

APM2616

October/November 2011

COMPUTER ALGEBRA

Duration 2 Hours

100 Marks

 EXAMINERS
 FIRST
 SECOND

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This paper consists of 3 pages.

THE USE OF POCKET CALCULATOR IS NOT PERMITTED.

Answer all the questions

There is a total of 100 marks. 100 marks will count as full marks

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QUESTION 1

Given that the following have been defined in a MuPAD session.

n positive integer

x array of n identifiers

g $n \times n$ matrix

write a MuPAD procedure called `mygam` that takes the above as input and outputs $n \times n \times n$ array defined by

$$C_{abc} = \frac{1}{2} \sum_{i=1}^n h_{ci} \left(\frac{\partial g_{ai}}{\partial x_b} + \frac{\partial g_{bi}}{\partial x_a} - \frac{\partial g_{ab}}{\partial x_i} \right)$$

where h is the matrix inverse of g .

[20]**QUESTION 2**

Write a MuPAD procedure, called `mydiff(f, x)`, that does not use the operator `D` or `diff`. The input parameters are f , which is a polynomial in x , which is an identifier. The output should be the first derivative of f with respect to x [Note that `degree(p, x)` returns the highest power of x in the polynomial p .]

[20]**[TURN OVER]**

QUESTION 3

Write LaTeX code, in the form of a complete document, for the following.

- 1 In what follows, Ω is a bounded domain of \mathbb{R}^3 with boundary Γ . We define the following

$$\mathbf{X} = \{\varphi \in \mathbf{H}^1(\Omega) \mid \varphi|_{\Gamma} = 0\}.$$

Poincaré inequality

$$\|\varphi\| \leq C_{\Omega} \|\nabla \varphi\|, \quad (1)$$

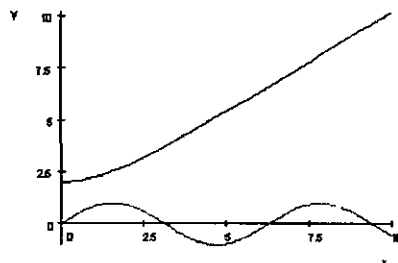
holds for $\varphi \in \mathbf{X}$.

- 2 Let

$$\phi(x) = \left[\sum_{n=1}^{\infty} \frac{\partial^n \varepsilon}{\partial x_n^n} \frac{1}{\sqrt{n}} \phi^{(n)}(x) \right]^{\frac{1}{n}} \quad (2)$$

Show that ε and ϕ are well defined for $x > 0$, in (1) and (2).

[20]

QUESTION 4

Write a MuPad code that produces the graph shown, containing plots of

(a) $y = \sin x$

(b) The solution of the differential equation

$$\frac{dy}{dx} = \frac{x}{y}, \quad \text{with } y(0) = 2.$$

[20]

[TURN OVER]

QUESTION 5

Let $f = (1 + a)^3 \sin x \cos x - e^{3x} \cos^2(2x)$. Use MuPad to find a representation of f in the form $f = a + bx + cx^2$ (a, b, c numerical constant) valid for $x \ll 1$. You may assume that $e^x = 1 + x + \frac{x^2}{2}$, $\cos x = 1 - \frac{x^2}{2}$, and $\sin x = x$ [20]

TOTAL: [100]