UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

APM2616

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COMPUTER ALGEBRA

Duration · 2 Hours

100 Marks

EXAMINERS :
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This paper consists of 2 pages

Answer all the questions

QUESTION 1

Given that the following have been defined in a MuPAD session

n positive integer

x array of n identifiers

g $n \times n$ matrix

write a MuPAD procedure called mygam that takes the above as input and outputs $n \times n \times n$ array defined by

$$C_{abc} = \frac{1}{2} \sum_{i=1}^{n} h_{ci} \left(\frac{\partial g_{ai}}{\partial x_b} + \frac{\partial g_{bi}}{\partial x_a} - \frac{\partial g_{ab}}{\partial x_i} \right)$$

where h is the matrix inverse of g [20]

QUESTION 2

The order p of a root x of a function f is the order of the smallest non–vanishing derivative at the point x For example, if $f(x) = x^2$, then the order of the root at $x = 0$ is 2 Write MuPAD code to find the order of the root at $x = 0$ of

$$f(x) = \exp(\sin x) - \sin(e^x - 1) - 1$$ [20]

QUESTION 3

Write LaTeX code, in the form of a complete document, for the following

1 In what follows, Ω is a bounded domain of $\mathbb{R}^3$ with boundary Γ We define the following

$$\mathbf{X} = \left\{ \varphi \in \mathbf{H}^1(\Omega) \quad \varphi_{/\Gamma} = 0 \right\}$$

Poincaré inequality

$$\|\varphi\| \leq C_\Omega \|\nabla \varphi\|, \tag{1}$$

holds for $\varphi \in \mathbf{X}$

2 Let

$$\phi(x) = \left[\sqrt{\sum_{n=1}^{\infty} \frac{\partial^n \varepsilon}{\partial x_n^n} \frac{1}{\sqrt{n}} \phi^{(n)}(x)} \right]^{\frac{1}{n}} \tag{2}$$

Show that ε and $\phi$are well defined for $x > 0$, in (1) and (2) **[20]**

QUESTION 4

Let rd denote the rounding function mapping a real point x to the nearest integer Write a MuPad program to plot the function

$$f(x) = \frac{|x - rd(x)|}{x}$$

on the interval $[1, 30]$ **[20]**

QUESTION 5

Let $f = (1+a)^3 \sin x \cos x - e^{3x} \cos^2(2x)$ Use MuPad to find a representation of f in the form $f = a + bx + cx^2$ (a, b, c numerical constant) valid for $x << 1$ You may assume that $e^x = 1 + x + \frac{x^2}{2}$, $\cos x = 1 - \frac{x^2}{2}$, and $\sin x = x$ **[20]**

TOTAL: [100]