# Tutorial letter 201/2/2018

## Mathematical Modelling

## APM1514

## Semester 2

## Department of Mathematical Sciences

Ths tutorial letter contains solutions for assignment 01.

BARCODE

Define tomorrow.

UNISA university of south africa

Solutions to assignment 01

1. (a) From $a_{n+1} = a_n^2$, and $a_0 = 2$

$$\text{for} \quad n=0; \qquad a_1 = a_0^2 = 2^2$$
$$n=1; \qquad a_2 = a_1^2 = (2^2)^2 = 2^{2^2}$$
$$n=2; \qquad a_3 = a_2^2 = \left((2^2)^2\right)^2 = 2^{2^3}$$
$$n=3; \qquad a_4 = a_3^2 = \left(\left((2^2)^2\right)^2\right)^2 = 2^{2^4}$$
$$n=4; \qquad a_5 = a_4^2 = \left(\left(\left(\left((2^2)^2\right)^2\right)^2\right)^2\right)^2 = 2^{2^5}$$

(b) Equilibruim points

from: $a_{n+1} = a_n^2$, we have

$$a = a^2$$
$$0 = (a^2 - a)$$
$$0 = a(a-1)$$

$\therefore a = 0$ or $a = 1$

(c)

$$a_1 = a_0^2, \quad a_2 = a_0^2, a_3 = a^{2^2}$$
$$a_4 = a_0^{2^4}, \quad a_5 = a_0^{2^5}$$

(d) $a_n = a_0^{2^n}$

2. (a) $A_{(n+1)} = \left(A_n + \frac{A_n}{100}\right) - 3000$

$A_{(n+1)} = 1.01A_n - 3000$

(b) equilibrium point

$$A = 1.01A - 3000$$
$$\frac{A}{100} = 3000 \therefore A^* = 300\ 000$$

(c) Since $A_0 = 200\ 000 < A^* = 3000\ 00$, it follows that the loan will be paid out completely.

(d) Yes, it will, and will not, for example

(i) will not change,

(ii) yes it will change to

$$A_{(n+1)} = 1.01A_n - 1600$$

(iii) yes it will change to

$$A_{(n+1)} = \left(A_n + \frac{2}{100}A_n\right) - 3000$$

3. (a)

$$\frac{dx}{dt} = \frac{2x}{t}$$

$$\frac{dx}{2x} = \frac{dt}{t}$$

$$\frac{1}{2}\int \frac{dx}{x} = \int \frac{dt}{t}$$

$\therefore \frac{1}{2}\ln(x) + A = \ln(t) + B,$ where $A, B$ are integration constants.

Thus $\ln(x) = 2\ln(t) + c,$ because $B + (-A) = C$, which is also an integration constant.

$$2\ln(t) + c$$

$$\therefore \quad x = e$$

(b)

$$\frac{dx}{dt} + 2x - 1 \quad = 0$$

$$\therefore dx + dt\,(2x-1) \quad = 0$$

$$\frac{dx}{2x-1} + dt \quad = 0$$

$$\therefore \int \frac{dx}{2x-1} + \int dt$$

$$\therefore \frac{1}{2}\ln|2x-1| \quad = -t + c$$

$\therefore \ln|2n-1| = -2t + D,$where $D = 2c$ which is still an integration constant.

$\therefore 2x - 1 = e^{-2t+D}$ OR $-(2x-1)\,e^{-2t+D}$

$\therefore 2x - 1 = e^{-2t+D}$ OR $2x = -e^{-2t+D} + 1$

$\therefore x = \frac{1}{2}\left(e^{-2t+D} + 1\right)$ OR $x = \frac{1}{2}\left(1 - e^{-25+D}\right)$

4.

$$\begin{aligned}
\frac{dy}{dt} &= 2\left(4-y\right),\ y\left(1\right)=1. \\
\therefore \frac{dy}{2\left(4-y\right)} &= dt \\
\therefore \frac{1}{2}\int \frac{dy}{\left(-y+4\right)} &= \int dt
\end{aligned}$$

$\frac{1}{2}\cdot\frac{1}{-1}\cdot\ln\left(-y+4\right)+A=t+B$, where $A$ and $B$ are integration constant

$\frac{1}{2}\cdot\ln\left(-y+4\right) \quad = t^1+C$ where $B+(-A)=C$

which is an integration constant also;

Thus we have

$$\begin{aligned}
\ln\left(-y+4\right) &= 2t+C \\
-y+4 &= e^{-2t+C} \\
\therefore -y &= e \\
\therefore y &= 4-e^{-2t+c}
\end{aligned}$$

Therefore

$$\begin{aligned}
y\left(1\right) &= 4-e^{-2(1)+c}=1 \\
\therefore 1-4 &= -e^{-2+c} \\
\therefore -3 &= -e^{-2+c} \\
\therefore 3 &=^{-2+c} \\
\therefore -2+c &= \ln\left(3\right) \\
\therefore C &= \ln\left(3\right)+2 \\
\therefore y &= 4-e^{-2t+\ln(3)+2}
\end{aligned}$$