

**APM2164**

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**COMPUTER ALGEBRA (APPLIED MATHEMATICS 216)**

Duration 2 Hours

100 Marks

**EXAMINERS****FIRST****SECOND****DR JMW MUNGANGA****PROF NT BISHOP**

This paper consists of 2 pages

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**QUESTION 1**

Implement a procedure Quad whose input is a function  $f$  (of one variable) and a list  $X$  of numerical values with  $x_0 < x_1 < \dots < x_n$

The call  $Quad(f, X)$  should compute a numerical approximation of the integral  $\int_{x_0}^{x_n} f(x) dx$  by means of the formula

$$\sum_{i=0}^{n-1} (x_{i+1} - x_i) f(x_i)$$

**[20]****QUESTION 2**

Write MuPAD code to draw a graph that plots the function  $f(t) = e^{\sin t}$  and  $g(t) = \frac{t^2}{1+t^2}$  for  $t \in [-1, 5]$ . The axes should be appropriately labelled and the scale in the vertical and horizontal directions should be the same. The graph of  $f$  is blue and that of  $g$  is green.

**[20]****QUESTION 3**

Write MuPAD code to solve the following system of recurrence relations

$$\begin{cases} x_{n+1} = 2x_n + y_n - z_n, \\ y_{n+1} = x_n - 2y_n - z_n \\ z_{n+1} = \frac{y_n}{2} + \frac{y_n}{3} - \frac{z_n}{4} \end{cases}$$

**[20]****[TURN OVER]**

**QUESTION 4**

Write a complete LaTeX document that produces the following

- 1 We define the metric connection  $C_{ijk}$  ( $1 \leq i, j, k \leq 4$ ) by

$$C_{ijk} = \frac{1}{2} \sum_{\ell=1}^4 h_{i\ell} \left( \frac{\partial \gamma_{k\ell}}{\partial x_j} + \frac{\partial \gamma_{j\ell}}{\partial x_k} - \frac{\partial \gamma_{jk}}{\partial x_\ell} \right)$$

- 2 The metric is

$$\gamma_{11} = \frac{1}{1 - \frac{2m}{r}}, \quad (1)$$

$$\gamma_{22} = r^2, \quad (2)$$

$$\gamma_{33} = r^2 \sin^2 \theta, \quad (3)$$

$$\gamma_{44} = - \left( 1 - \frac{2m}{r} \right) \quad (4)$$

Substitute (2), (3) and (4) into (1)

[20]

**QUESTION 5**

The order  $p$  of a root  $\alpha$  of a function  $f$  is the order of the smallest non-vanishing derivative at the point  $\alpha$ . For example if  $f(x) = x^2$ , then the order of the root at  $x = 0$  is 2. Write MuPAD code to find the order of the root at  $x = 0$  of

$$f(x) = \exp(\sin x) - \sin(e^x - 1) - 1$$

[20]

[TOTAL: 100]