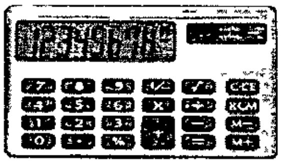
UNIVERSITY EXAMINATIONS



UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM1514

May/June 2018

Mathematical Modelling

Duration 2 Hours

100 Marks

EXAMINERS

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Use of a non-programmable pocket calculator is permissible

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 4 pages

Answer all questions

[TURN OVER]

QUESTION 1

1 For each of the following difference equations, classify the equation as either autonomous or not, and as first-order or not

(a) $a_{n+1} = 2 + a_n$ (2)

(b) $a_{n+1} = a_n + (a_{n-1})^n$ (2)

2 Write down a_0, a_1, a_2, a_3 and a_4 when the difference equation and initial value are as given below

(a) $a_{n+1} = \frac{a_n}{2},\ a_0 = 1$ (2)

(b) $a_{n+1} = 2a_n(a_n + 3),\ a_0 = 4$ (4)

3 Find the equilibrium values and the general solution to the following difference equation

$$a_{n+1} = (a_n - 1)^2 \qquad (10)$$

4 A difference equation is given by

$$a_{n+1} = a_n^3$$

(a) If $a_0 = 2$, find a_1 and a_2

(b) Find all the equilibrium values of the difference equation

(c) Is $a_n = a_0^{3n}$ the general solution to the difference equation, justify your answer? (5)

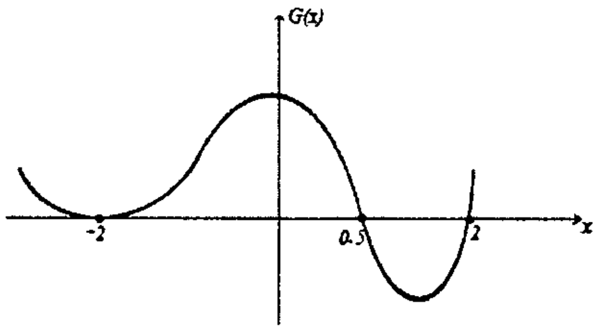
[25]

QUESTION 2

1 Assume that for the system

$$\frac{dx}{dt} = G(x)$$

a graph of the function G looks like this



Draw the phase line of the system, and explain what will happen to $x(t)$ as t increases if

(a) the initial value is -1, and

(b) the initial value is -3 (5)

2 Draw the phase line of the following systems List all the equilibrium points and state which are stable and which are unstable In the system, a and b are assumed to be positive constants,

$$\frac{dx}{dt} = a(x - b) \tag{10}$$

3 Consider the model

$$\frac{dP}{dt} = 2P + 4P^2,$$

here $P(t)$ denotes the size of a population at time t

(a) Is this a logistic model? Justify your answer!

(b) Draw the phase line of the model (10)

[25]

QUESTION 3

1 Assume that a population grows according to the Malthusian model, with $k = 0\ 01$ and $P_0 = 10000$, with time t measured in years

(a) Find the values of $P(5)$ and $P(50)$ (5)

[TURN OVER]

(b) Find the values of t for which $P(t) = 100000$ (5)

2 The Microtus Arvallis Pall is a species of rodent that reproduces very rapidly Assume that there are two rodents (male and female) present at time $t = 0$ Let the time units be months and let $k = 0$ per month Calculate the number of rodents at the end of 2,6 and 10 months respectively, if the Malthusian model applies (15)

[25]

QUESTION 4

Consider the system

$$\frac{dx}{dt} = 2xy + x - x^2$$
$$\frac{dy}{dt} = xy - 2y$$

where x and y denote the sizes of two interacting populations

(a) How does the x and y species behave, respectively, in the absence of the other species? (5)

(b) Describe the type of interaction between the two species (e g competition, predator/prey, etc) (5)

(c) Draw the phase diagram and use it to predict the outcome of the system if initially $x_0 = 2,\ y_0 = 2$ (15)

[25]

TOTAL: 100 Marks