Tutorial letter 203/2/2018

Mathematical Modelling

APM1514

Semester 2

Department of Mathematical Sciences

Ths tutorial letter contains solutions for assignment 03.

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Assignment 3 Solutions Second Semester 2014
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1 a) equilibrium points: $\frac{dP}{dt} = 0$.

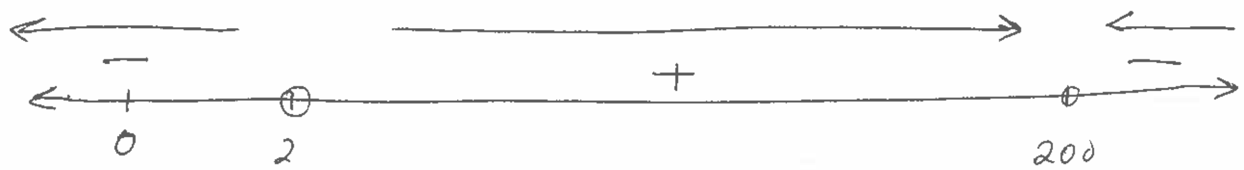
$$0 = 4(P-2)(200-P)$$
$$\therefore \quad P = 2 \quad \text{or} \quad P = 200$$

Phase line:

$$\frac{dP}{dt} < 0 \quad \text{iff} \quad P < 2 \text{ and } P > 200$$

and

$$\frac{dP}{dt} > 0 \quad \text{iff} \quad P > 2 \text{ and } P < 200$$



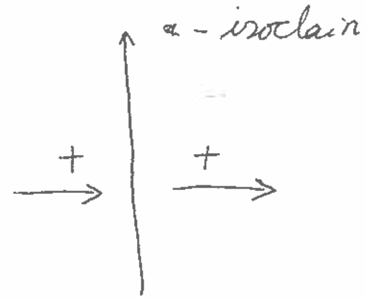
b) if $P(0) = 1$, then the system will decrease asymptocally from $P = 2$,
if $P(0) = 150$, then the system will increase asymptoeally towards $P = 200$.

c) No, because if $P(0) = 2010$, then the system will then decrease towards $P = 200$, and will stay there i.e. $P = 200$ is stable point. So there is no initial point that can course the population to grow or increase without bounds. Also as shown in (b) above, if $P = 1$ then the population will decrease asymptotically reaching negative values

2. a) Given $\frac{dx}{dt} = x^2$; $\frac{dy}{dt} = -y$

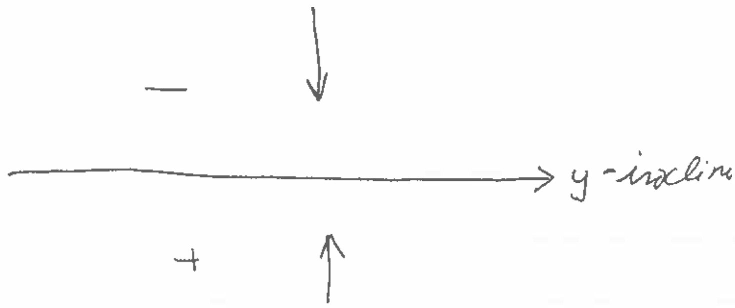
<u>x - isocline</u>: $\frac{dy}{dt} = 0$

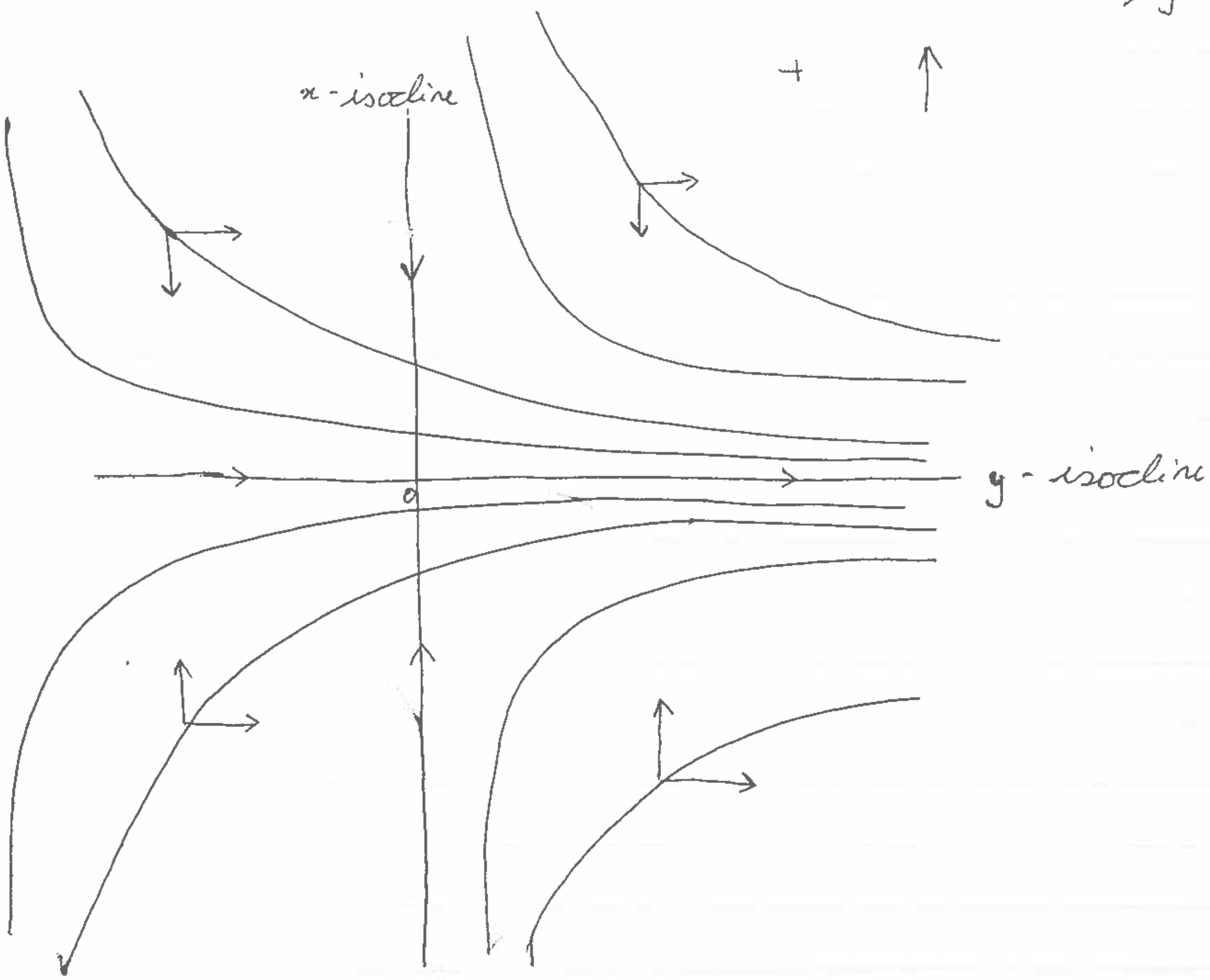
$\therefore y = 0$



<u>y - isocline</u>: $\frac{dx}{dt} = 0$

$x^2 = 0$, $x = 0$





<u>equilibrium points</u>: are points when both

$\frac{dx}{dt} = 0$, and $\frac{dy}{dt} = 0$ hold.

That is at $(0,0)$.

<u>Stability of the equilibrium points:</u>

The equilibrium point is not stable.

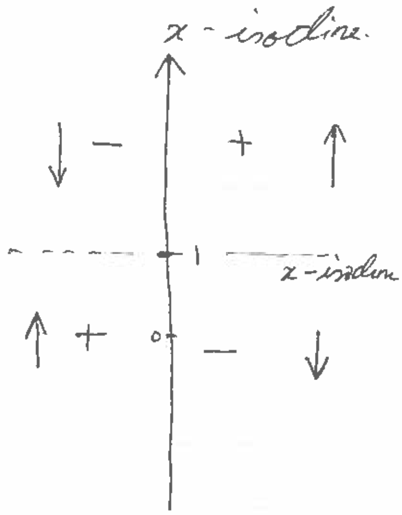
<u>Describing the outcome of the system.</u>

The system if we start at $x=-1$, will decrease asymptotically from aboure the y-isclane and will at the same time increase asymptotically from belowe the ye-isoclaine.

b) Given $\dfrac{dx}{dt} = x(y-1)$, $\dfrac{dy}{dt} = 2y$

<u>x-isocline:</u> $0 = x(y-1)$

$\therefore x = 0$, or $y = 1$



$\dfrac{dx}{dt} > 0$ iff $x < 0$ and $y < 1$

$x > 0$ and $y > 1$

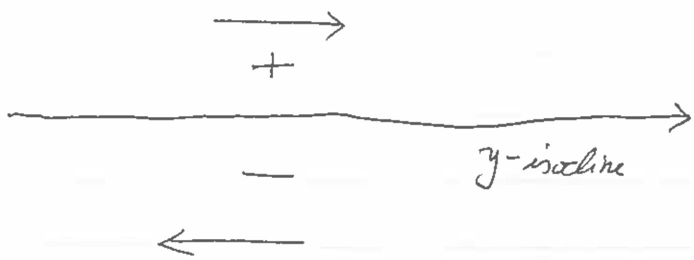
and

$\dfrac{dx}{dt} < 0$ iff $x < 0$ and $y > 1$

$x > 0$ and $y < 1$

<u>y-isocline:</u> $0 = 2y$

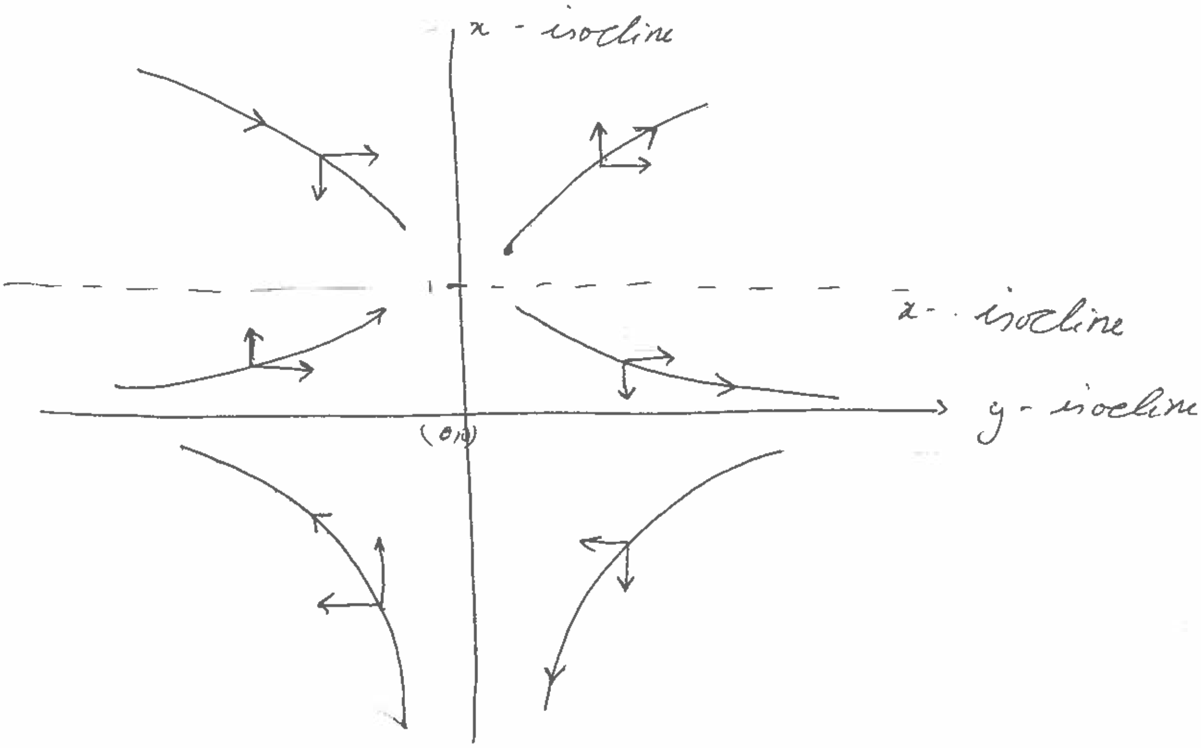
$\therefore y = 0$



$\dfrac{dy}{dt} > 0$ iff $y > 0$

and

$\dfrac{dy}{dt} < 0$ iff $y < 0$



<u>equilibrium points:</u>

(0,0) is the only equilibrium point when

$\frac{dx}{dt} = 0 = \frac{dy}{dt}$ (i.e when $\frac{dx}{dt}$, and $\frac{dy}{dt}$ intersect)

<u>Stability of the equilibrium points:</u>

The equilibrium point is not stable.

<u>Describing the outcome of the system:</u>

The system will never approach asymptotically the equilibrium point (0,0). For example if the system is disturbed from the equilibrium point then it will never return from it again. That is the point will move away from the equilibrium point.

3. $\dfrac{dx}{dt} = (1 - x^2 - 2y)x$ [LEFT AS AN EXERCISE FOR THE EXAM]

$\dfrac{dy}{dt} = (1 - 2x - y)y$

x - isocline: $\dfrac{dx}{dt} = 0 \Rightarrow (1 - x^2 - 2y)x = 0$

$\therefore x = 0$ OR $1 - x^2 - 2y = 0$

$\therefore x = \pm\sqrt{1 - 2y}$

Thus: $\dfrac{dx}{dt} < 0$ iff $-x^2 - 2y > 1$, and $x > 0$

$\Rightarrow -x^2 > 1 + 2y$

$x^2 < -1 - 2y$

$\therefore x < \pm\sqrt{-1 - 2y}$

but we must have $-1 - 2y > 0$

$\Rightarrow -2y > 1$

$\therefore y < -\frac{1}{2}$

Therefore: $\boxed{\dfrac{dx}{dt} < 0 \text{ iff } x < \pm\sqrt{-1 - 2y},\ y < -\frac{1}{2}, \text{ and } x > 0}$

Similarly: $\dfrac{dx}{dt} < 0$ iff $-x^2 - 2y < 1$, $x < 0$

$\Rightarrow -x^2 < 1 + 2y$

$x^2 > -1 - 2y$

$\therefore x > \pm\sqrt{-1 - 2y}$

and $-1 - 2y > 0 \Rightarrow y < -\frac{1}{2}$

Therefore: $\boxed{\dfrac{dx}{dt} < 0 \text{ iff } x > \pm\sqrt{-1 - 2y},\ y < -\frac{1}{2}, \text{ and } x < 0}$

also: $\dfrac{dx}{dt} > 0$ iff $-x^2 - 2y < 1$, and $x > 0$

$\Rightarrow -x^2 < 1 + 2y$

$\therefore x^2 > -1 - 2y$

$\therefore x > \pm\sqrt{-1 - 2y}$

and $-1-2y>0 \Rightarrow y<-\frac{1}{2}$

Therefore: $\boxed{\frac{dx}{dt}>0 \text{ iff } x>\pm\sqrt{-1-2y}\text{, and } x>0}$

Similarly: $\frac{dx}{dt}>0$ iff $-x^2-2y>1$, $x<0$

$\Rightarrow -x^2>1+2y$

$\therefore x^2<-1-2y$

$x<\pm\sqrt{-1-2y}$

but we must have $-1-2y>0$

$\Rightarrow -2y>1$

$\therefore y<-\frac{1}{2}$

Therefore: $\boxed{\frac{dx}{dt}>0 \text{ iff } x<\pm\sqrt{-1-2y}\text{, } y<-\frac{1}{2}, x<0}$

<u>y-isocline</u>: $\frac{dy}{dt}=0 \Rightarrow (1-2x-y)y=0$

$\therefore y=0$, OR $1-2x-y=0$

$\therefore y=1-2x=-2x-1$

$\frac{dy}{dt}<0$ iff $-2x-y>1$, and $y>0$

$\Rightarrow -y>1+2x$

$y<-2x-1$

but we must have: $-2x-1>0$

$-2x>1$

$x<-\frac{1}{2}$

Therefore: $\boxed{\frac{dy}{dt}<0 \text{ iff } y<-2x-1,\ x<-\frac{1}{2},\ y>0}$

Similarly: $\frac{dy}{dt} < 0$ iff $-2x - y < 1$, $y < 0$

$$\Rightarrow -y < 1 + 2x$$

$$\therefore y > -1 - 2x$$

but we must have: $-1 - 2x < 0$

$$-2x < 1$$

$$x > -\tfrac{1}{2}$$

Therefore: $\boxed{\frac{dy}{dt} < 0 \text{ iff } -2x - y < 1,\ x > -\tfrac{1}{2},\ y < 0}$

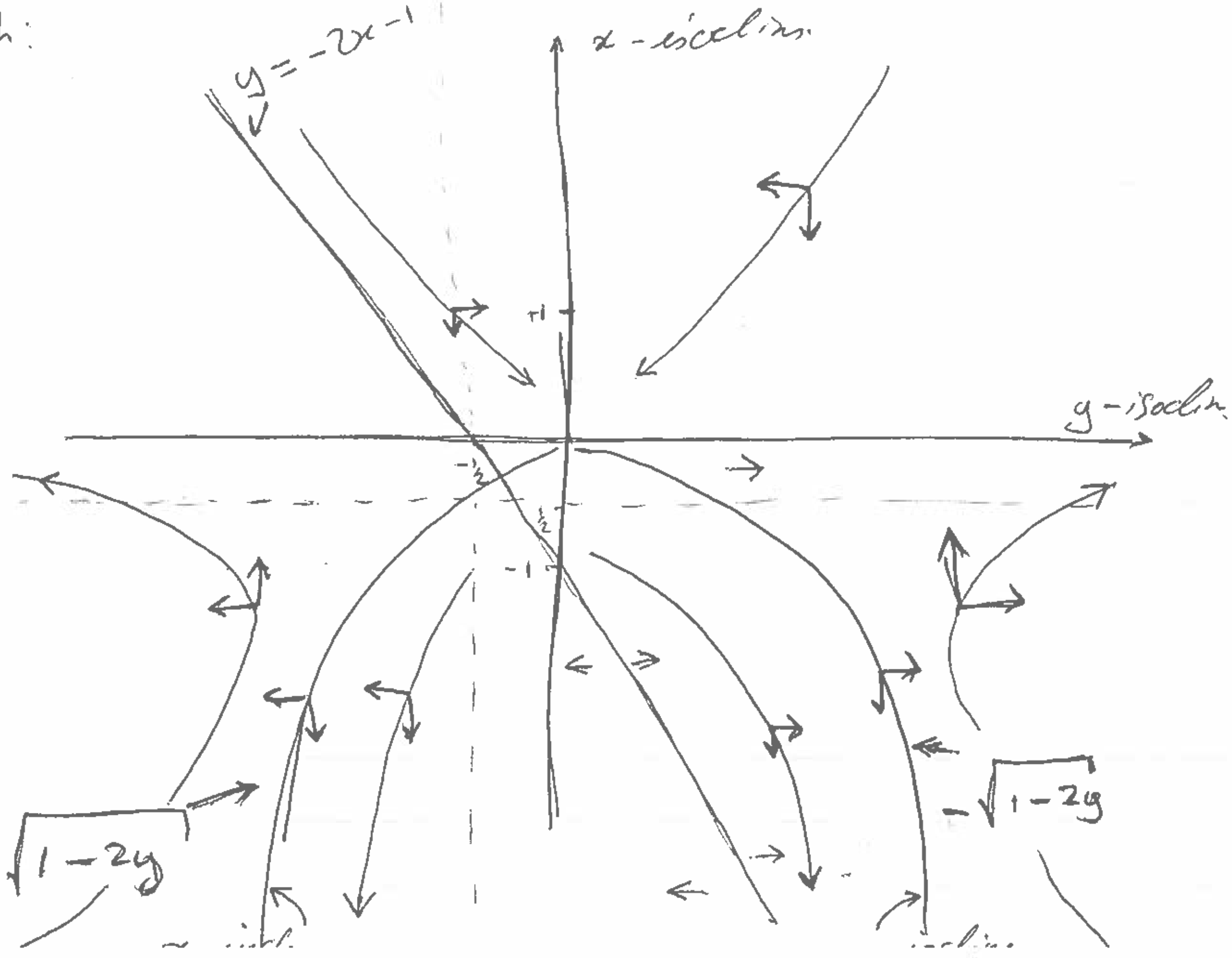
and $\frac{dy}{dt} > 0$ iff $-2x - y < 1$, and $y > 0$

$$\Rightarrow -2x - 1 < y$$

but we must have $x < -\frac{1}{2}$

Therefore: $\boxed{\frac{dy}{dt} > 0 \text{ iff } -2x - y > 1,\ x < -\tfrac{1}{2},\ y < 0}$

a) Graph:



b) Yes, coexistence is possible because of the equilibrium point at $(x,y)=(0,0)$ which in this is unstable. This means that coexistence will be be unstable.

c) If the initial values of the system are $x(0)=3$, $y(0)=2$, Then the system will move towards the equilibrium point $(0,0)$ as it is evident from the first quadrant in the graph.

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