UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1513

October/November 2013

APPLIED LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS

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Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 4 pages

Answer all questions

QUESTION 1

1 On Matlab/Octave command line, assign values to the variables a and b, for example $a = 3$, and $b = 5$, and write statements to find the sum, difference, product, and quotent of a and b (5)

2 What does the following Mathlab/Octave statements do

(a) 1 + 1 5 (2)

(b) 1 + [1 5] (2)

(c) 1 5′ (2)

(d) [1 5]′ (2)

3 Given the following vectors

$$a = \begin{bmatrix} 2 & 4 & 5 \end{bmatrix},$$

$$b = \begin{bmatrix} 6 & 2 & 2 \end{bmatrix},$$

Calculate

(a) $a.*b$, (2)

(b) $a./b$, (2)

(c) $a.\hat{}b$, (3)

4 Given the vector [1 2 3 4 5] Write Matlab/Octave statements that uses different vector array operations to produce the following vectors

(a) [2 4 6 8 10] (1)

(b) $\left[\frac{1}{2}\ 1\ \frac{3}{2}\ 2\ \frac{5}{2}\right]$ (1)

(c) $\left[1\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}\right]$ (1)

(d) $\left[1\ \frac{1}{2^2}\ \frac{1}{3^2}\ \frac{1}{4^2}\ \frac{1}{5^2}\right]$ (2)

[25]

QUESTION 2

1 Suppose we have R1000,00 saved in the bank, with interest compounded at the rate of 9% per year Write a Matlab/Octave code to determine how much will your bank balance be after one year (5)

2 Given the following systems of equations

$$\begin{aligned} 3x + 2y - z &= 10 \\ -x + 3y + 2z &= 5 \\ x - y - z &= -1 \end{aligned}$$

(a) Write Matlab/Octave code to solve the above equations (3)

(b) How would you calculate the residual to the above system of equations (2)

3 Define what is

(a) Overdetermined system (2)

(b) Underdetermined system (2)

(c) Conditioned system (2)

4 A formular for the population of the USA is

$$P(t) = P_0 - ae^{-0.02(t-1800)}$$

where t is the date in years Some actual data is as follows

Data	Population
1800	5308000
1820	9638000
1840	17069000
1870	38558000
1900	750995000
1930	122775000
1950	150697000

Write a Matlab/Octave code that find value of P_0, and a, and that give a best fit of the formula to the data and then plot the graph showing the function $P(t)$ against time as a continuous line, together with the given data points as discrete points (9)

[25]

QUESTION 3

1 What does the following Mathlab/Octave do

(a) plot(x,y,'- -') (2)

(b) plot(x,y,'0') (2)

(c) plot(x,sin(x),x,cos(x),'om- -') (3)

2 Write a Matlab/Octave code to evaluate the following series $\sum_{n=1}^{\infty} U_n$ in which U_n is not known explicitly but is given in terms of a recurrence relation You should stop the summation when $|U_n| < 10^{-8}$ $U_{n+1} = U_n^2$, with $U_1 = 0\ 5$ (3)

3 (a) How do you find the dominant eigenvalues and eigenvectors of the following matrix, using both eig and power_method Also give the power method code (4)

$$\begin{bmatrix} 2\ 78134 & -1\ 92133 & 0\ 49361 & 1\ 36719 & -1\ 01429 \\ 0\ 01505 & -0\ 205733 & 090338 & 1\ 78026 & -082406 \\ -0\ 08714 & 0\ 60601 & 2\ 97786 & -0\ 14047 & -0\ 75094 \\ 0\ 21244 & -2\ 47759 & 0\ 98023 & 4\ 23356 & -1\ 20758 \\ -0\ 13665 & -1\ 16892 & 0\ 45369 & 0\ 91525 & 1\ 71296 \end{bmatrix}$$

(b) Also give the power method code (10)

(c) How can one diagonalized matrix A? (1)

[25]

QUESTION 4

1 The Milko Diary can receive no more than 100 000 litres of milk per day Due to a long-term contract, at least 10 000 litres of each day's milk must be used for chase manufacture The balance can be used for bottled milk butter or yoghurt At today's market prices, the contribution to profit and firxed cost of each litre of milk, when put to these uses, is as follows

Butter	R2 50
Cheese	R2 00
Bottled Milk	R1 50
Yoghurt	R3 00

The butter equipment can handle up to 50 000 litres of milk per day, and the milk equipment up to 40 000 litres Part of the yoghurt and cheese processing uses the same equipment and this imposes a limit on the combined usage of 50 000 litres per day The buttere and cheese packaging equipment can handle a combined usage of at most 55 000 litres per year

What mix of products should the company produce so as to maximize profit? (20)

2 Find the maximum value as well as the point at which the maximum occurs of

$$L = 2x_1 + 3x_2 + 4x_3 + 3x_4$$

subject to the constraints

$$\begin{aligned} 1.5x_1 + 2x_2 + 1.5x_3 + x_4 &\leq 30 \\ 1x_1 + 2x_2 + 1x_3 + 3x_4 &\leq 45 \\ 5x_1 + 4x_2 + 7x_3 + 2x_4 &\leq 65 \\ 6x_1 + 3x_2 + 7x_3 + 4x_4 &\leq 60 \\ 8x_1 + 4x_2 + 8x_3 + 2x_4 &\leq 70 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

(5)

[25]

TOTAL: 100 Marks