APM1514

October/November 2015

MATHEMATICAL MODELLING

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	DR AS KUBEKA	
SECOND	DR B HUNTLEY	DR JM MANALE

Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 3 pages

Answer all questions

[TURN OVER]

QUESTION 1

1 Consider the difference equation

$$B_{n+1} = B_n - B_n^2 + 1$$

(a) For which values of B_0 will the values of B_n not change? Justify your answer

(b) Is the following statement true or false? Justify your answer "If we have $B_0 = 0$, then because of the +1 on the right hand side, the values of B_n will grow without bound"

(15)

2 Find the solution $x(t)$ to the following problem

$$\frac{dx}{dt} = (10 - x)\,t, \qquad x(2) = 0$$ (10)

[25]

QUESTION 2

A population grows according to the logistic model We know that the size of the population in year 2000 was 1000 the population five years later, in 2005 was 600, and the rate of change in the population at the beginning of year 2000 was −50 population members per year and at the beginning of year 2005 it was −6 population members per year

(a) Find the values of a and b, and find the limit population value (10)

(b) Prove that the logistic model does not have a constant doubling time (5)

(c) In the logistic model, it is always assumed that the parameters a and b are strictly positive Explain what kind of a model we would get in each the following cases by explaining what the outcomes of the model are from all possible (non-negative) initial values for different values of the remaining parameter

(i) If we take $a = 0$ but $b > 0$ in the logistic model (5)

(ii) If we take $b = 0$ but $a > 0$ in the logistic model (5)

[25]

[TURN OVER]

QUESTION 3

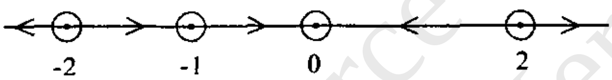
(a) Explain briefly what is meant by

- (i) A stable equilibrium point (2)
- (ii) Separation of variables (2)
- (iii) Proportional population growth (2)
- (iv) Newton's law of cooling (2)

(b) Consider the model

$$\frac{dy}{dt} = F(y)$$

where the phase line of the model looks as follows



- (i) Draw a rough sketch of the function F (9)
- (ii) Draw a rough sketch of the solution curve $y(t)$ if the initial value is $y(0) = 1$ (8)

[25]

QUESTION 4

A farmer breeds fish in a tank and each month buys a certain amount of fish. Let $N(t)$, $t \geq 0$ denote the amount of fish in the tank at time t, with t measured in months. Assume that the system can be described by the model given by the differential equation

$$\frac{dN(t)}{dt} = aN - bN^2 + h$$

where a, b and h are positive constants.

(a) Explain the meanings of the constants a, b and h. (5)

(b) Describe how the outcome of the system depends on the initial population size N_0. Under which choices of values a, b, h and N_0 would the population die out? (20)

[25]

TOTAL: 100