UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA

APM1514

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MATHEMATICAL MODELLING

Duration 2 Hours

90 Marks

EXAMINERS .
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Closed book examination.

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This paper consists of 3 pages

Answer **ALL** questions

QUESTION 1

1 Consider the difference equation

$$B_{n+1} = B_n - B_n^2 + 1$$

(a) For which values of B_0 will the values of B_n not change? Justify your answer!

(b) Is the following statement true or false? Justify your answer! "If we have $B_0 = 0$, then because of the $+1$ on the right hand side, the values of B_n will grow without bound "

(15)

2. Find the solution $x(t)$ to the following problem

$$\frac{dx}{dt} = (10 - x)\,t, \quad x(2) = 0. \tag{10}$$

[25]

QUESTION 2

1. Which of the following statements are true and which are false? Justify your answers!

(a) In a predator-prey system, the prey species will always die out (2)

(b) Assume that population A is Malthusian with growth constant k_A and population B is Malthusian with growth constant k_B If $k_A > k_B$ then the doubling time of population A is shorter than the doubling time of population B (4)

[TURN OVER]

(c) Assume that population A is Malthusian with growth constant k_A and population B is Malthusian with growth constant k_B If $k_A > k_B$ then the size of population A will always be larger than the size of population B (2)

(d) If a differential equation $dx/dt = f(x)$ has more than one equilibrium points then at least one of them must be unstable. (2)

(e) In the logistic model, if $a = b$ then the population has only one equilibrium point (2)

2 (a) Draw a phase line of an autonomous differential equation of the type $dx/dt = f(x)$ with the following property Some solutions converge towards the point $x = 0$, and some solutions converge towards the point $x = 1$ (8)

(b) Draw a phase line of an autonomous differential equation of the type $dx/dt = f(x)$ with the following property All solutions grow without bound (5)

[25]

QUESTION 3

A population grows according to the logistic model We know that the size of the population in year 2000 was 1000, the population five years later, in 2005, was 600, and the rate of change in the population at the beginning of year 2000 was -50 population members per year and at the beginning of year 2005 it was -6 population members per year

(a) Find the values of a and b, and find the limit population value (10)

(b) Prove that the logistic model does not have a constant doubling time (5)

(c) In the logistic model, it is always assumed that the parameters a and b are strictly positive Explain what kind of a model we would get in each the following cases, by explaining what the outcomes of the model are from all possible (non-negative) initial values for different values of the remaining parameter

(i) If we take $a = 0$ but $b > 0$ in the logistic model (5)

(ii) If we take $b = 0$ but $a > 0$ in the logistic model (5)

[25]

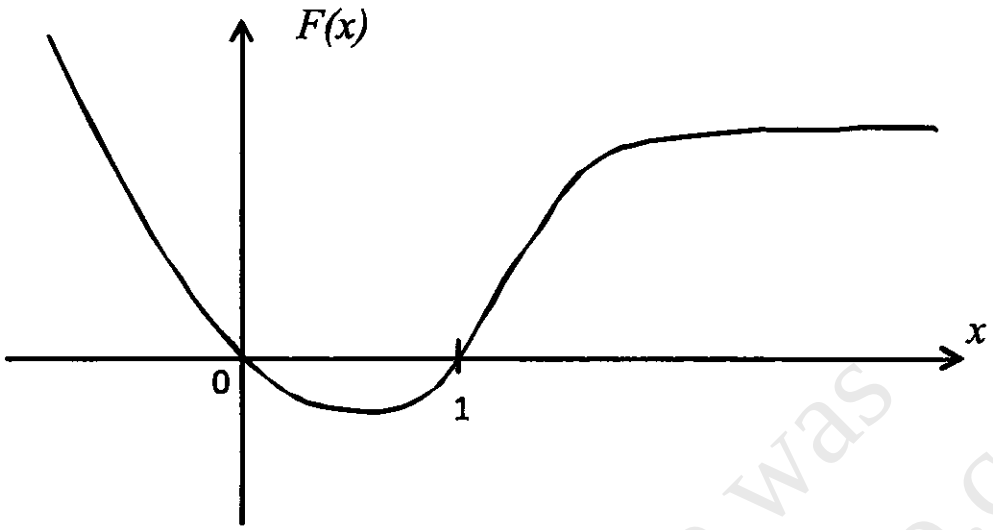
QUESTION 4

Assume that we are given an autonomous differential equation

$$\frac{dx}{dt} = F(x)$$

[TURN OVER]

where a sketch of the function $F(x)$ looks like this



(a) Can any solutions to the differential equation grow without bound as t increases? Justify your answer! (5)

(b) For which initial values will the value of x decrease? Justify your answer! (5)

(c) Do any solutions converge towards 1, as t increases? Justify your answer! (5)

[15]

TOTAL: 90