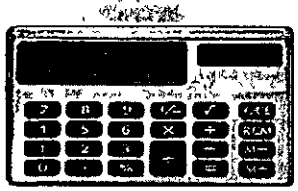
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1514

October/November 2016

MATHEMATICAL MODELLING

Duration 2 Hours

100 Marks

EXAMINERS :

FIRST DR AS KUBEKA
SECOND DR B HUNTLEY DR JM MANALE

Use of a non-programmable pocket calculator is permissible

Closed book examination.

This paper consists of 4 pages

Answer all questions

[TURN OVER]

QUESTION 1

1 For the following difference equation

$$a_{n+1} = 2 + a_n$$

classify the equation as either autonomous or not, and as first-order or not (4)

2 Determine the values of a_0, a_1, a_2, a_3 and a_4 when the difference equation and initial value are given by

$$a_{n+1} = \frac{a_n}{2} - 1, \; a_0 = 1 \tag{3}$$

3 Find the general solution to the following difference equations

$$a_{n+1} = a_n - 8, \; a_0 = 1 \tag{5}$$

4 For the following system, find the equilibrium points, if they exist

$$a_{n+1} = a_n^2 - 4a_n - 2 \tag{3}$$

5 Write down the difference equation for the following proportional population models, when time n is measured in years

(a) A population with 2000 births and 800 deaths with 10 000 individuals per year (5)

(b) A population with 0,002 births and 0,01 deaths per population member per year (5)

[25]

QUESTION 2

1 Assume that a deposit of R50 000 is made into an investment account which pays interest at the rate of 5% per month How many months will it take until there is more than R60 000 in the account? (6)

2 Solve the following initial value problem

$$\frac{dy}{dt} = \frac{2t}{y} \text{ with initial condition } y(1) = -2 \tag{5}$$

3 Assume that a population of a town was 200 000 in the year 1950 and 2000 in the year 2000 Assuming that the Malthusian model is valid, determine

(a) In which year the population size will be 200

(b) In which year the poputon size was 20 000 individuals

(6)

[TURN OVER]

4 Assume that a radioactive substance has a half-life of 2000 years

(a) Find the approximate value of the constant of decay, k

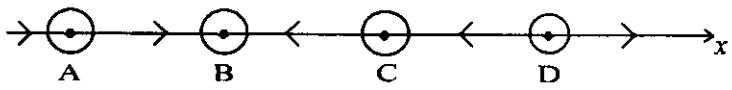
(b) If 10 grams of the substance remains today, when was there 50g of the substance?

(c) If 10 grams of the substance remains today, when will there be 1g of the substance left? (8)

[25]

QUESTION 3

1 Assume that the phase line of a system is as given below

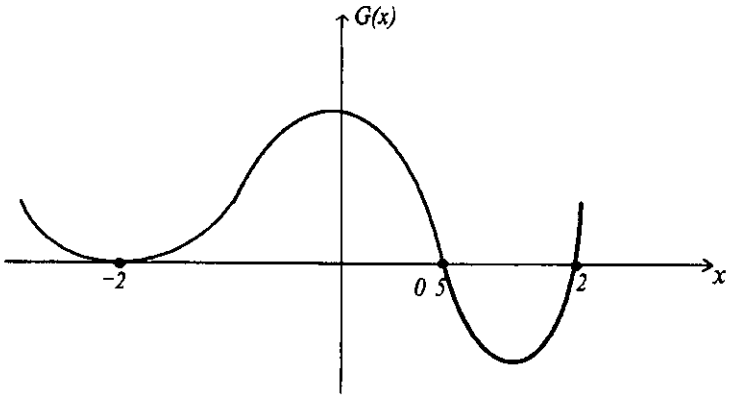


Classify all the equilibrium points as stable or unstable (5)

2 Assume that for the system

$$\frac{dx}{dt} = G(x)$$

a graph of the function G looks like this



Draw the phase line of the system, and explain what will happen to $x(t)$ as t increases

(a) if the initial value is -1, and

(b) if the initial value is -3

(10)

[TURN OVER]

3 Let a system be given by

$$\frac{dx}{dt} = x(a - x)$$

where a is a real number

(a) What value of a should be chosen to make $x = 1$ an equilibrium point?

(b) How should a be chosen to guarantee that any solution starting with $x_0 < 0$ will decrease?

(c) Explain why, regardless of the value of a, no solution to this system can ever increase without bound

(10)
[25]

QUESTION 4

1 A tank initially contains 200 litres of liquid A Liquid B is pumped into the tank at the rate of 4 litres per minutes The mixture is stirred continuously and the tank is kept full at 200 litres at all times The contents of the tank is pumped out at the same rate of 4 litres per minute Let $X(t)$ denote the amount (in litres) of liquid B in the tank after t minutes

(a) Write down the differential equation for $X(t)$ and its initial value

(b) Solve the initial value problem in (a) to find the solution $X(t)$ for all t

(c) How long will it take for the tank to contain liquid A and liquid B in equal quantities?

(15)

2 A body with an initial temperature of 100°C is placed in a tank of water, the temperature of which is held at 30°C The body cools down to 70°C in 15 minutes

(a) Write down the differential equation for Newton's Law of cooling for the system

(b) Solve the equation using the information given, and

(c) Calculate when the body's temperature will be 40°C

(10)
[25]
TOTAL: 100 Marks