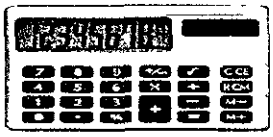
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1514

May/June 2015

MATHEMATICAL MODELLING

Duration 2 Hours

90 Marks

EXAMINERS

FIRST DR AS KUBEKA
SECOND DR JM MANALE

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

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This paper consists of 3 pages

Answer all questions

QUESTION 1

(a) The population of a town obeys the Malthusian model, with $k = -0\,023$ and time measured in years

(i) How long does it take for the population to reach 50% of its original size? (5)

(ii) If the town has 10 people today when was there 10 000 people? (5)

(iii) How long does it take for the population to decrease by 1000 people, if the initial population was 20 000? (5)

(b) A logistically growing population has growth constants $a = 0\,01$, $b = 0\,005$, with t measured in years, and the initial population size is $P_0 = 20$ The solution in the logistic model is given by

$$P(t) = \frac{a}{b + \left(\frac{a}{P_0} - b\right) e^{-at}}$$

(i) What is the initial rate of growth of the population? (5)

(ii) For which population size does the population grow at the fastest possible rate? (5)

[25]

[TURN OVER]

QUESTION 2

A cake is baked in an oven, the temperature of which is kept at 220° Initially the cake has the temperature 25° Assume that Newton's law of cooling/heating up applies

(a) Explain why we do not have enough information to find the value of the constant k (10)

(b) If $k = 0\ 08$ (time measured in minutes), how long does it take for the cake to reach the temperature of 100°? (5)

(c) If $k = 0\ 08$ (time measured in minutes), what is the rate of change of the temperature of the cake after 60 minutes? (5)

(d) At what time is the rate of change of the temperature at its highest? (5)

[25]

QUESTION 3

A population grows according to the logistic model We know that the size of the population in year 2000 was 1000, the population five years later, in 2005, was 600, and the rate of change in the population at the beginning of year 2000 was -50 population members per year and at the beginning of year 2005 it was -6 population members per year

(a) Find the values of a and b, and find the limit population value (10)

(b) Prove that the logistic model does not have a constant doubling time (5)

(c) In the logistic model, it is always assumed that the parameters a and b are strictly positive Explain what kind of a model we would get in each the following cases, by explaining what the outcomes of the model are from all possible (non-negative) initial values for different values of the remaining parameter

(i) If we take $a = 0$ but $b > 0$ in the logistic model (5)

(ii) If we take $b = 0$ but $a > 0$ in the logistic model (5)

[25]

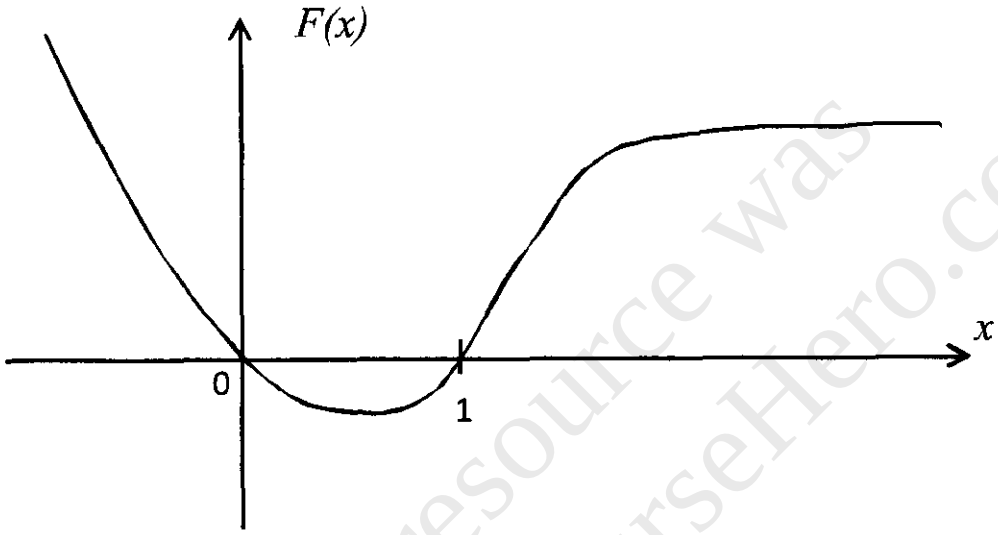
[TURN OVER]

QUESTION 4

Assume that we are given an autonomous differential equation

$$\frac{dx}{dt} = F(x)$$

where a sketch of the function $F(x)$ looks like this



(a) Can any solutions to the differential equation grow without bound as t increases? Justify your answer! (5)

(b) For which initial values will the value of x decrease? Justify your answer! (5)

(c) Do any solutions converge towards 1, as t increases? Justify your answer! (5)

[15]

TOTAL: 90