7)

(ii) Find the exact solution by Gaussian elimination

i.e. by Analytic means and by writing using either Octave or Matlab

(6)

[25]

END OF PAPER

TOTAL: 100 Marks

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