UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM2616

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COMPUTER ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS:
FIRST: DR JMW MUNGANGA
SECOND: DR R MARITZ

This paper consists of 3 pages

Answer all the questions

QUESTION 1

We define the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ by

$$f(x) = \begin{cases} 3x+1 & \text{for } x \text{ odd,} \\ \dfrac{x}{2} & \text{for } x \text{ even} \end{cases}$$

Write a program which on input x_0 returns the smallest index i with $x_i = 1$ **[20]**

QUESTION 2

Compute the general solution of the system of linear equations

$$\begin{cases} a+b+c+d+e=1 \\ a+2b+3c4d+5e=2 \\ a-2b-3c-4d-5e=2 \\ a-b-c-d-e=3 \end{cases}$$

How many free parameters does the solution have? **[20]**

QUESTION 3

Determine the solution $y(x)$ for each of the following initial value problems

(a)

$$y' - y\sin x = 0, \quad y'(1) = 1$$

(10)

[TURN OVER]

(b)

$$2y' + \frac{y}{x} = 0, \qquad y'(1) = \pi.$$

(10)

[20]

QUESTION 4

Let rd denote the rounding function mapping a real point x to the nearest integer Plot the function

$$f(x) = \frac{|x - rd(x)|}{x}$$

on the interval $[1, 30]$ **[20]**

QUESTION 5

Write a complete LaTeX script that reproduces the document printed on page 3

Please take note that manual references and manual labelling of equations, pages and citations will not be allowed.

Suspension of Flexible Fibres In a Newtonian Fluid

JM MUNGANGA

May 26, 2010

1 The dissipation Inequality

1.1 Free Energy

If the free energy is denoted by Ψ, then the local form of the second law is the dissipation inequality[1]

$$\rho\Psi \leq \mathbf{T} \quad \mathbf{D} \tag{1.1}$$

1.2 Motion of Fluid

The motion of the fluid is described by the function $\mathbf{x} = \varphi(\mathbf{X}, t)$, in which $\mathbf{x}$ and $\mathbf{X}$ denote reference and current position, respectively in the fluid We assume that $\varphi(\mathbf{X}, t)$ is smoothly invertible in its arguments, and define the deformation gradient $\mathbf{F}$ by

$$\mathbf{F} = \nabla_{\mathbf{X}}\varphi \tag{1.2}$$

If the inverse $\mathbf{F}^{-1}$ of $\mathbf{F}$ exists, then

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} \tag{1.3}$$

For any sufficiently smooth functions $\mathbf{F}$ and $\mathbf{A}$,

$$\Psi(\mathbf{F}, \mathbf{A}) = \Psi_F \cdot \mathbf{F} + \Psi_A : \mathbf{A} \tag{1.4}$$

Here Ψ_F and Ψ_A are the derivatives of Ψ with respect to $\mathbf{F}$ and $\mathbf{A}$

Substitution of (1.1), (1.2) and (1.3), into (1.4) gives

References

[1] Coleman BD and Noll W, The thermodynamics of elastic materials with heat conduction and viscosity. *Archive for Rational Mechanics and Analysis*, **13** (1963) 167-178.

[20]

TOTAL: [100]