UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

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COMPUTER ALGEBRA

Duration 2 Hours

100 Marks

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Closed book examination

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This paper consists of 3 pages

Answer all the questions

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QUESTION 1

MuPAD cannot solve directly the differential equation

$$\begin{cases} \frac{dy}{dx} = y(x)\cos x^2 - \sin x^3 \\ y(0) = 2 \end{cases}$$

Write MuPAD code to find a series solution of the form

$$y(x) = 2 + y_1 x + y_2 x^2 + \quad y_9 x^9 + O\left(x^{10}\right)$$

for the differential equation

Hint Express $\cos\left(x^2\right)$ and $\sin\left(x^3\right)$ as series, and in the differential equation equate coefficients of x^0, x^1, $, x^8$

[20 marks]

[TURN OVER]

QUESTION 2

Write MuPAD code to create a set, then create new sets with

(a) One member removed (5 marks)

(b) One new member (5 marks)

(c) Union of the two sets above (5 marks)

[15 marks]

QUESTION 3

Implement a procedure Quadrature whose input is a function f (of one variable) and a list X of numerical values $x_0 < x_1 < \quad < x_n$ The call Quadrature (f, X) should compute a numerical approximation of the integral

$$\int_{x_0}^{x_n} f(x)\,dx$$

by means of the formula

$$\sum_{i=0}^{n-1} (x_{i+1} - x_i)\, f(x_i)$$

[25 marks]

QUESTION 4

Write MuPAD code to draw a graph that plots the function $f(t) = e^{\sin t}$ and $g(t) = \frac{t^2}{1+t^2}$ for $t \in [-1, 5]$ The axes should be appropriately labelled, and the scale in the vertical and horizontal directions should be the same The graph of f is blue and that of g is green **[20 marks]**

QUESTION 5

Write LaTeX code, in the form of a complete document, for the following

1 In what follows, Ω is a bounded domain of $\mathbb{R}^3$ with boundary Γ We define the following

$$\mathbf{X} = \left\{ \varphi \in \mathbf{H}^1(\Omega) \quad \varphi_{/\Gamma} = 0 \right\}$$

Poincaré inequality

$$\|\varphi\| \leq C_\Omega \|\nabla\varphi\|, \tag{1}$$

holds for $\varphi \in \mathbf{X}$

[TURN OVER]

2 Let

$$\phi(x) = \left[\sqrt{\sum_{n=1}^{\infty} \frac{\partial^n \varepsilon}{\partial x_n^n} \frac{1}{\sqrt{n}} \phi^{(n)}(x)}\right]^{\frac{1}{n}} \tag{2}$$

Show that ε and φ are well defined for $x > 0$, in (1) and (2) **[20 marks]**

TOTAL: [100 marks]