

APM2616

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COMPUTER ALGEBRA (APPLIED MATHEMATICS)

Duration 2 Hours

100 Marks

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This paper consists of 2 pages

Answer all the questions

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QUESTION 1

MuPAD cannot solve directly the differential equation

$$\begin{cases} \frac{dy}{dx} = y(x) \cos x^2 - \sin x^3 \\ y(0) = 2 \end{cases}$$

Write MuPAD code to find a series solution of the form

$$y(x) = 2 + y_1x + y_2x^2 + \dots + y_9x^9 + O(x^{10})$$

for the differential equation

Hint. Express $\cos(x^2)$ and $\sin(x^3)$ as series, and in the differential equation equate coefficients of x^0, x^1, \dots, x^8

[20]**QUESTION 2**

Write MuPAD code to create a set, then create new sets with

- (a) One member removed. (5)
- (b) One new member (5)
- (c) Union of the two sets above (5)

[15]**[TURN OVER]**

QUESTION 3

Implement a procedure Quadrature whose input is a function f (of one variable) and a list X of numerical values $x_0 < x_1 < \dots < x_n$. The call Quadrature (f, X) should compute a numerical approximation of the integral

$$\int_{x_0}^{x_n} f(x) dx$$

by means of the formula

$$\sum_{i=0}^{n-1} (x_{i+1} - x_i) f(x_i)$$

[25]

QUESTION 4

Write MuPAD code to draw a graph that plots the function $f(t) = e^{\sin t}$ and $g(t) = \frac{t^2}{1+t^2}$ for $t \in [-1, 5]$. The axes should be appropriately labelled, and the scale in the vertical and horizontal directions should be the same. The graph of f is blue and that of g is green [20]

QUESTION 5

Write LaTeX code, in the form of a complete document, for the following:

1. In what follows, Ω is a bounded domain of \mathbb{R}^3 with boundary Γ . We define the following

$$\mathbf{X} = \left\{ \varphi \in \mathbf{H}^1(\Omega) \mid \varphi|_{\Gamma} = 0 \right\}$$

Poincaré inequality

$$\|\varphi\| \leq C_{\Omega} \|\nabla \varphi\|, \quad (1)$$

holds for $\varphi \in \mathbf{X}$

2. Let

$$\phi(x) = \left[\sum_{n=1}^{\infty} \frac{\partial^n \varepsilon}{\partial x_n^n} \frac{1}{\sqrt{n}} \phi^{(n)}(x) \right]^{\frac{1}{n}} \quad (2)$$

Show that ε and ϕ are well defined for $x > 0$, in (1) and (2) [20]

TOTAL: [100]