APM1513

May/June 2013

APPLIED LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS .
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Closed book examination

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This paper consists of 4 pages

ANSWER ALL THE QUESTIONS
ALL CALCULATIONS MUST BE SHOWN

QUESTION 1

(a) How does one create a variable in Octave/Matlab? (1)

(b) What does the following statement do? (3)

```
B1=ones(2,3)
```

(c) Use the concept in (b) to explicitly construct

(i) A row vector with entries 3 3, 1 7, 2 1 (2)

(ii) A row vector where the values change by equal increment (2)

(d) Given $U2, V2, V3$, and $A2$ as

$$U2 = \begin{bmatrix} 1\ 00000 \\ 0\ 80000 \\ 0\ 60000 \\ 0\ 40000 \end{bmatrix},$$

$$V2 = [1\ 00000\ 0\ 80000\ 0\ 60000\ 0\ 40000],$$

$$V3 = [1\ 0000\ 2\ 00000\ 3\ 00000\ 4\ 00000],$$

$$A2 = \begin{bmatrix} 1\ 00000 & 2\ 00000 & 3\ 00000 & 4\ 00000 \\ 5\ 00000 & 6\ 00000 & 7\ 00000 & 8\ 00000 \\ 9\ 50000 & 11\ 50000 & 13\ 50000 & 15\ 50000 \\ 1\ 00000 & 0\ 00000 & 0\ 00000 & 0\ 00000 \end{bmatrix},$$

(i) What is $A4 = [A2 \quad U2]$ (List entries), (3)

(ii) What is $A4 = [A2, \quad V3]$ (List entries), (3)

(iii) What is $A5 = A4$ (List entries), (2)

(iv) Find the values of the following elements of a vector or a matrix

(1) in a row vector, V3(2), (1)

(2) in a column vector, U2(4), (1)

(3) in a matrix, A2(2,3), (2)

(4) in a matrix, A2(3,2) (2)

(v) What do the following notations do?

$$A2(2,\ ),$$

$$A2(\ ,3)$$

Write down the matrix entries in each case (3)

[25]

QUESTION 2

(a) A stone is thrown vertically upward with an initial speed u, its vertical displacement s after an elapsed time t is given by the formula

$$s = ut - \frac{gt^2}{2}$$

where g is the acceleration due to gravity Air resistance is ignored Calculate the value of s over a period of 12 3 seconds at intervals of 0 1 seconds, and plot the distance-time graph over this period by writing Matlab/Octave code that implements and solves the problem (8)

(b) Consider the sequence

$$x_n = \frac{a^n}{n!} \quad n = 1, 2, 3,$$

where $a = 10$ (a constant) and $n!$ is the factorial function
If we try to compute x_n directly, we can get into trouble, because $n!$ grows very rapidly as n increases, and numerical error overflow can occur However the situation is neatly transformed if we notice that x_n is related to x_{n-1} as follows

$$x_n = \frac{ax_{n-1}}{n}$$

which then implies that we now no longer have numerical overflow problems

Find x_{20} using the last formula
Write Matlab/Octave code that implements and solves the problem (5)

(c) In a series form, the natural logarithm of 2 (i e ln(2)) is given by

$$1-\frac{1}{2}+\frac{1}{2}-\frac{1}{4}+\frac{1}{5}+\quad+(-1)^{n+1}\frac{1}{n}+$$

Write Matlab/Octave code that implements and computes the sum of the first 999 terms using a for loop (5)

(d) The initial heat distribution over a steel plate is given by the function

$$u(x,y)=80y^2e^{-x^2}-0\ 3y^2$$

write Matlab/Octave code which plots the surface u over the grid defined by

$$-2\ 1\le x\le 2\ 1,\quad -6\le y\le 6$$

where the grid width is 0 5 in both directions (7)

[25]

QUESTION 3

(a) Write Matlab/Octave code for the Gauss-Seidel method to solve the linear system of equations $Ax = b$ Input should be a matrix `A`, a column vector `b` and the initial guess `xold` (10)

(b) Given the following system of equations

$$10x_1+7x_2+8x_3+7x_4=32$$
$$7x_1+6x_2+6x_3+5x_4=23$$
$$8x_1+6x_2+10x_3+9x_4=33$$
$$7x_1+5x_2+9x_3+10x_4=31$$

write an M-file called "iterative_linear_solver m" to solve this system with maximum permitted value of the relative error `ToL<0 005`, the initial estimate of the solution `xinitial=[0000]`, the maximum number of permitted iterations `MAX_it=50` , and the `method` to be used for obtaining an iterative solution, which is the `Gauss_Seidel` method (code) you have written in (a) Your M-file should state `A`, `b` as initial matrix inputs from the equation above (15)

[25]

QUESTION 4

(a) Write Matlab/Octave code for the power method The code must accept any square matrix as input (10)

(b) Modify the power method in (a) so that the stopping condition is changed to

$$\left|\frac{Ax_n-\mu_n x_n}{|x_n|}\right|<\mathit{tolerance}$$

(5)

(c) Given the matrix

$$A=\begin{bmatrix}1&-3&3\\3&-5&3\\6&-6&4\end{bmatrix}$$

find both the eigenvalues and eigenvectors of A using the command `eig` (3)

[TURN OVER]

(d) Given the objective function

$$L = 40x_1 + 60x_2$$

subject to the constraints

$$2x_1 + x_2 + \leq 70$$

$$x_1 + x_2 \leq 40$$

$$x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \leq 0$$

write Matlab/Octave code to maximize L (7)

[25]

TOTAL MARKS: [100]